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CURRENT APPROACHES TO DECISION MAKING IN COMPLEX SYSTEMS: III

VOLUME II, CONFERENCE POSITION PAPERS

THIRD CONFERENCE, RICHMOND, SURREY, ENGLAND
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The Third Richmond Conference centered on foundational problems in decision-making. The conference is reported in two volumes. Volume I (ARI Research Note 80-10) contains an overview of each of the major papers presented, with the author's commentary and the discussion by conference participants, plus several general discussions. Volume II provides the major papers themselves: Observable Components of the Decision Process and a Revised Theoretical Posiion, by Gordon Pask; Decision Making as an Event-Search: Traffic on a Multi-dimensional Structure, by R. H. Atkin; Decision: Foundation and Practice, by

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Brian R. Gaines; Competing Modes of Cognition and Communication in Simulated and Self-Reflective Systems, by Stein Braten; On the Spontaneous Emergence of Decision Making Constraints in Communicating Hierarchical Systems, by John S. Nicoiis; and also a paper by Maria Nowakowska, on a new model of decision under risk.

CURRENT SCIENTIFIC APPROACHES TO DECISION MAKING IN COMPLEX SYSTEMS: III
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CONTENTS

VOLUME 1

OBSERVABLE COMPONENTS OF THE DECISION PROCESS AND
A REVISED *THEORETICAL* POSITION

OBSERVABLE COMPONENTS OF THE DECISION PROCESS AND A REVISED THEORETICAL POSITION

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Third Richmond Conference on Decision Making in Complex Systems.

Observable components of the Decision Process and a Revised Theoretical Position

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contents continued ......

Preface

Perhaps the disquiet over "decision", which is a central theme of this paper, reflects a personal and completely unjustified aversion to games. Let it be known that I do like watching games, in attractive surroundings, and at a safe distance; for I can dress the players in rich garments. Poker is played on a Mississippi sternwheeler, horses are raced in the vicinity of the Royal Enclosure at Ascot, at Newmarket, also; roulette is for Monaco, or, in more flamboyant guise, Las Vegas; chess is redolent of pipe smoking Dons; I accept it, because Dodson took the psuedonym of Lewis Carroll. With chance it is the same, it conjures up the Prebendary of St Pauls, Whitworth, formerly fellow of All Souls, concocting his 1000 exercises (later published, as Choice and Chance (1901)). If you strip away these apparitions from a game, reduce it to the arid bones of an abstract tree of choice, or place it in normal form, as a rectangular array, the joy of it is gone.

Others, of a less incurably romantic disposition, do not feel so strongly on this score. Taking an impartial stance, even I can see that clarity is gained, and a great deal of mathematical elegance is achieved, by working outwards from this simple paradigm. All the same, there is some sense in my emotive reaction "I can't see why they bother to play a game like that, at all" (where "they" means the players or participants).

The fact is, Decision Theories which are rooted, with few exceptions, (Beer, or Ackoff, for example) exclusively in a game-like image of things do not really talk about the participants.
These theories either assume or assert some characteristics which the participants have (the descriptive variety of theory) or that they ought

to have (the normative variety). Under one banner or the other, theorists proceed apace to manufacture abstract superstructures, several of which count amongst the most beautiful scientific accomplishments of the last few decades. Most of these have been debated during the previous conferences, but some stand out as landmarks, even if we limit our attention to recent developments, since the publication of Von Neumann's and Morgenstern's Theory of Games and Economic Behaviour (1944, second edition 1947).

There are two classical papers by Ward Edwards (1954 , 1961) which introduce the community of behavioural and psychological scientists to the game theoretic, or decision theoretic, notions developed in the context of economics and sociology . The later review ends with a consideration of the paradoxical game "Prisoner's Dilema", which gave rise to much of Anatol Rapaport's brilliant work and to an ongoing debate involving his student and associate over the years, Nigel Howard (the latest papers appeared, quite recently, in Behavioural Science) . There is an early symposium (1952) under the aegis of the Ford Foundation, the military agencies, and the RAND Corporation, written up and edited by Thrall, Coombes, and Davies as Decision Processes (1954), and the experimental report by Davidson, Suppes and Seigel, Decision Making (1957) both of which treat issues as significant today as they were in those days (for example, the 1957 book ends with an account, in terms of preference chains, of choice between incomparable alternatives). Luce and Raiffa's classic, Games and Decisions (1957) is still about the most comprehensive and thoughtful overview of issues that are surprisingly up to date. For example, the authors comment upon one of the truly perspicuous (but little known) pieces of game theoretic enterprise, Braithwaite's inaugral lectures as Knightsbridge Professor, at

Cambridge, published as The Theory of Games as a Tool for the Moral Philosopher (1955).

Actually, what are people doing with this game and decision paradigm. Some state their intention clearly ; for example, the work of Swets and Tanner, or of Broadbent,(1971) in Decision and Stress (to cite only two instances), is directed towards explaining mental data processing as though it were some kind of signal in noise detection, a kind of noise perturbed decision. These are explicitly metatheories about the human operator qua object, and the usefulness of these theories (for example, of the Receiver operating curve as a predictive instrument, in human data processing), is well acknowledged . Here, and in like minded developments, there is no essential commitment to the effect that someone really is "playing" a game; it is just as though this were the case, it is our choice, as external observers # or experimenters, to view the situation in that way.

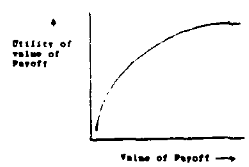
On the other hand, the majority of theories, both descriptive and normative theories of decision, are committed, like it or not, to a tacit (and since it is tacit, quite commonly forgotten) agreement, on the part of persons and teams, to act as players or decision makers**. The expedient of stripping a game situation down to the bare bone paradigm also carries with it the idea that choice amongst alternatives is, somehow, elementary or unitary; that it is normal and natural to "guess" (a somewhat mysterious operation except in the "20 Questions" Television show which is contrived to foster guessing, just as one armed bandits are made for this same purpose).

"Task specific" theories, such as Brunswick's "lense model" may, perhaps, count as intermediary cases but all of them can be construed in this manner.

** In particular, Decision Analysis carries this commitment very strongly, and here it is made explicit.

The assumptions (in a sense, the axioms) of the paradigm are as follows. There are players A, B,... (one may be nature in a "game against nature) who find themselves in states where they can choose between alternatives, modelled as exclusive and exhaustive sets of moved, which situation may occur once, or repeatedly, (in "iterative" games expanded as "tree structures" to select "move sequences" or "strategies"). The contingent or (in the limit of pure choice, by one person) the uncontingent, outcomes are known, or may become known (depending upon the conditions) or may be known probabilistically, so that expectations can be determined. The players or participants find it reasonable to make a deterministic or probabilistic choice amongst moves, strategies, or mixed probabilistic combinations (as in the Von Neumann and Morgenstern solutions). Regarding probabilities, they are subjective probabilities for the Bayesian, (although rationally modified by the data at successive trials). In contrast probabilities are in some sense objective for the frequentist.

The players or participants must entertain preferences or personal values over the possible outcomes and these preferences must obey certain rules (for example, transitive ordering) and may or may not (again depending upon the circumstances) correspond to a utility function ie. a graph like



Where "Payoff" may be money, or some other commodity (or a whole batch of different ones, mapped onto a common "evaluated" property). In order to compound these quantities, for calculating expectation, likelihood, and other derived figures, it is essential that <u>preferences</u> and <u>probabilities</u> are independent.

I appreciate that these axioms of the paradigm are relevant to the casino, and to some real life situations. I repeat my genuine and considerable admiration for the elegant formal structures that are built up around the paradigm and, insofar as the paradigm is realistic, salute their indubitable predictive power.# However, it seems to me that this paradigm (which has been sketched afresh, in order to accommodate greater liberality of interpretation than is usually allowed) is not often applicable; it does not seem that choice, in the game-like, or decision sense, is often a unitary act. If matters of consequence are involved, it is hazardous to suppose that choice might be an elementary, or unitary act.

Moreover, looking back over the literature and melding it with discussion at the previous conferences, many of the "mainstream" decision theorists seem to entertain a similar scepticism. A large amount of the work, both formal and empirical has been devoted to expansions, elaborations, or enrichments of the paradigm, intended to retrieve that raucous, rich fabric of reality for which the paradigm is one, and only one, abstraction.

Efforts are made to retain the axioms upon which the sophisticated mathematical development hinges. But, with that caveat, nearly everyone is end-

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The literature is replete with "mathematical elegance" as witnessed by a couple of arbitrarily selected papers. Both Nichols, A.L. "Coalitions and Learning" pp 391-403, and Gilette, R. "Collective Indecision" pp 383-391, are quite characteristic of this attitude, both of them in Behavioural Science, Vol 22, No 6. 1977. They are excellent papers, but it is difficult to see their psychological relevance and the experimental studies referred to in Nichols' paper are of the "contrived" type.

eavouring to build the games played by Homo Ludens (Huizinger 1949) and the games played by children and their seniors back into the roots and origins of thought. Being realists, they would like to retrieve the games and decisions epitomised in Flowers of Delight (De Vries, 1968, Dobson; a charming anthology of late 18th, early 19th century, nursery-literature where games come mixed, as they really are, with moral admonitions and the magic of the hydrogen balloon at Raneleigh Gardens). Or they would like to decide how to run a business, or a country; operations with similar complexity, and style.#

These entirely understandable endeavours, typical of the synthetic essays of reductionism, are bound to fail because the building blocks are inadequate. The act of abstraction that models the poker players on a showboat, or the Oxford Dons, within this paradigm, is optimal for a very special class of decision but it cannot be generally employed. Embedding the skeletal paradigm in elaborate taxonomies (decision under uncertainty, risky choice, bargaining, and so on), is liable to obfuscate the issues at stake.

Perhaps it is better to reserve the paradigm for those special situations where it does fit; few of them involving human decision

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Disillusionment with the current state of the art, as practiced in the decision process of operational research, is reflected in a recent paper by J D Tocher (1977), "Planning Systems ", Phil, Trans, R. Society London, A 287,p425-442. In the same volume De Jong JJ "Setting objectives in a business Enterprise, a Cybernetic Approach", 493-507 and Ozbekhan H, "The future of Paris", p523-544 show a trend towards a participant and system approach (very different approaches, incidentally) Again, in the same volume, Lefkowitz I "Integrated Control of an industrial system" pp443-465, and White G W T and Simmons M D "Analysis of Complex Systems" pp405-423, vere towards the distributed regulation paradigm recommended in Section 3 of this paper

as such ; after that to start afresh by examining the reality of command and control, or management, or the games that are played, if not in earnest, then at least for pleasure. By "reality", I mean both intellectual (or psychological) reality, as well as "concrete reality"; in fact, _a fortiori_, the intellectual reality of the participants (not just the reality of an unbiassed external observer). The synthetic converse; building up structures and measures from inadequate analytic reduction statements, axioms, units, or whatever, is, in a sense, shown to be absurd by the conundrums which plague the more philosophically astute mathematicians. The point is made by Martin, in his very difficult, but very wise, book, _Intension and Decision_ (1963).

The main opposition to this proposal is likely to come from the experimentalist, stripped of what seems to be a neat and tidy experimental situation already equipped with formal tools. But, as soon as experimenters move from picayune choice tasks to relatively complex tasks they, too, must admit, by token of the results obtained when the system is complex, that the neat and tidy paradigm is a sham.

One cannot write so critically, if nothing is suggested in place of the paradigm denied tenure. So, reserving the elegant mathematics for where it _does_ apply, this paper provides a rather different kind of experimental situation (which exteriorises many aspects of conceptual activity as stretches of behaviour). It is not ideal, but it _does_ stand a chance of furnishing data about realistically complex decision.

For example,

(a) It is _not_ assumed that alternatives are _given_ (though they may be, in special cases). Usually, actions _become_ alternative-like only where processes are set in motion at an ongoing cost.

(b) It is <u>not</u> assumed that decision-situations are tree-like (though they may be, in special cases). It is quite possible that processes set in motion (generally several at once) may (or may not) interact ; they may or may not be autonomous (if not, they can be monitored by feedback, and modified).

(c) On the whole, we are more intimately concerned with subjective probabilities (degrees of belief, and the degree and content of doubt) than frequency probabilities , though frequencies are one amongst several useful indicators of the aleatory component in a situation. Inference from one situation to another, and reasoning based upon frequency of occurrence are valuable insofar as one situation is comparable to another (an analogy of <u>form</u>) *or may be dealt with* in a similar manner (an analogy of <u>method</u>),

(d) Analogies, either recognised or abduced (invented) are the basis for most (all) decision making. An analogy is more than a *similitude*. *It is* a (more or less specific, but well determined) <u>similarity</u> taken <u>together with</u> at least one <u>distinction</u> (there may be an indefinite number of valid distinctions).

(e) In respect to subjective probability, the proposed methodology is in accord with Phillips (1976), or Raiffa (1970). ≠ But the lack of a tree-like structure leads to obvious mathematical difficulties and these, in turn, call for a differently based theory of how beliefs, and the tangible evidence supporting them, are compounded. At least one candidate theory (not my own) is proposed as operationally sound.

≠ A propos tree structures; see Raiffa, Chapter 9, Section 2 (when does a tree become a bushy mess" and the concluding line " - a bit of art can be added to science". Some other possibilities are proposed; as to whether they constitute art or science I do not know but suspect they are both.

(f) The experimental situation exteriorises definitely <u>subjective</u> but sharp, or exact, events; these, and fuzzy quantities surrounding them, may be (objectively, or impartially) scrutinised by an external observer.

(g) The notion of value or worth is assigned to activities or process preferred by the participating decision makers. It follows that value is considered as a reflective (participant sensed) quantity, which is one aspect of an agreement (unlike, for example, length or velocity). It is possible for an external observer to act as a participant and, under this circumstance, preference, value, etc, may be quantified but in a relativistic manner, only.*

(h) One, possibly perplexing, feature of the experimental paradigm, or methodology, is that a decision process is seen systemically as implicating the participants, their task environment, and their exteriorised representation of the task environment. Under these conditions, the participants should <u>never</u> guess (of course, they do entertain and occasionally exteriorise imperfect or incomplete descriptions and intentions, and act upon them; that is not guessing, ie. imitating some external chance device). It is also true that the familiar chance devices (dice and the like) are usually amongst the worst oracles which may be consulted in order to resolve indeterminancy.

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*Social tokens, such as money, have a commonly <u>agreed</u> value, shared by an external observer (acting as a member of that society). Here the agreement may be quite <u>general</u>; but it is still true that "the worth of money" is of a different kind to the "weight of some pound (or dollar) notes".

The experimental (or operational) methods are attached to a theory; to some extent, the methods grew out of the theory. Regarded as a theory of learning, conceptualisation and thinking, this theory (Conversation Theory) has enough empirical support to warrant a fairly confident manner of exposition. It is a quantifiable but relativistic and reflective theory of the <u>participants</u> (hence, the comments on subjectivity made in clause (d) above).

With respect to decision making, the theory is embryonic; it is open to amendment and in its amended form, is in need of very considerable development.

In conclusion, I would like to thank colleagues, too numerous to mention, at System Research, Brunel and the Open University, and in other places. As a particular acknowledgment about half of the material in Section 4 is extracted from Progress Reports No 3 and No 4 jointly authored by Robin McKinnon Wood, Dave Ensor, Nick Green and myself and the Decision System which is described has been developed under the sponsorship of the Army Research Institute for the Behavioural and Social Sciences, Grant DAERO 76-G-069. The THOUGHTSTICKER system, which is conjoined with it, in the "planning sessions", has been developed jointly under the sponsorship of this grant, our SSRC Research Programme HR 2708/2; "Learning Styles, Educational Strategies and Representations of Knowledge: Methods and Applications", and AFOSR Grant 78-3520, "The Influence of Learning Strategies and Performance Strategies Upon Engineering Design".

1 Introduction

This paper is an exposition of two, closely related, lines of argument. One theme sets out (and describes a realisation of) a means for exteriorising decision on the part of individuals, or teams; it is argued that such a realisation is minimal, at least in the context of command and control, management, or complex man-machine systems. The other theme is a discussion, as yet incomplete, of what decision *is*.

Both of the two lines of argument depart, more or less radically, from the empirical and theoretical paradigms of decision theory.

On the empirical side, it is maintained that the choice behaviour observed in the majority of psychological and decision oriented studies is insufficient to give an adequate picture of an underlying decision process; that, in order to obtain such a picture, it is necessary to exteriorise normally hidden conceptual events as stretches of complex and many-faceted symbolic behaviour (ie. the use of a language, although usually a non verbal language.)

On the theoretical side, we question the orthodox idea of choice amongst a set, or series, of *given* alternatives over which there are realistically distinguishable probabilistic and preferential choice-criteria; at any rate, the orthodox paradigm, even if elaborated by inferential aids, such as computer programs able to carry out Bayesian inference or by the employment of multiattribute utility scales, is of local, rather than general, relevanc

In developing *both* themes emphas.. will be placed upon the notion of decision as a *process*, akin to problem formulation, problem solution, or thinking which may be distributed over the individuals in a team, or the components of a man-machine system.

1.1. Agreement to Participate, Goals and Representations

"Decision " surely implies an agreement, by a decision maker (individual, team, or man-machine-system) to participate in an activity (consisting in decision, or culminating in decision), which takes place in a situation such as military command and control, or management. Both the activity and the situation are circumscribed by a usually incomplete description of conditions it is desired and agreed to achieve or maintain. Occasionally, the description is neat enough to constitute a "goal", like maximising "payoff", or the mean value of payoff. More often, the term "goal" is misleading, insofar as it suggests a single mindedness and degree of specificity that seldom exists, in practice.

Consider, for example, the description of a mission to protect trade routes between several island-countries, with specific means of transportation (sea, aeroplane, rail across a causeway, and so on). The routes need protection because the trade is disturbed by marauders which, maliciously or not, disrupt traffic. The mission is to be accomplished by manoeuvring vehicles which can intercept marauders that endanger the trade. The vehicles, though they may communicate and cooperate, have limited resources (for example, fuel is made available at a certain, possibly variable, rate). The decision maker (one or more individuals) agree to undertake the mission by acting as commander(s) of (groups of) vehicle(s).

The mission, as described, has no fixed duration (though it is quite realistic to suppose, as part of the agreement, that any individual has a fixed term of service). Nor is it clear that the mission is really one mission. Responsibility may be, and may have to be divided between commanders with their own contributory missions.

Nor is the description itself really fixed. Even if roles are assigned to each participant in a strict hierarchical structure, the norms and duties are interpreted idiosyncratically. Even if geographical boundaries are delineated by a map, with a commonly agreed interpretation, the geography is individually perceived. For example, different trade routes are perceived to have, and consequently do have, varying significance. Distances on the map are not the same as perceived distances, measured by the yardsticks of danger, difficulty and the kudos attached to engaging in a particular expedition. Insofar as there are strange, partially known, or apparently haphazard, events, any description that properly accommodates these events is bound to change, and, generally, to change quite differently for different participants.

Finally, the set of possible activities, bounded by the number and control characteristics of the vehicles, and fuel available to the vehicles, are augmented by personal (if you prefer the word, "superstitions") features that add to, though do not contravene, the operating manual description. To phrase it otherwise, if captains did not personalise their craft, if commanders did not ("superstitiously" as well as factually) personalise their units, if stockbrokers did not personalise their ploys, then they would not be decision makers, except in the trivial, and irrelevant, sense that an autopilot may be said to decide. It follows that the customery demarcation between descriptions and prescriptions is hazy. Any prescription for what can be done involves a description of what may be done and, vice versa, descriptions may be used, in a process, as a prescriptive formula. Where processes are involved, it is safer to speak of a representation sometimes used descriptively and sometimes prescriptively.

1.2. Some Consequences of Statements about Decision

With these dogmatic statements, I am tacitly characterising "decision" (as a process distinct from the activity of an autopilot or programmed regulator) and do not anticipate too many quibbles on this score.

The view is fairly uncontentious if "decision" is qualified as "good decision" or "effective decision" If an autopilot, for example, could manoeuvre the vehicles to protect the trade routes, effectively, then surely this, or some other device, would be assigned to the job.

It is undoubtedly true that certain aspects of the job should be, and are, relegated to control mechanisms; all credit to the clever people who design the contraptions, and make decisions in the process of design. It is also true that almost any decision process, viewed grossly, from outside, will <u>resemble</u>, or <u>look like</u>, the operation of a regulator or controller. But this does not mean that the decision process <u>is</u> such an operation and I strongly maintain it is <u>not</u> (though maintaining, also, that during most complex decision processes, the decision makers will decide to employ regulators which do not, themselves, decide, in order to achieve, or to aim for, stipulated ends).

It is worth re-emphasising the generally dynamic character of decision making. Clearly, the "mission" is satisfied by an activity; by intentions that are actualised, or reified. This characterisation is believed to be universally applicable. It is something of an accident that decision theory, in its "traditional" form, emerged from contrived situations that are designed to be truncated and to yield static outcomes (like dice throwing, or choosing between alternative lotteries). These are very special cases though it is mathematically appealing to image the normally active form by a static picture (as Howard, in his elegant metagame theory, images the interplay of the hypotheses and anticipations of participants by an hierarchy of metagames). In command and control, or in management systems, it is impracticable to rely entirely upon these static images. Quite generally, whatever the situation, decision is a process.

1.3. Doubt

One critical property of a decision process, which distinguishes it from any other conceptual process, is that the conduct of a decision process leads to an awareness, a doubt, and the resolution of doubt. All the underlined words (awareness, doubt and resolution) are used in a technical sense, to be explicated in the sequel. For example doubt, is many faceted and, as a rule, its resolution changes the kind of doubt experienced . If all the several kinds of doubt are considered, then the aggregate doubt is a degree of awareness. In particular, we are unaware of automatic or robotic processes.

Surely, we can be aware that we have acted automatically; that such and such a behaviour is automatic and yields results . We can be aware of deciding to act automatically, and may be more or less doubtful of the outcome. But the process responsible for the automatic behaviour does not, itself, product doubt. We are, literally, unaware of it. This is true, a fortiori, for well learned skills, or well learned intellectual skills which count as concepts. We are not aware of walking, or of adding numbers, unless, of course, a mistake is made and is noticed or pointed out.

1.4. Independence, Coherence (or agreement) and distinction (or predication).

Another characterisation of the decision process is brought out by the following question. "How is a decision picked out from the flux of an ongoing decision process?".

Some reorientation is needed to answer the question and I (at least) find it necessary to put aside a number of deeply ingrained presuppositions, such as the idea that sets of "exclusive and exhaustive alternatives" are given or that trials (as in throws of a dice) are independent unless otherwise stated.

Consider, instead, two or more a priori independent systems; intuitively, the most familiar example is two or more independent people, A, B.

Let them be in doubt about something, T. and let their doubt be resolved . Stated in mechanistic terms, suppose there are independent processes in A's brain and B's brain, focussed upon T, which initially cannot be jointly executed ie. they are incoherent processes. By dint of transactions between A and B, let coherent execution of A's T-relevant process and B's T-relevant process be rendered coherent. This is an A, B, agreement in respect of T and, to this degree A and B (the a priori independent systems) become dependent. The information transfer required to effect coherence is what A and B experience as awareness (its degree as a doubt, its content as the processes shared in the agreement) . However, A and B remain distinct, in respect of T; literally, they have (or are) different perspectives with respect to T. In order that this shall be so the T agreement (or, equisignificantly, the T coherence or T dependency) between A and B must be supported by a predication or the creation of a distinction which is computed by A and B. It may also be called a decision in respect of T by A and B; such units of agreement, dependency, or coherence supported by distinctions are the units that can be winkled out of a decision process as particular decisions.

A representation of this decision process contains static symbols that designate actively computed distinctions; for example, of T itself. This, generally, is the only way to dissect particular decisions from an ongoing process such as command and control or management. We shall retain the image of A and B reaching agreements supported by distinctions, throughout the paper, though the characterisation of A and B will be liberalised to encompass, for example, different teams or different points of view entertained simultaneously, by one person. At this point, also, we insist, once and for all, that the decision process implicates not only A and B (however characterised) but their task environment (A and B are not usually solipsists).

The A, B, paradigm is useful, since it allows me to bring notions from Conversation Theory (the author and others) to bear upon decision making. It is also, so far as I can see, the minimal situation to examine.

This view is in sharp contrast to classical decision theory and comparable disciplines that identify decision as selection and suggest that a decision process is a very complex arrangement of selections, performed by a complex machine. According to the present (conversation theoretic, agreement - maintained - by distinction) approach the entrenched suppositions of classical decision theory lead to a description of events from which, if taken seriously, all vestiges of <u>decision</u> have evaporated.

1.5. The unity of Decision and other Mental Operations

So far as command and control situations are concerned, it is almost platitudinous to comment that decision making involves learning. For example, the representation is learned . In general decision processes are inextricably merged with other conceptual operations such as learning , thinking, problem formulation and problem solving,which also give rise to an awareness and resolution of doubt. To this extent, the previous demarcation of decision process is convenient but arbitrary. In the sequel, it will be assumed that these other mental operations are invariably implicated,though attention is focussed upon participants who have agreed to take part in, and from time to time reach agreement about, a mission centred upon a given task environment, open or not.

2. The Character and Quantification of a Decision Process

This section and the next are concerned with an overview of decision by individuals and teams. Decision has various components such as planning and implicitly giving a description of the task environment, taking points of view,or perspectives, selecting methods (as a limiting case, a plan seen to be reasonable from a particular point of view),and anticipating outcomes.

Specifically, we consider various kinds of stable agreement, in the context of a man-machine-system able to accommodate a "mission" such as the "mission" of Section 1.1. ; agreements over plans and descriptions, the role of consciousness (and ,to some extent, what consciousness _is_), the notion of stability applicable under such circumstances, the genesis and resolution of doubt about methods and outcomes, the influence of conceptual style upon the decision process (or, at any rate, _how_ style is theoretically related to a decision process within the present framework).

2.1. Summary and Development

It has been argued that:

(a) Decision is a _process_, engendering awareness of doubt and its resolution.

(b) Decision makers _agree_ to participate in the process. Decision makers also agree, from time to time, about plans, actions, methods, and the like. The _logic_ of _agreement_ is a coherence logic and may be regarded as an extension of Rescher's coherence logic. Coherence and veridiciality (factuality) are _compatible_, but _not_ identical.

(c) The agreements reached in a decision process are supported by distinctions which can be used to tag "particular", or "unitary", decisions and are inscribed in a representation of the process.

(d) The activity and the situation in which decision makers agree to participate are both represented in an often idiosyncractic manner, over and above certain commonly interpreted features.

It is difficult, if not impossible, to demarcate parts of this representation, as descriptive and other parts as prescriptive ; rather, the representation, which is the focus of the initial agreement to participate, may be used descriptively and used prescriptively.

(e) Certain actions, that, superficially, resemble decision are not (except trivially) deemed to be a decision process; notably, the operation of an autopilot. Clearly, human decision makers are not always bound to decide. The human decision maker can, for example, act like an automatic and autonomous regulator but, in doing so, this person is not deciding.

(f) The decision process is a component of mental activity and is invariably associated (at least in complex command and control systems) with thinking, problem formulation and problem solving.

Certain skills, relevant to the task environment both are and must be overlearned, as in (e). The well known positive correlation between decision performance and task specific performance is neither surprising nor accidental, but simply necessary.

(g) These contentions are not, generally, compatible with standard, decision theoretic, paradigms; thus, for example, it is usually impracticable (maybe impossible) to estimate probabilities and preferences independently, in the course of decision. Further, the relevant alternatives frequently emerge from the representation of a decision domain, without prejudice to the existence of concrete alternatives (for example, that a vehicle cannot simultaneously move left and right).

(h) It is wise to remain uncommitted on the score of decision size. A commander or an industrial magnate seems to make large and important decisions, whereas the helmsman, at the wheel of a yacht, is engaging in a relatively unintellectual activity. Supposing that all these people really are "deciding", within the present "terms of reference". we cannot, without further data, determine whether one or the other is doing "more decision". There is no real justification for the view that the commander and the magnate "decide more"; because they have a more abstract and formal background, though the possesion of intellectual skills is an interesting fact in its own right and one determiner of conceptual style; a quantity which may be estimated quite accurately and reliably. We may, also, estimate the responsibility taken by an individual or a team, in the context of a task or a mission (like the mission in Section 1.1.) and relative to other individuals or teams who might have acted differently (given the same opportunity, as more or less responsible decision makers). This matter is taken up in Section 5 of the paper.

2.2. Exteriorising the Decision Process for Observation

In Section 2.1.(a) Decision was defined as a process engendering the awareness of doubt and the resolution of this doubt (to produce a belief or agreement supported by one or more distinctions). The definition is entirely subjective (as the decision maker is identified as a "you" or an "I" or a "they" or a "we" but not as an "it" or a "that". To hark after a genuinely objective (meaning "it referenced") decision process would, in the present framework, be doomed to failure. Regulators, such as autopilots, are properly "it referenced" but, just because of that, they do not decide.

The underlying philosophy deserves attention. That something or other is subjective does not imply that it cannot be delineated or quantified; nor, barring a peculiarly blinkered vision of science, does the subjectivity of something or other place it beyond the compass of scientific enquiry. However, as it stands, the awareness, the doubt and the belief attending a decision process are esoteric, and it is natural enough to seek for an other-than-private way of discussing the decision making of an individual or team.

A clue as to what is required is given by Clause (b) of Section 2.1, that decision makers agree to participate in a situation and an activity. The clue leads to the consideration of agreements over areas of doubt (or belief), supported by predications or distinctions. These agreements most certainly include agreements over a situation, an activity, and its representation (either used descriptively, or prescriptively). But the agreements involved in the decision process are more numerous. Insofar as the representation is used prescriptively, there must be an agreement over means (or methods); insofar as the representation is used descriptively, an agreement over goals or conditions to-be-maintained. That is not all. If a specific method is intended (ie. there is an agreement to adopt a specific method), then there is an agreement over a plan (over how to do something). If a condition is specified, there is agreement that this condition should be maintained by any method; given a method or not, a doubt to be resolved regarding the conditions likely to prevail; given a method, a doubt to be resolved about whether or not the method will achieve the desired condition. There are agreements over roles, within the representation that is agreed, and over perspectives, or points of view, with respect to the agreed representation. Finally, there are agreements to change the representation, and, if it happens (due, as later, to a singularity or bifurcation) that the representation must be changed, then there are agreements about how to change it.

To exteriorise awareness, the doubt and the resolution of doubt, leading to a system of beliefs, we shall arrange for a dialogue, or conversation (as in Section 1.4) between two or more participants A, B.. who are in a position to reach agreements in an appropriate language, L. The participants are a priori independent (equisignificantly, they are a priori asynchronous), entities who become partially dependent (equisignificantly, locally coupled, locally synchronous), when they do reach agreement. For example, A and B may designate two or more members of a team or, with equal cogency, A and B may designate teams (or subgroups or roles).

2.3. Observable Awareness and Consciousness

Using the word "consciousness" in the careful manner advocated by McCulloch (that participant A is conscious with participant B of something which they call T) the private "awareness" of doubt and its resolution is converted, by this expedient, into A,B, consciousness of some (possibly complex) entity, T, whenever A,B reach an agreement in language, L (ie. an L agreement) over T. Thus, consciousness has a degree or quantitive index which is the A,B, doubt; it has a content which is the set of coherent beliefs entertained by A and B; that is, what they agree about and denote "T". If attention is restricted to stable and coherent beliefs, then the logic of agreement becomes an extended logic of coherence and stable agreements can be recognised by a standard method of observation, ie. such stable agreements and the decision process in which they are reached are "it referenced", and thus objective, observables.

Stable agreements (in contrast to ephemeral or evanescent indications of accord) can be secured by defining the notion of stability and providing rules of L usage that are bound to satisfy this definition. The standard condition for observing a stable agreement is shown in Fig 1 (later, what stability is), where the

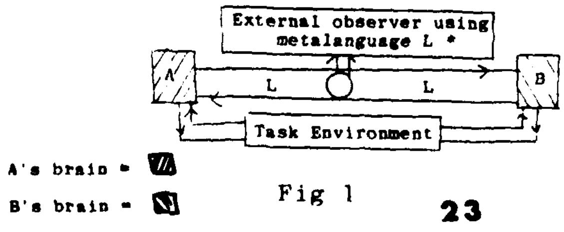

Fig 1

participants (here, A and B) engage in an L conversation regarding a task such as the mission of Section 1.1 and an external observer, using a metalanguage, L * over L as his means of expression, detects stable agreements (occurring as "I referenced" or "You referenced" statements in L). Within L * these agreements are "it referenced" statements with a veridicial (or factual) truth value rather than a coherence (or agreement) truth value assigned to the agreement by A and B in their language, L, and in contact with the task environment.

The arrangement is refined with great practical benefits by requiring that A and B converse, through a computer regulated interface consisting of a <u>representation</u> of the task environment, using L, but constrained by part of the interface to the rules of L usage.

This arrangement is shown in Fig. 2 and the interface contains both the representation and the equipment used to ensure logical L usage. Notably, the computer regulated interface can include various control systems, decision aids, records of useful information and that the representation may represent not only the task environment, but also A's image of B and B's image of A; their mutually conceived roles in the decision processes. Further, the control mechanisms may be employed to act upon the task environment (in order to perform control operations prescribed by A and B) or (and this is a departure from tradition) as prescribed by the <u>representation</u> of the task environment, to which they, in turn, provide data.

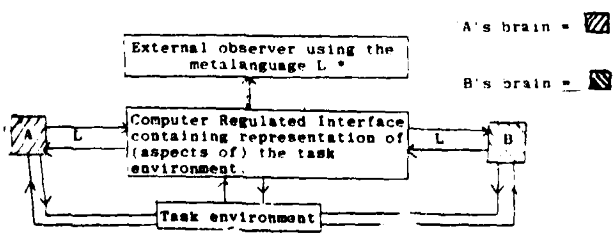

Fig 2

To implement the arrangement shown in Fig 2 it is convenient since all or part of L is a non verbal language, to use graphical and mechanically interpretable symbols. However, L must retain the richness of spoken language; it must be possible to personally address questions and commands in L, to give instructions, precise or incomplete, to offer descriptions, complete or incomplete, and to express metaphors that designate L analogies (ie. the semantic interpretation of an L metaphor contains, as any valid analogy, a similarity and a distinction).

Given the properties sketched in the last paragrpah, and spelled out in other publications, the representation of the task environment can be completely expressed in L, even though L is a non verbal language.

What is the representation of the task environment? Where does it come from?. The answers to these

I shall simply provide the answers that apply to a configuration in use at this laboratory, pointing out that there are other possible answers (our configuration is, however, one of the minimal configurations able to exteriorise a genuine decision process).

2.4. A Non Trivial Team Decision Making task

A concrete implementation of a system capable of carrying out the "mission" of Section 1.1. has been developed as a result of a 2 year project in which various less sophisticated arrangements were tried out and found inadequate, for one reason or another, to exteriorise the process of team decision making. A complete account is furnished in the Final Scientific Report of DAERO 76G069 and in subsequent reports of DAERO 79G0009 which are available as detailed accounts. The following comments provide an overview that highlights salient and peculiar features of the Team Decision System (TDS), as it exists. They certainly do not furnish a

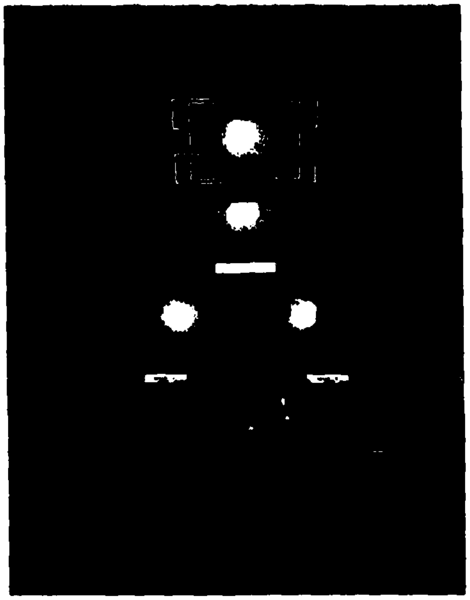

Plate I

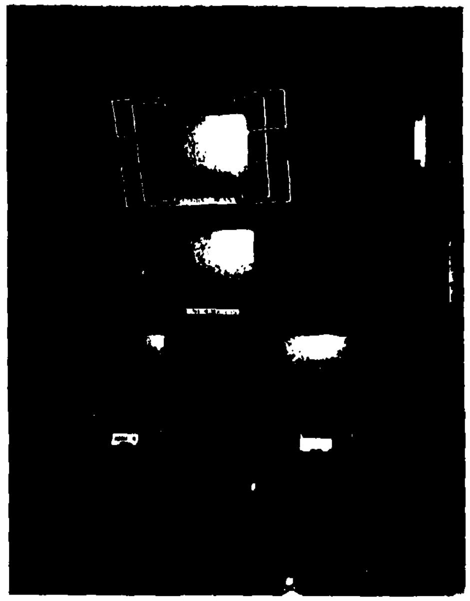

Plate II

Configuration of the Team Decision System complete description, if only because the system is being continually updated as research proceeds. The existing configuration is shown in Plate I and Plate II.

The scenario involves an environment called "Space" which (initially) is a *torroidal surface*. Disposed about space are 4 starbases which have energetic economies maintained by trade. The trade is carried out by transporting resources or manufactured articles in barges that ply between the "Starbases" . Left on its own, the economy will thrive but there is, unfortunately, a source (possibly, haphazard, possible malicious, deterministic but too complex to learn fully) of marauding entities "Klingons", which have the property of interupting trading barges and pirating their contents; given this source the starbase economy would collapse unless something is done to eliminate "Klingons" when they are nuisanceful.

Other features of space are (a) the presence of fixed and innocuous "stars" the clusters of "stars" providing a grain or geography to space, and (b) the existence of annular, symmetric lines of weakness such that if for any reason, excess energy is dissipated in their neighbourhood, space is "cracked" (for example, the torus into cylinders, the cylinders into a plane, the plane into half plane etc). Navigation of barges and (as later) spacecraft is impeded by "cracks". in space and "space" is implemented on an LSI 2 with 32k core and discs; part of the system block outlined in Fig 3(i).

Because the economy is disturbed by Klingons, one or two commanders are hired (the subjects) and each is in command of two a-priori-independent spacecraft. In fact, each spacecraft is a Z80 microprocessor with 56k storage, so that each commander has two independent consoles for display and response by means of which the two spacecraft are controlled. This control activity gives direction and velocity to the spacecraft and permits the following operations; "Mining" (offensive Klingons) repairing "holes" or "cracks" in space, investing energy resources, and search for specific information. Upon initialisation, each spacecraft is provided with a constant amount of energy. But any spacecraft must be in continual motion (except for an occasional docking operation). All of "mining", motion and information search, (apart from a local scan showing space in the neighbourhood of a craft), involve the expenditure of energy. Fresh energy is obtained by docking at a starbase for a restricted interval and in their role of mercenaries, the commanders may invest spare energy in the economy of one or more of the starbases, as well as refuelling their craft.

Apart from the movement and information search the primary operation is Klingon elimination, by depositing "mines" in their vicinity. The energy of a "mine" is absorbed by any "Klingons" in its vicinity and the "energy dosage" is accumulated until a threshold value is reached and the "Klingon" is demolished. "Excess energy" (if any) impinges upon space. Hence it is easy to expend "mine" energy improvidently. If this occurs, at any point, a hole is made in space, and unless repaired, is a small impassible gap. If too much energy is dissipated, by "mines" laid in the neighbourhood of a line of weakness, then space is cracked along that line (Fig 3(ii))

Repair operations that patch up space is one mandatory type of cooperation. To repair a hole any two independent spacecraft may come together (and pay, in energy units, for the repair operation). To repair a crack at least one spacecraft of each commander must be brought together the two commanders must (jointly) pay for the repair.

The view from the spacecraft is strictly limited each commander sees, gratis, a limited "window" containing the objects in that vicinity with the spacecraft in the centre, and the windows may or may not overlap Fig 3(iii).

Global information can be purchased by the commanders, at energetic cost, but is delivered through a separate display (representing kinds of object in all of space (Fig 3(iv)).

Interaction with the environment is limited and the updating of the environment state and the spacecraft state is based on the partial information also available, gratis, to the commanders (current motion and action, the local scan is an image of space in the vicinity of any one spacecraft). Apart from this local synchronisation, at the parallel interface, the spacecraft are asynchronous and independent so that conflict may arise and conflict resolution is a real issue.

This is true, a fortiori, at the essential bifurcations, signified in this case, as cracks in space, when genuine and not arbitrary, decision is required. For apart from the limited interchange with the environment of space, it is the commanders who locally synchronise the activity of the spacecraft to render it overall coherent.

The rate of *Klingon intrusion* is a variable which can be given values that render the task non existent (rate is zero) or on the other hand, quite impossible rate (too high). Most of the experiments have been run under differing rate values, all of which have been chosen to render the task impossible under most circumstances. In practice, the task can be performed if, and only if, the spacecraft (two of them to each of the two commanders), are given tactics, ie. series of conditional instructions that allow the spacecraft an autonomy to act on their own.

For practical purposes, tactics are unlimited in length (actual maximum about 60 instructions and tests) and an unlimited number of them may be stored (in each spacecraft microprocessor) for access (a) by another tactic (b) automatically, as later, or (c) as though they were commands to move in a given direction, to mine, to repair etc. Finally, tactics are programs consisting in usually iterated "If ..then.. else" statements. These statements combine the potentialities of the craft, for movement, Klingon elimination, repair, information search ie. the original commands together with conditionals. Unless provision is made in the tactic for global inform ation ,the conditions tested apply to the craft window (the window displayed in the scan on each spacecraft display). In the interests of uniformity the commands "to move in a given direction", to "repair", or whatever are defined, also, as tactics.

Since "no movement" is impossible there is no circumstance under which "no tactic" exists. The space-craft once commanded, or better, given some tactics, will carry them out as an automonous unit.

(This, at least, is the existing situation; when this paper was written there was only a fixed menu of tactics, which was not too satisfactory).

Under conditions of Klingon intrusion that render the use of tactics (for most purposes, other than unconditional commands), mandatory, the commanders exter iorise much of their normally hidden conceptual operations in tactics of which some, at least, are used. The resulting behaviour, together with spacecraft positions and the condition of the environment, is logged (at the point when the spacecraft are interacting with their environment). Summary records, of energies, Klingon density, etc. are printed out as data (Appendix 2), although the current system records all relevant events as well as summaries.

Certainly, the tactic expedient does externalise large segments of normally private mental operations as recordable intentions or behaviours. But is that degree of exteriorisation enough to uncover decision making?

Not quite; for the commanders might ,in principle, simply leave their craft alone. In any case, it is desirable to have **reasons** for doing things on record together with why certain actions are takenor certain operations (the tactics stored in the microprocessor) are introduced.

To close this gap, a further role is introduced namely, a "supervisor" who, from time to time interrogates both of the commanders, through the central display and keyboard (this is the only use made of the keyboard; other responses are routed by the control boards, one to each spacecraft).

The "supervisor" is, in fact, a computer program run in the environment computer. It comes into operation (and the entire system is "frozen" whilst it is in control) when certain information conditions have been satisfied by the commanders and their environment; when they have both experienced and reacted to, enough events.

The design is as follows:

We know, a priori, certain syntactic forms of question to be asked in an interrogation about whatever has happened. for example, what , where, how , why, how likely, what is the size of, questions.

Further, question forms can be assigned to 8 slots in an interrogation session (a technically convenient but otherwise arbitrary number). Hence, a series of interrogation ~~sessions~~ sessions each of 8 questions, can be mapped out and a question form assigned to each slot in each session.

Thus:

	Session 1	Session 2	Session n
Slot 1	Where	Is there	
Slot 2	How often	Why	
Slot 3	Why	How often	
Slot 4	How likely	Where	
Slot 5	How many	How Many	
Slot 6	What	How likely	
Slot 7	When	How Likely	
Slot 8	Which	How large	

is a typical distribution of question forms. Each question form contains at least one blank to be filled in.

Thus, for a question form constructed in the usual manner there are blanks, to be filled by whatever data of a given data type may be encountered For example, there are question forms like the following, with blanks denoted U, V, T, R, S, ...

"How many U were there in the vicinity of Spacecraft X and How many U in the vicinity of Spacecraft Y"
U ≐ Klingon, or hole, or trade route (as typical data types) or
"Why did you change U for Spacecraft V"
U = tactic, V = spacecraft X or Spacecraft Y,'
or an economic enquiry such as:
"Which is the most prosperous R or S"
R = one starbase, or trade route, S = another starbase or a trade route.
or the unconditional form
"Where is X in Space and where is Y in Space"
Reply by coordinates on the <u>global</u> picture of <u>use</u> of space.
or likelihood estimations:
"What is the chance of finding U in the left upper quadrant of your local "scan"?
U = Crack, Hole, Barge, Klingon, Starbase or other spacecraft.

<u>Blanks</u> to be filled by data types are listed as the content of the <u>slots</u> in the question forms listed as components of the sessions 1, 2, ... n. The program fills in, and, if necessary, overwrites the blanks as soon as data of an appropriate type is provided by an event in the "window" neighbourhood of spacecraft X or the "window" neighbourhood of Spacecraft Y, that is, by events that have actually been experienced by the commanders although not necessarily perceived as relevant.

The first (after that, the next) interrogation session is initiated when the blanks for both commanders are filled by relevant data collected by the blank filling operation, since the last interrogation, so that, on presenting questions (to be answered, together with a confidence estimate), the blanks have been filled.

Appendix 1 shows a record of the questions with blanks filled-in (perhaps differently, for the commanders, although both commanders are interrogated), printed with the commanders responses and the factual data and tactics for the last occurrence of the salient events. The process is repeated, for subsequent interrogation sessions. In the current system, factual or behavioural data and tactics are recorded **at** each interaction between the spacecraft and the environment. But, as in this sample, the questions and responses are recorded only at the (comparatively few) interrogation sessions.

This method is extremely powerful, and so far as I know, original. Still it is not quite enough because the responses should be combined with the tactic specifications and continually embedded in a representation common to the commanders (that is some part of the representation must be common to the commanders; in many respects they can see their environment quite differently; depending, for example, upon whether they are oriented to Klingon elimination or to making kudos out of investing in the starbase economy).

The building of a representation is discussed in the next subsection of this paper

2.5. Representation

Decision making takes place in the framework of a task (and decision maker) representation which is built up by an act called "planning" though it could equally well, be called "describing the world as it is seen". Subjects are required to engage in this activity from time to time, as a decision process goes on. The borderline between tactic construction and planning is hazy (for tactics are represented and may be derived from a representation); It exists only because of technical limitations upon the present system. As it stands, the machinery used for running the environment is, also used for constructing or executing plans.

2.5.1. Planning Languages

Goguen and his associates have devised a peculiarly lucid language for describing plans. That is, given some transcript material and a means of discourse analysis, he describes the plans, conspiracies, or whatever, that may have been thought up by the people engaged in planning conversations, meetings, and the like. It is, for example, possible to express the idea of a goal, of an actor, of doing and of done by, or to be done by, in this graphical notation as well as the idea of condition holding (the predicates in Goguen's formulation). The main connections are "If ... then ... do X and do Y and"... or "Do at least one of X, Y, ..." and a special qualification of the type "Do X, next Do Y, next ..." (in order to maintain or to satisfy a goal relation).

There is a family resemblance between this planning language and the language employed in the system under immediate discussion. The differences are partly due to the different domains of employment (ie. post hoc description of plans that have been made, in contrast to the present domain of decision where descriptions are not really distinguishable from plans being made). Thus, in the present language, it is possible to describe (say) a mission, or a vehicle or the actions of a vehicle; similarly, to prescribe (for example) the tactics or immediate actions of a vehicle; similarly, to prescribe the strategy or strategy class appropriate to the mission.

Tersely, the language L includes this graphical language, and the inscriptions in a representation are legal L expressions.

A statement of which L expressions are "legal" and of why they are "legal" must await the discussion of stability (Section 2.8) as the term is used in respect to agreements between the participants. Note, here (for the record, and as a cue to recall) that the rules of L usage permit the inscription of stable agreements, (in the sense to be indicated in Section 2.8).

2.5.2 Overview of the Formal Character of the Representation

Without invoking stability, it is possible to consider the types of representation that are available as formal entities. A certain amount of jargon is unavoidable (though it is explicated fully at a later point).

(a) The "representation" is a described array of orders of entailment meshes, their condensations and their pruning fields. In general, these pruning fields cover the task environment or have it as part (not usually, all) of their semantic interpretations.

(b) An entailment mesh is a directed graph in which the nodes stand for stable L agreements (that have been or might be reached by A and B); the directed arcs stand for L derivations that carry one stable agreement into another. Apart from nodes in the graph that represent analogies, all derivations are locally cyclic or rederivable and may be multiply cyclic (derivable by many different paths),

(c) If a perspective, or point of view, is adopted (as it must be in stating an intention, or converting part of the representation into an action), the cyclic mesh is opened out into a nearly hierarchical structure, called a pruning, under whatever node in the mesh corresponds to the perspective or point of view.

(d) The mesh may be pruned under any node. A pruning is a class of plans. A selective pruning is one plan.

That is, a selective pruning, alias a plan, becomes identical with a tactic of Section 2.4 if the nodes stand for commands or "unconditional tactics" (as they are defined in Section 2.4). Similarly, a pruning is a coherent class of plans (alias tactics) and may be constructed as such. Vice, versa, if any entailment mesh representation is activated, then some or all of these tactics may be "executed". This formulation is closely allied to the Admiralty Acts Representation. Gregory C (1979).

(e) Supposing a mesh of order $\underline{m}$, any of its prunings may be systematically and mechanically condensed to yield one node in a mesh of order $\underline{m} + 1$, and so on for higher orders, $\underline{m} + \underline{r}$, $\underline{r} > 1$. As a rule, only the condensations of prunings under nodes of the order $\underline{m}$ mesh that have more than local cyclicity are of interest. Other than local cyclicity indicates an essential redundancy in the derivational structure at order $\underline{m}$ and the derivations in the mesh of order $\underline{m} + 1$ abstract such redundancies, (they do not simply image the relations between several distinct nodes, selected as perspectives, already expressed in the mesh of order $\underline{m}$).

(f) A mesh at order $\underline{m} + 1$ (or $\underline{m} + \underline{r}$) may be pruned at that order.

(g) Conversely, any condensation may be expanded In general, expansions are not unique (thus, expansion of a node at order $\underline{m} + 1$ may yield the pruning of order $\underline{m}$, from which it was derived by condensation, or a class of prunings of order $\underline{m}$, which will include the one from which it was derived).

(h) The expansion of nodes in a mesh of order $\underline{m}$ yields a pruning or a class of prunings at order $\underline{m} - 1$ or, in general, $\underline{m} - r$. Notice this is distinct from a pruning, or hierarchicalisation, at order $\underline{m}$.

If a pruning under a given node is expanded prior to condensation, the expansion calls for additional information from the participants and is obtained (if at all) by adding to the representation. The importance of this idea of expansion without prior condensation is as follows (and it needs emphasis).

In general, it would be necessary to associate each node with something that it stands for; namely, a process such as the motion of a vehicle (Fig. 4(a)) representing one spacecraft.

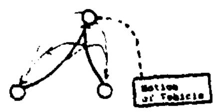

Fig 4(a)

Usually, processes are executed, and thus interpreted, independently, unless otherwise stated (by an analogy in the mesh). Using logical terminology, processes are usually realised (given a semantic) in distinct "universes of interpretation"; for example, vehicle X, in contrast to vehicle Y (which are, by analogy, perhaps <u>similar</u> in control characteristic but different in position, course, energy). The construction is shown in Fig. 4(b) and its reliance on independent processes is stated in terms of "universes of interpretation" in Fig. 4 (c).

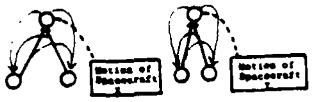

Fig 4(b)

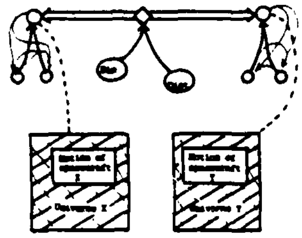

Fig 4(c)

This principle, if applied to each node, certainly leads to great redundancy.Thus (Fig. 4 (d)) Spacecraft X motion may all be obtained from the environment of spacecraft X and the course of spacecraft X as given in the entailment mesh. However, distinct universes are preserved (Fig. 4 (e)) and appear in the mesh only in terms of the distinctions (Dist (X,Y)) which,together with the similarity (Sim),make up any analogy that may be asserted between these vehicles (Fig. 4 (f)).

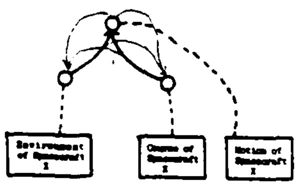

Fig 4(d)

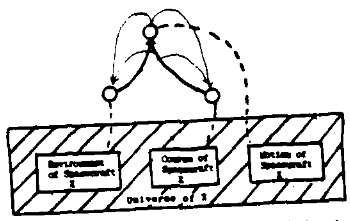

Fig 4(e)

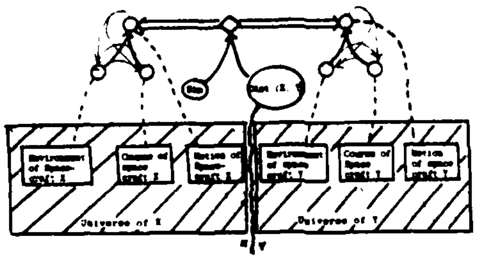

Fig 4(f)

Are these process models needed?

Some of them are, since the mesh has no other interpretation. But we might, as in Fig. 4 (g) open out the mesh by adding further nodes so that processes contract to events that are, for any participant and some particular purpose or perspective, regarded as unitary events. This operation is precisely expansion without prior condensation and it leads, in a loose way, to a uniform representation.

The construction is tricky, however, for the following reasons. There is an indefinite number of unitary events for any participant, certain finite subsets of which are sufficient to retrieve the entire structure if the Dist (X,Y) are stated. It does not follow that events are, in any sense, absolute (any of an indefinite number of subsets might be chosen as unitary events). There is, for example, no reason to believe (Fig. 4 (g)) *that even for one participant* and perspective, the unitary events proper to X are also proper to Y. The events that seem unitary will, usually, depend upon the perspective adopted and the number of distinctions, Dist (X,Y), that are made to support the asserted similarities of analogies (the Simi) if any at all are stated.

The difficulties are real but have less consequence than might be expected when it comes to practical applications and the implementation of experimental systems. The representation is expanded (without prior condensation) to any entity in the system that has real values (for example, a vehicle does, willy nilly, have values of motion, direction, energy). These do not, of course, specify an adequate operational vehicle (with a sensible motion, a sensible tactic, or whatever) but values exist in the system and can be picked up as part of a (very bad) vehicle description/plan/representation (in fact, so bad that the vehicle is moving slowly in an initially specified direction, as a result of which it will run out of energy). The representation can be refined (a further expansion) and it must be refined if the vehicle is to do anything of value (without moment to moment supervision) but, logically, this expansion is not demanded even though it is operationally mandatory.

Other entities in the system exist, ie. have values, willy nilly (for example, intruding marauders, called "Klingons", barges journeying between fixed loci called "Starbases"). Their values include energy, location, direction, and, in the case of "Starbases", economic resources and trade carried out (by means of "barges"). If these entities are represented as nodes, there is no logical demand for a refined representation, even though operational efficiency depends upon such a refinement.

However, there are also entities created by the participants which do not have values excepting those assigned by the participants, for example, "Starbase Governors" and "Vehicle Tactics". A "Starbase Governor" is a plausible "superstition" (an important "superstition" for some participants, also). Here, we logically require either that the participant specifies a process representing the Starbase Governor and a domain upon which it operates, having variables with a finite set of possible values, or that the mesh is expanded to properly derivable nodes, that stand for variables having, in Zadeh's sense, possible values. In the case of a "Vehicle Tactic", we do not logically require expansion (since vehicles have values, as above) though it is operationally desirable.

Let us return to the initial theme.

We thus have a possible array of entailment meshes of various orders. To satisfy the initial statement, of (a), there is some r for which an array can be based upon an entailment mesh of order m that has nodes that are condensations of entities (in general processes that include all the entities in task environment). The description of this array is a combination of predicates asserting distinctions in all the L analogies designated by nodes in the array of meshes.

2.6. Origin of the Representation

Having said (indigestibly, but tersely) what a representation is, we can address the question of how it comes into being.

One possibility (in our particular configuration) is that a representation exists before A and B, the participants, come on the scene; further that this representation satisfies clause (a) of Section 1.5.

Another possibility is that no representation exists before A and B make their appearance and that a representation is built up by forming a static inscription of the L agreements reached between A and B as a result of their dialogue. In this case, the stability of agreements (which can be expressed in terms of the derivational structure relating the corresponding nodes in an entailment mesh) is guaranteed as a result of the computer regulation of the interface, which ensures L legality (and, in the required sense, stability), as in Section 1.1.

In between, A and B may find an initial representation of part of the task environment, which does not satisfy clause (a) of Section 2.5. However, the participants can amend and add to this representation, as they decide about the task environment.

So, for example, the participants who agree to undertake the "mission" of Section 1.1 may find an incomplete representation of the geography, trade routes, and of what their vehicles can do. It should be emphasised that the representation, though accurate so far as it goes (and it is incomplete) is not unique.

As a rule, the incomplete, initial representation does not suit A and B. Moreover, it is inadequate to determine the mission or even to control the vehicles that carry out the mission. By adding to and amending the representation in the course of their dialogue (recall, the exteriorisation of a decision process) A and B may or may not arrive at a representation that satisfies Clause (a) of Section 2.5.

Usually, in experiments, things are arranged so that even after a great deal of decision the representation does not satisfy Clause (a) of Section 2.5. For, if it did, then this representation would determine the mission and the participants would not need to decide.

1.7. Accommodating an Evolving Representation

In order to depict the amendments and additions made by the participants, Fig. 2 requires modification as shown in Fig. 5.

First, the L agreements of A and B must operate, given the L legality conditions maintained by computer regulation, upon the representation at the interface.

Next, if that is so and if control operations (such as the motion of vehicles, the collection of data) can be initiated through the interface, there is no need for A and B to act directly upon the task environment.

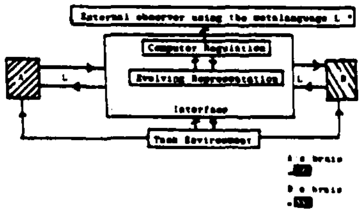

Fig 5

For example, a vehicle able to intercept marauders and protect a trade route exists <u>for A and B</u> insofar as it is represented; it has control characteristics and tactics <u>for A and B.</u> If so, then they need only appeal to <u>this</u> either in order to perform a simple manoeuvre or to ordain a particular (and represented) tactic.

This notion leads to two new extensions of Fig 5 . Of the two, Fig 6 is fairly commonsensical. A further inanimate (though slightly unconventional) device is added to the picture, and called C. It has the status of a participant insofar as it can, like A and B, initiate actions or obtain data. But it cannot (as A and B can), change the representation. In the most recent implementations of the system C can overgeneralise the agreements reached by A and B so as to promote inventive behaviour and C can point out places where the representation is incomplete and interrogate A and B (in much the same way that they question each other in reaching an agreement). But, even here, C is not allowed to alter the policies of A and B, as reflected in their perspectives and decision styles (though it is true that A and B may change their styles as an indirect result of experience in the system).

The other modification of Fig 5, shown in Fig 7, is usually considered to be contentious though there are no fundamental difficulties provided that one has an open minded attitude towards specifying an individual.

So far, A and B have represented people or teams, characterised as independent except for their interaction in dialogue. This indentification of A and B remains valid, but a further possibility is added (the construction shown in Fig 7, which is compatible with Fig 5 and Fig 6)

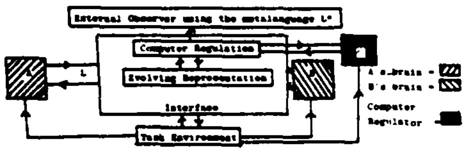

Fig 6

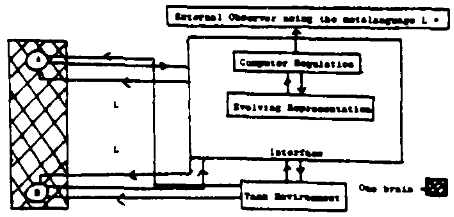

Fig 7

In Fig 7 the participants A and B are shown as initially independent organisations in one person, who adopts different roles (A, B) such as Minsky's "hypothesis poser" and "critic". Normally, A and B reach agreement (reach conceptual coherence) by an internal dialogue which may, however, be exteriorised, insofar as the conceptual transactions are "pulled out" as L transactions through the interface (at the "price of" making use of this interface). That is, A and B represent coherent systems of belief , characterised by distinct but simultaneously entertained perspectives.

2.8. Stability Indices

The notion of stability appropriate in the context of the many faceted agreements that are proposed as the observables in a decision process, is formally expressed by a system which is "organisationally closed and informationally open" (Varela and Maturana or Goguen and Varela).

In terms of conversation theory, we formulate this matter in conceptual (rather than biological) terms as a system which is productive as well as reproductive. (Equally well, constructive and reconstructive). Just as a set of biological productions must maintain or compute boundaries (of a cell, for example) if these productions are to have autonomy so a productive and reproductive conceptual process must compute distinctions insofar as it is a stable and integral unit. Such a system is called a stable concept (in conversation theory; the author and others) and it is argued here that a decision process is a special case of a process involving procedurally defined stable concepts denoted $\underline{Con}_A$, $\underline{Con}_B$ ... insofar as they belong to A,B, ..., ie. decision is a special kind of thought.

A concept (unqualified) is defined as a wholly or partially coherent class of mental procedures or interpreted programs that are open to execution and some of which are always undergoing execution in any participant A,B, ... who is aware. The production and reproduction of $\underline{Con}_A$ or $\underline{Con}_B$ depends upon the (again continual) execution of operations that are distinguished from concepts, only as a matter of convenience, because they act upon concepts. There is empirical evidence that these operations belong to classes of Description Building or DB operations and Procedure Building or PB operations and that both types of operation are involved in the production or the reproduction of a concept, ie. the mental mechanism responsible for organisational closure, and thus stability.#

If T_A and T_B are descriptions entertained by participants A,B, then, using "Ex" to mean "execution of" and " $\Rightarrow$" to mean "production"

$$\underline{Ex}\ (\underline{Con}_A T) \Rightarrow T_A \text{ or } \underline{Ex}\ (\underline{Con}_B T) \Rightarrow T_B$$

Where the execution may imply an internal awareness (or imagery) of something, or a behaviour that satisfies or maintains some relation, such that

$$T_A \succcurlyeq T^* \preccurlyeq T_B$$

In conversation theory, a coherent class of concepts, together with the context in which they are productive and reproductive, is a "P Individual" or "the P Individuation of A, B, ..."; the procedural specification of a participant.

(meaning, there is an isomorphism # between all or part of T_A and T_B, namely ,the part T*) and where T (a topic) is the union of T* over all pairs and n tuples of a class of participants A,B, ... Topics exist ,insofar as concepts are stabilised; thus, T exists insofar as $\underline{\mathrm{Con}}_A(T)$ and $\underline{\mathrm{Con}}_B(T)$ are stabilised.

From the auxilliary postulate of types <u>DB</u>, <u>PB</u>, of productive and reproductive operations, it is possible to write the productive systems concerned in the stabilisation and the potential development of $\underline{\mathrm{Con}}_A(T)$, $\underline{\mathrm{Con}}_B(T)$ as

$$\underline{\mathrm{Ex}}\ \underline{\mathrm{DB}}_A\ (P_A, Q_A) \Rightarrow T_A \quad \text{or} \quad \underline{\mathrm{Ex}}\ \underline{\mathrm{DB}}_B\ (R_B, S_B) \Rightarrow T_B$$

(where P_A, Q_A, are A descriptions; R_B, S_B, are B descriptions).

together with

$$\underline{\mathrm{Ex}}\ \underline{\mathrm{PB}}_A\ (\underline{\mathrm{Con}}_A Q,\ \underline{\mathrm{Con}}_A Q,\ T_A) \Rightarrow \text{Procedure in } \underline{\mathrm{Con}}_A T.$$

$$\underline{\mathrm{Ex}}\ \underline{\mathrm{PB}}_B\ (\underline{\mathrm{Con}}_B R,\ \underline{\mathrm{Con}}_B S,\ T_B) \Rightarrow \text{Procedure in } \underline{\mathrm{Con}}_B T.$$

(where $\underline{\mathrm{Con}}_A P$, $\underline{\mathrm{Con}}_A Q$ are A's concepts of P_A, Q_A, and $\underline{\mathrm{Con}}_B R$, $\underline{\mathrm{Con}}_B S$ are B's concepts of R_B, S_B; noting that the execution of these concepts give rise to descriptions P_A, Q_A, or R_B, S_B, just as the execution of $\underline{\mathrm{Con}}_A T$, $\underline{\mathrm{Con}}_B T$ give rise to descriptions T_A, T_B).

Isomorphism , $(\leftrightarrow)$, or partial isomorphism $\geq$, not $\subseteq$ or $=$, since A is not the same participant as B, so that T_A, T_B are distinct.

A rough sketch of the minimal unit called a stable concept of T is shown in Fig. 8, and the formally minimal requirements for organisational closure are shown in Fig. 9 and Fig. 10 (from Pask, 1977). The notation used in Fig. 9 and Fig. 10 includes the sign " $\longrightarrow$ " which stands for "collection and return of products".

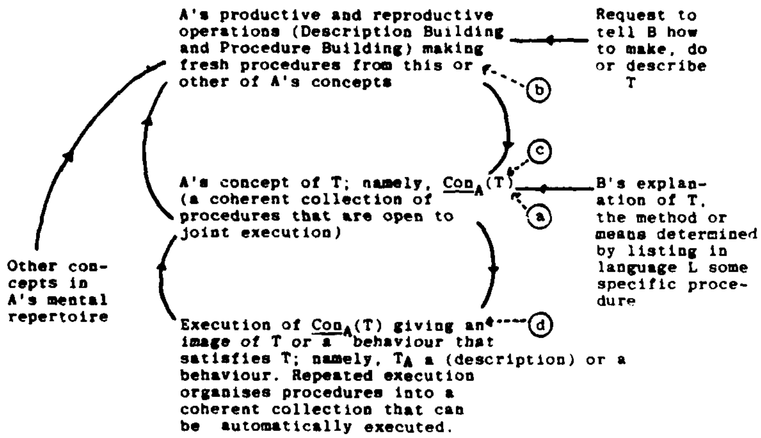

Fig. 8

The organisationally closed system is also informationally open, since the DB and PB operation classes are specified and recognised empirically in terms of agreement forms and products. There are many other products obtainable from the application of these operation classes and different participants A,B, may have quite different DB and PB operation classes, provided that the productive scheme and its specificity is satisfied. Hence, the subscripts in DB_A, PB_A and in DB_B, PB_B . In this respect, the production system units are artificially isolated but some isolation is realistic insofar as organisational closure relies upon the computation of a distinction $P_A \neq Q_A \neq T_A$ (or any other A description) and $R_B \neq S_B \neq T_B$ (or any other B description), these being eigen values (in the non linear, extended, sense of Von Foerster) for the system of productions, under indefinite iteration, regarded as Fuzzy, non linear, eigen operations.

Returning to Fig. 8 and recalling the idea that a concept, a fortiori a stable concept, is a partially or wholly coherent class of procedures, we shall interpret decision making, and the resolution of doubt, as a process that leads to stabilisation; in various stages and by various patterns.

Suppose, for example, that A has no concept for T but that participant B (or even the symbolic environment), provides a method or procedure for realising a described condition (T). In this case, the procedure must be compiled for execution in A's brain; a process involving <u>information transfer</u>, in the sense of Petri or Holt*, between $\underline{DB}_A$, $\underline{PB}_B$ operations and the given listing or procedural specification <u>as it is being compiled.</u> This possibility is shown as (a) in Fig. 8 and the sheer necessity of this organisation is evident on recalling that Petri or Holt information transfer is equisignificantly the emergence of local synchronisation of two a priori asynchronous systems A and B or the local coupling of two a priori independent systems. Here, the information transfer is <u>between</u> A and B, the participants.

Once a procedure is compiled for execution, it is a (limiting case) concept, and it is stabilised by $\underline{DB}_A$ and $\underline{PB}_A$ operations that add further procedures to the concept (a wholly or partly coherent class). In general, such procedures are not coherent in the original set. In this case, information transfer must occur <u>within</u> A (between the procedures in $\underline{Con}_A(T)$) in order to compile them so that they <u>can be</u> coherently executed. Similar comments apply if B (or the environment)

---

* This fundamental definition of information is crucial to our argument and involves notions of independence as well as temporality. Information, in this sense, should not be confused with information <u>measures</u> either combinatorial (Ashby, Conant) or statistical (Shannon and Weaver) algorithmic (Kolmograff, Loefgren) logical (Bar Hillel, Chiaraviglio) or physical (Gabor and Macky, Brillouin).

provides a fresh procedure when the concept $\underline{Con}_A T$ <u>exists</u>. The information transfer, in either case, is shown as (c), in Fig. 8.

Next, supposing $\underline{Con}_A(T)$ *exists and is* stabilised, B may ask A how to make, do, or describe T. In this case there is an A,B, information transfer required in order to construct or reconstruct a specific L listing and to engage the specific <u>PB</u> operations that will do so. This transfer is shown as (b), in Fig. 8.

The execution of a partially incoherent $\underline{Con}_A(T)$ also gives rise to information transfer between otherwise incompatible procedures, but there is no such information transfer if $\underline{Con}_A(T)$ is rendered entirely coherent, by dint of execution and reproduction. Nor, in the latter case (which is typical of a well learned and automatically performed skill, either a perceptual motor or an intellectual skill). However, suppose that a "mistake" is made (for example, if A's concept of driving a motor car leads to a behaviour that crashes the vehicle, or A's concept of addition does not tally with the result obtained by an adding machine). If so, an information transfer, shown as (d) in Fig. 8, is produced insofar as the "mistake" is remedied by adding procedures that enlarge $\underline{Con}_A(T)$.

Such information transfers <u>are</u> doubt, in a general sense indicating (as before) many species of doubt. If the information transfer is <u>within</u> A, it is an <u>awareness</u>; If the information transfer is <u>between</u> A and B, it is a consciousness. Its <u>degree</u> (only quantified in the latter case) is the doubt; its <u>contents</u> are the operations involved in the transfer.

Recall, (Fig. 7) that A and B are <u>participants</u>; they may be distinct perspectives, simultaneously adopted by the same person .

2.9. Agreement over Concepts that are Stable.

The canonical, sharp valued, observations available to the external observer of Fig. 1, Fig. 2, Fig. 5, Fig. 6, Fig. 7 are agreements over stable concepts between participants A and B. These are known, in Conversation Theory (the author and others), as <u>understandings</u>, but, in the context of decision making (thus limiting consideration to a subset of conceptual processes), they are better labelled as stable choices of a joint A,B, view of their environment (to see things from the point of view of a particular topic).

To exhibit the dynamics involved suppose that A has the stable concept $\underline{Con}_A(T)$ of Fig. 9 (where T_A is derived from P_A and Q_A), whereas participant B has the stable concept $\underline{Con}_B(T)$ of Fig. 10 (where T_B is derived from R_B and from S_B). Either T is given environmentally or T_A, T_B are generated by $\underline{DB}_A$ and $\underline{DB}_B$ operations.

To evidence agreement over a stable concept means that A's concept and B's concept, though distinct as $\underline{Con}_A(T)$ and $\underline{Con}_B(T)$, are open to "coherent execution"

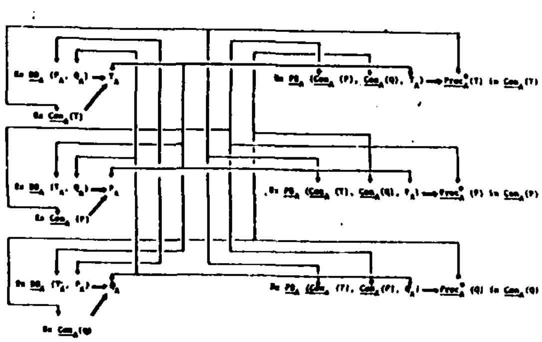

Fig 9

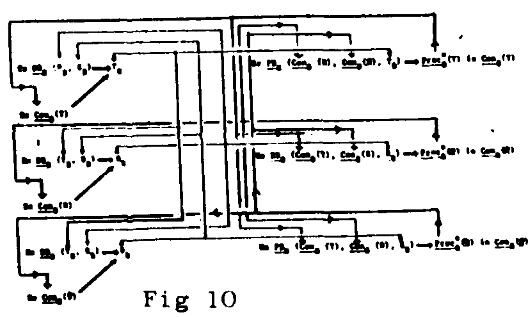

Fig 10

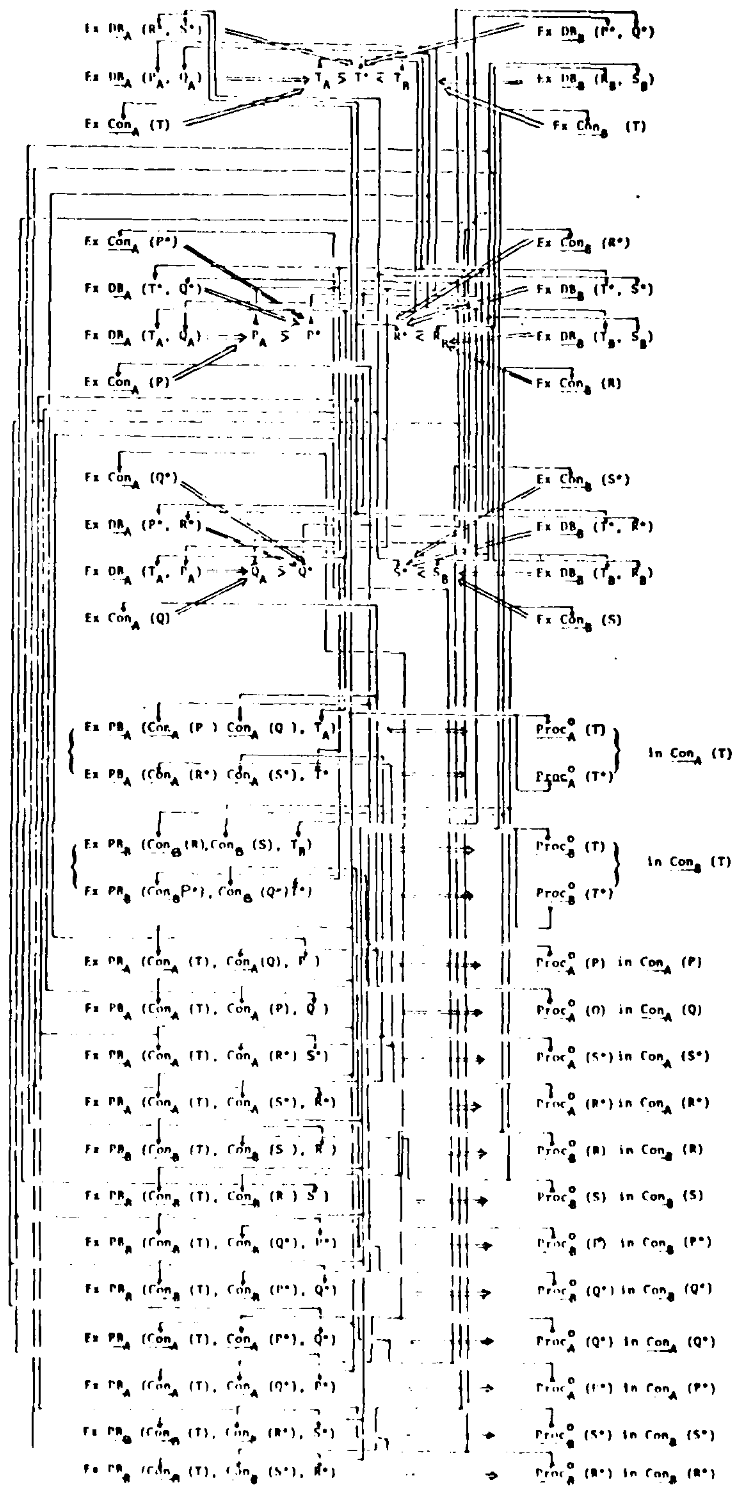

Fig 11. L-agreement over common understanding of topic T. A derives T from P and Q. Participant B derives T from R and S. An agreement may be complete or partial depending upon the isomorphic part (for example, T*) of topic and the similarity of method.

or "joint execution" to produce, at least, the commonly agreed *description* T* *or a behaviour that gives rise to or maintains* T*. That is, A is able to incorporate a procedure derived, by B, from R_B, S_B and representative of $\underline{\text{Con}}_B(T)$, into $\underline{\text{Con}}_A(T)$; further, to produce and reproduce it; conversely, B is able to incorporate a procedure derived, by A, from P_A, Q_A and representative of $\underline{\text{Con}}_A(T)$, into $\underline{\text{Con}}_B(T)$; further to produce and reproduce this procedure.

The minimal production scheme for agreement over a stable concept is shown in Fig. 11, which makes explicit the local synchronisation or coupling of A and B. Fig. 11 may be regarded as a picture of the process, a decision process, that leads from the initial condition of Fig. 9 and Fig. 10 to the common representation shown in Fig. 12 (the <u>commonly agreed</u> stable concept of T is executed to produce T*, either by A or by B).

Note:
Z is a variable indexing participants here Z = A or Z = B

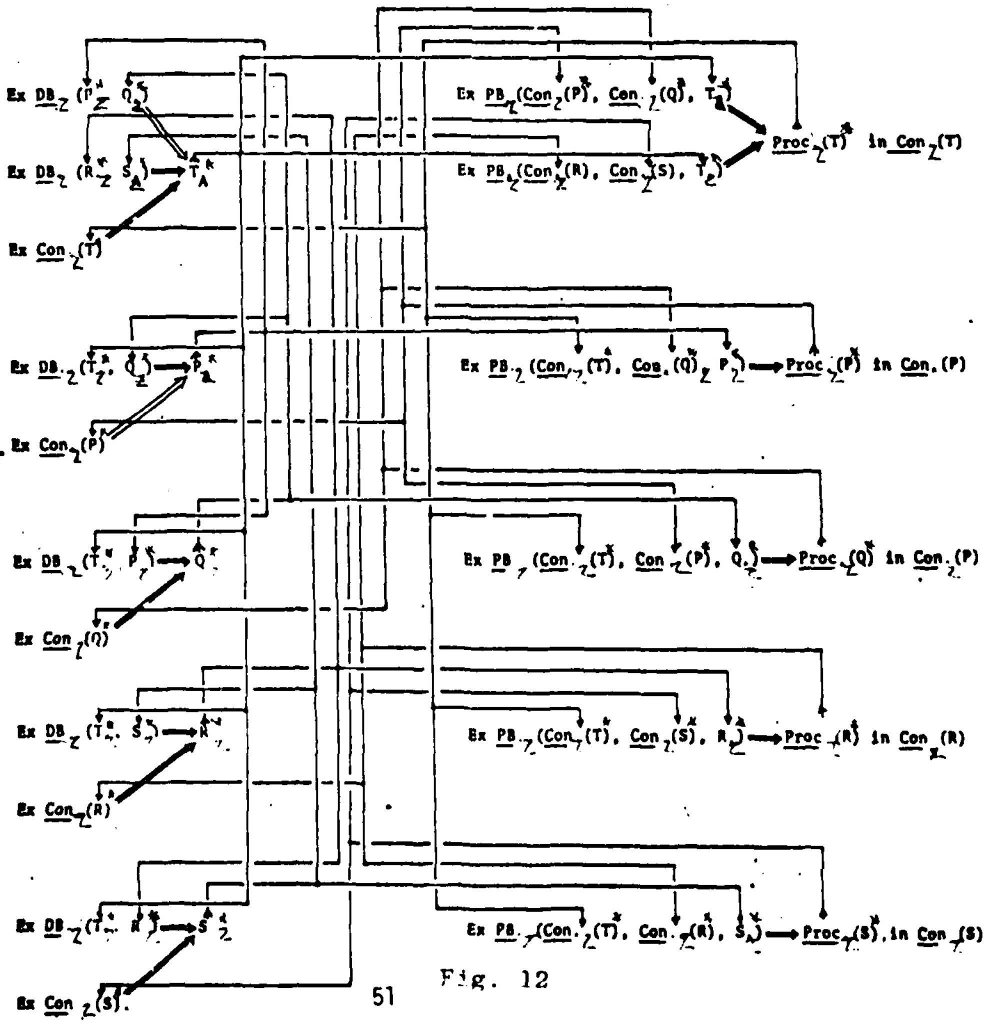

Fig. 12

or "joint execution" to produce, at least, the commonly agreed description T* or a behaviour that gives rise to or maintains T*. That is, A is able to incorporate a procedure derived, by B, from R_B, S_B and representative of $\underline{Con}_B(T)$, into $\underline{Con}_A(T)$; further, to produce and reproduce it; conversely, B is able to incorporate a procedure derived, by A, from P_A, Q_A and representative of $\underline{Con}_A(T)$, into $\underline{Con}_B(T)$; further, to produce and reproduce this procedure.

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This kind of decision making process is of primary importance, since it depicts the mental mechanism of planning, or constructing a description of the task environment, at any rate in the manner of Section 2.5. To see this, we note that a node in an entailment mesh can be inscribed (under the computer regulation and rules assumed to prevail in the previous discussion) if, and only if, there is such an agreement. But no nodes (standing for a topic) can exist legally in isolation; hence, the minimal structure which represents (and in the machinery is encoded as) Fig. 9, is shown in Fig. 13; of Fig. 10 in Fig. 14 and of the result of an agreement over a stable concept between A and B (namely, Fig. 12), in Fig. 15.

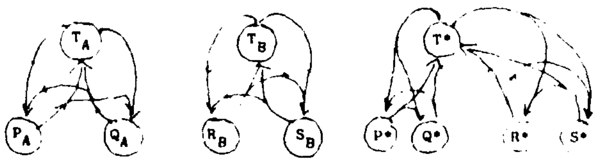

Fig. 13 Fig. 14 Fig. 15

Notice, that the _plans_ derived from these meshes are contingent upon the perspective adopted (assumed to be T) but that _once_ inscribed other plans are possible by pruning (Section 2.5) from other perspectives. Some possibilities are shown, for example, in Fig. 16.

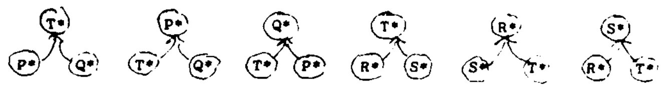

Fig. 16

The untramelled choice of perspective indicates a special kind of decision making (or, for that matter, of thinking) concerned with abstract or universally interpretable tasks. Though it has great value, such decision is relatively uncommon in practice. Most task environments have a many faceted interpretation, inscribed as the _Dist_ predicates, of analogies in an entailment mesh (Section 2.5).

As pointed out in Section 2.5 _Dist_ predicates are indications that information (usually about some concrete universe) must be involved in the stabilisation of an analogical concept; for example, the inscriptions shown in Fig. 4 (c) or Fig. 4 (f). Only under special circumstances is the similarity in an analogy an isomorphism and, if not, than the plans which may be obtained by prunings (Section 2.5) and selective prunings are limited.

Agreement over an analogical topic involves the production scheme of Fig. 17, which depicts the _process_ whereby A and B (people _or_ perspectives) with conceptual organisations shown as the initial

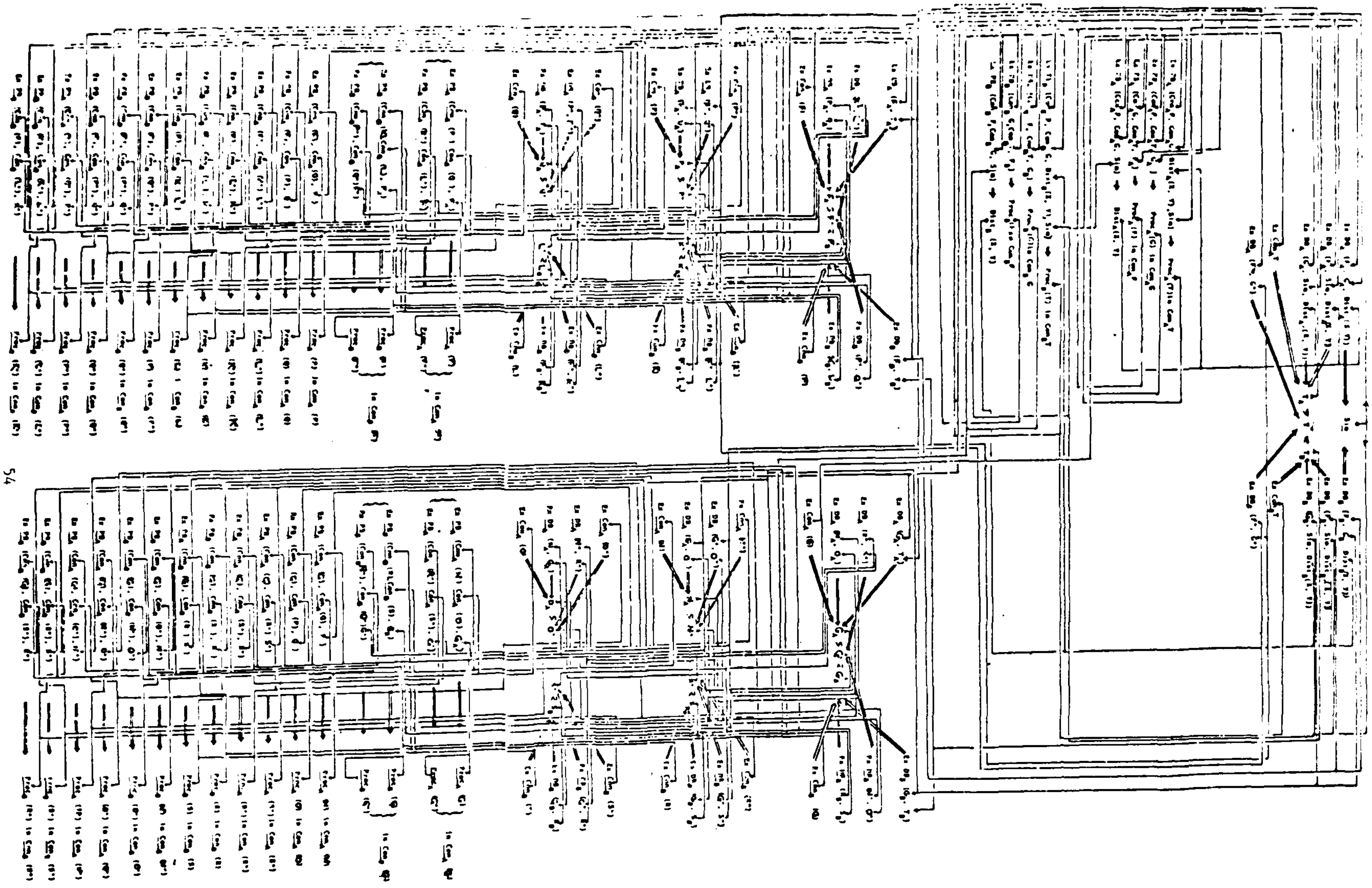

Fig 17: Production scheme, representing stable concept of analogy under perspective A and perspective B. For graphical clarity the analogical topic T of Fig 18 is written T, but it is not at all the same T as in Figs 12 to 15

inscription of Fig. 18 reach a coherent organisation shown as the terminal inscription of Fig. 18. #

The notation employed in Fig. 18 is a shorthand convention which avoids an otherwise complicated picture; any analogy is represented by a diamond-like node at the centre of double arrows; the full form is shown in Fig. 29 (which is the structure introduced into an entailment mesh, if a shorthand statement is accepted by the system).

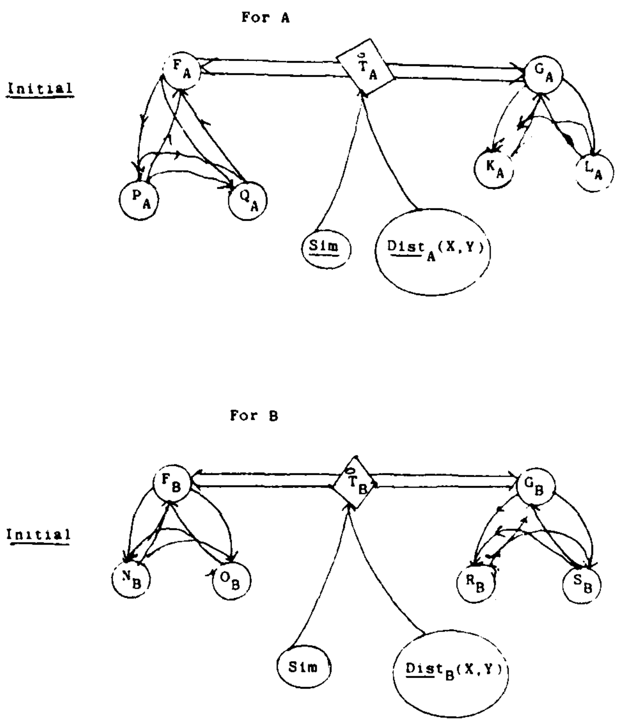

Fig. 19 (also see next page)

The inscriptions and production schemes of Fig. 11 to Fig. 18 justify the view that a conversation between participants A and B (either people, as in Fig. 1, 2, 5, 6, or Perspectives, of one person, as in Fig. 7) is the minimal unit for sharp valued observation of a decision process, the "its" or "objects of sharp valued external observation" being stable agreements between the participants or perspectives.

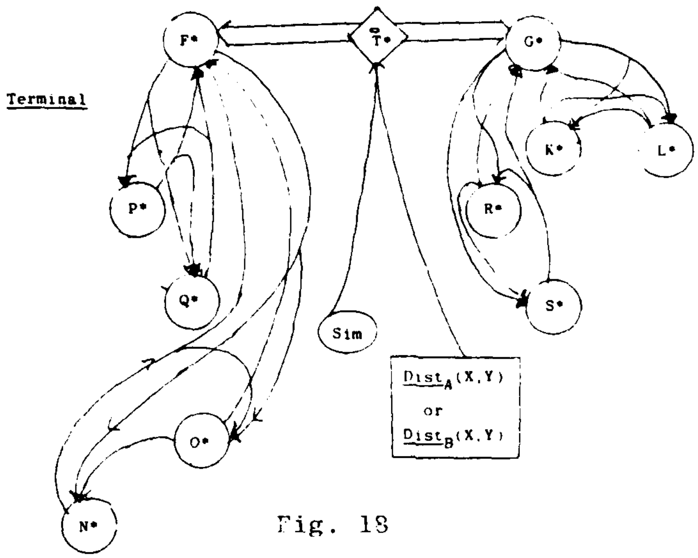

Fig. 18

It is worth noting that analogies may hold between any number of topics. Further, a distinction is made between analogies of <u>form</u> and analogies of <u>method</u>. The former type is shown in Fig. 18. The <u>Simi</u> that supports the analogy is a mapping between descriptions of topics without committment to the methods employed to recognise or create the similarity. However, if <u>Simi</u> is replaced by a derivational structure, then this derivational structure determines a method, which the analogically related topics have in common and the analogy is based upon a similarity of method.

There is an important difference between the conceptual operations involved in Fig. 11 (or Fig. 12, 13, 14) and those involved in the scheme of Fig. 17 (or Fig. 18) or, as a rule, any kind of analogical process. Both sequences require a distinction which may, equisignificantly, be regarded as taking a perspective and isolating some topic which is decided about as independent of other decisions that might be made. In the latter case, however, at least two distinctions (between universes X and Y in the illustration) are made and at least two perspectives are initially entertained by each participant. The process of keeping X and Y independent (apart from the analogical similarity, Sim), is perpetuated and continued (an indefinitely large number of distinctions might be made) and the act of predication (or distinction) is embedded inside the L expression to symbolise the fact. It is in this sense (that the L expression denoting an analogy is an instruction to continually compute distinctions which support the similarity, Sim) that information must be generated wherever an analogical topic is agreed, or its concept stabilised.

Notice that any invention, or creation, manifest as the development of an entailment mesh involves analogical reasoning; even if the mesh, as is usual, represents a concrete task environment the method of representation is literally invented, something (say P_A, Q_A, or R_B, S_B) that exists is said to be like something (say T_A or T_B) that is added. However, in *the case of a formal derivation leading to an agreement* that $T_A \ngtr T^* \nless T_B$ the distinction becomes absorbed as a personalised inequality of relations (Section 2.8, that "$P_A \neq Q_A \neq T_A$, or any other A description"; that $R_B \neq S_B \neq T_B$ or any other B description", and is expressed in the L syntax (formally, coherence truth requires no factual support other than the intellectual machinery of A and B, using the derivational logic). In the case of an analogy inscribed in the mesh, the distinction remains to assert the similarity between factually distinct universes (concrete or intellectual).

2.10. Doubt and Degree of Belief

Given the primary decision process, just outlined (and there are many variations), the participants may decide about methods, or between outcomes. Once again, an agreement is involved, but it is not agreement over a stable concept and is a Fuzzy# (perhaps probabilistic) observable rather than a sharp valued observable even if (as will be assumed), the participants continually use or construct an entailment mesh as an inscription of their plans, or their images, of the task environment.

I do not know how many categories of doubt there are and imagine there are very many of them. At least, this sentiment is consistent with the view expressed (in Section 2.8, and summarised in Fig. 8) that doubt, insofar as it is entertained by one participant (or perspective), A, is the degree of A's awareness and insofar as it is shared by A and B is the degree of A's consciousness with B (or vice versa), of some joint decision process.

Whatever else, doubt is not an homogenous quantity but is structured, like its converse, belief (though it may be fuzzily quantified by various kinds of confidence estimate or valuation). It is, for example, possible to make a useful, though incomplete, distinction between the following kinds of doubt, and within each category, to examine the microstructure of conditional and contingent doubt when the decision process takes place within the assumed framework (Figs. 1, 2, 5, 6, 7).

(a) Given an entailment mesh representing a task environment (and assuming that the mesh is, momentarily, not being changed) there is doubt on the part of a participant about the perspective, or several perspectives, to adopt. Some one or more perspectives must be adopted in order to act (including the intellectual act of conceiving the mesh from one or more points of view). Conventional measurement is hampered by the one or more condition.

In the sense of Zadeh, Gaines and others. Fuzzy systems and structures have a well specified manipulative calculus and are not, in themselves, imprecise entities though, as a rule, they represent situations that are ambiguous or vague or imprecise.

It would be easy to estimate the participants doubt about which of several perspectives (given in an entailment mesh) to adopt, if we imposed the experimentally common assumption that one and only one perspective, point of view, or focus of attention is adopted (in some sense,"at once"). But the evidence we have strongly suggests that this "hidden assumption" of experimental psychology breaks down, at any rate for complex tasks, and may be overly naive for any task (as in (b) below). This kind of doubt, or, conversely, this kind of belief, is succintly called a doubt about purpose and intention (equally, however, and more precisely under certain circumstances, it appears as a doubt about role and identity).#

(b) Whereas in (a) the entailment mesh is assumed to be fixed, let us next assume that it is to be modified or developed. In this case, (for each of possibly several perspectives) there is a doubt about the proper distinctions to make, and how to make them.

(c) Both (a) and (b) suppose something about the entailment mesh; that it is fixed, or not. In fact, the supposition is replaced by a doubt about the adequacy of the mesh, as a representation of the task environment. Is the mesh properly analogous to the actual task environment?

Frequently, the adequacy or otherwise of the entailment mesh is determined by factual evidence either deterministic, or statistical. Consequently it may be useful, at this stage, to invoke frequencies and other objective, or "it referenced" guidelines.

For example, and very pertinently, the agreement (Section 1.1. and Section 2.1) to participate in a decision process; a certain activity and a certain situation. This overall agreement, or contract, is as properly open to doubt as the minutiae that are agreed, doubted, or decided given that participation does take place.

Somewhat contrary to current practice, the circumstance of reviewing the adequacy of a representation is the one and only point in the process of decision at which the evidence is weighed up or statistical data collection is invoked. This is obvious, at the level of a decision process regarded as an ongoing whole, not so obvious if the process notion is built up from elementary parts. After all, the representation (entailment mesh or not) presents, amongst other things, the participants image of an objective (to them it-referenced) task-environment, tied down to observation (by them) of data (to them) so that it, (the representation) is what may be changed by evidence. It will be changed if the hypotheses or plans derived from their current representation appear to be inadequate or if a change in the task environment indicates that the representation is inadequate as a description.

There are, however, difficulties insofar as statistical inference rules do not usually apply to (pruned) entailment meshes. Clearly, this can be seen as an inadequacy of the entailment mesh representation; but I do not think this vision of things is correct. Precisely the same difficulty besets decision analysis (which is a conversational attempt to build an adequate representation for a decision process) and gives rise to a dilemma, namely

(A) If the methods of decision analysis are rigorously enforced to provide an essentially tree-like (or set of tree-like) representations obtained (by an operation akin to pruning, from a "black box" model); for example, if Ward Edward's or Raiffa's paradigms are pursued rigidly, then the decision maker does not seem happy with his representation; it does not "suit" him.

(B) If the methods are not so rigidly applied, then (by token of concensus over previous conferences) the representation that does suit the decision maker is often intractable at the level of (standard) probabilistic rules. Further, it may turn out that the (evidential) descriptive component of the representation is counterfactual, biased, and in other ways unsatisfactory except to the decision maker, who may (by token of the same concensus) use it very profitably.

A modicum of compromise is possible. In the first place it looks as though the truth candidancy of data are in fact (and independently of conversation theory) judged on criteria of coherence truth (Rescher) rather than veridiciality, or factuality, alone. As soon as coherency is extended from propositions to process representations like entailment meshes (or those of the liberalised decision analysis) the representations appear quite rational. I happen to prefer entailment meshes because they are relatively neat and manipulable.

But what of the probabilistic rules? Phillips, very perspicaciously, points out that Bayesian Inference is one coherence scheme over a probabilistic domain. However, it will not work (for structural reasons) over representations of the type "entailment mesh" or "pruned entailment mesh". The reason for this is given by Atkin's analysis (in his paper for the conference, which I have seen in draft form) that the probabilistic rules are applicable only over structures that, in the present sense, do not support a decision process (they have zero "obstruction vector"). On the positive side, his search for a faithful, and coherently manipulable, representation of likelihood may provide rules which satisfy Phillip's paradigm (and maybe the Bayesian ethos) but are not the standard rules.

Finally, we must accept that representations usually represent more than the task environment and can be inadequate on other grounds.

It is always possible to adopt an indefinite number of representations that are veridicial; as a rule, only some of these suit the decision maker.

(d) Given that a mesh exists and that perspectives are momentarily fixed there is, for each perspective, a doubt about the description. For example, in the case of A and the topic name T ,"is T*, the moiety agreed with B ,isomorph to some part of T_A as computed by the production $\underline{DE}_A$ $(P_A, Q_A) \Rightarrow T_A$?" "or is T* (or T_B) a proper analogue of T_A ?"

(e) Given the resolution of this (DB type) doubt there is a doubt (of PB type) regarding how to satisfy the description, ie. a doubt about method. If $\underline{Con}_A$(T) does not exist ,then the doubt in question is associated with the execution of PB operations. If $\underline{Con}_A$(T) exists, in A's repertoire, this doubt is minimised if $\underline{Con}_A$(T) is one procedure . Continual execution of the productive and reproductive operations lead to a multiplicity of methods (as in a well learned skill; how do you drive a motor car, for example, even though you do so successfully).

(f) Suppose that a description is determined, as in (d), and consider a doubt about the values of variables that are descriptors in the scheme ,(as a rule, some values are given, other values are to be obtained). This doubt (of which "multiple choice" or confidence estimation amongst a set of "exclusive and exhaustive alterations", is a familiar parody) is a doubt about outcomes.

Doubt about outcomes may be experienced even if no method exists. Several cases can be distinguished (of which (I) and (II) below are significant, whereas the "guessing" of a type considered in the next clause (g) is dismissed as trivial and irrelevant to the serious conduct of a decision process, at any rate, as "decision" is interpreted in this paper.

(I) An analogy of form (or description) is discovered or recognised ,between the firmed up description and some other description (so that outcomes can be placed in correspondence). For this other description, there are procedures that determine outcomes. These procedures are executed and an outcome corresponding (in the f•rmed up description) to the outcome determined in the other description is selected. This is a possibly fallible but often useful expedient (that the current set of outcomes are like the outcomes in a different set).

(II) An analogy of method is established, such that the method or procedure can be transferred and applied to produce an outcome in the firmed up description. Again, this is a potentially fallible but often useful expedient (that the means employed in a different field are like those that should be employed, if any existed).

(g) If no method exists (as supposed in clause (f) any applicable means which is irrelevant to the situation is brought to bear) usually because something must be done. The justification for this act, known as "guessing", is to equate irrelevance with randomness and to believe that such a "guess" is better than nothing in avoiding biasses and preconceptions. Alternatively, an external random device, such as a dice or coin toss is consulted. This does not seem to be an adequate justification unless the task environment is specially contrived, as in gambling, to have a random structure (where randomness is a very special interpretation of indeterminancy).

It must, of course, be conceded that some "guesses" are "informed guesses" and they are defensible as being "better than chance", which is perfectly true. The present contention is as follows; insofar as a guess is "informed", the "informedness" is due entirely to process of the kind described in (f) and that the guessing part, pure and simple, is based upon a confusion between "irrelevance" and "independence" together with a naive interpretation of "chance" in any but "specially contrived" task environments.

(h) Doubt "because things move too fast", is a final and important category. Even if guessing (in the sense of (f)) is excluded, the methods at hand may, though incomplete, be sufficient to select a subset of plausible or possible values. Even if an adequate method exists, it may take too long to execute.

In a decision process, viewed in the present framework, a decision maker must relinquish control at the point where guessing,(as characterised in (f)) begins.

Just as the participants in our Decision System are periodically required to plan (to construct representations at a planning session), so, also, any participant is periodically interrogated to sample the varieties of doubt (c), (d), (e), (f), (g) and (h) (the latter, in a special sense). The varieties (a) and (b) are sampled either in planning sessions or by examining open content, fixed format, communications between the participants (or of reflective IPM type questions, in the case of one person with more than one perspective).

These, and other operations, are performed by the machine, C, of Fig. 6. We should, in this context, reemphasise the statement that such a categorisation of doubt is possible only under the stated conditions; that the primary decision process of Section 2.8 is exteriorised for observation in a generally evolving entailment mesh representation.

The account of the process of sampling doubt suggests a degree of preordination that does not really exist. Surely, the Decision System (on which this paper is empirically based) is not as flexible as it should be, ideally. For example, there are constraints imposed by planning sessions and by interrogation. But both events are process dependent, not arbitrarily preprogrammed events. Further, they are not, organisationally speaking, as distinct as they are said to be for the purpose of easy exposition. Data from interrogation, for example, gives values to nodes in the representation (especially those representing the plausible but not preexisting entities noted and exemplified by "Starbase Governors" in Section 2.4 (h)) and machine C, apart from acting as a supervisor "uses" the representation to determine what happens in the task environment (Fig. 6); indirectly, also, when interrogation will take place.

2.11. Commentary Upon Changes and Conditions of Doubt

Conversation Theory, the background against which we consider the decision process and the doubt it engenders, is a global or general theory. Its predictive capability appears at the points where it is specifically applied. Predictive statements of a standard kind are thus contingent upon the occurrence of particular events. If such an event takes place (which is very different from contriving matters so that it seems to, or even does, take place) then standard predictions are possible.

If, for example, the participants focus upon a situation not previously encountered and if they stick with, or repeatedly return to learn about, this initially unfamiliar situation (encoded in their representation of the task environment as a topic, T); if, also, doubt of type (c) (Section 2.10) is zero, the entailment mesh representation is fixed and perspectives are fixed (so that doubt of type (a) in Section 2.10 is zero), then varieties of doubt (Section 2.10) labelled (d), (e), (f) , will, by hypothesis, vary in a particular way; moreover, on examining the records, there is evidence that the predicted variation does take place.

Recalling that (d)is a doubt about a description or class of prescriptions (plans), that (e) is a doubt about method, and that (f) is a doubt about otucomes, the covariation of standard estimates (distinct information measures) of the doubt. To illustrate the point, the doubt about a descriptive framework is estimated as the information measure attached to answering (by confidence estimation) "why" questions or "How do you derive questions" reduced to "which topic description" form. The doubt about method is estimated (similarly) over answers to "How" questions. Finally, doubt about outcomes is estimated (again, similarly) over replies to "which" questions, "which of several outcomes deemed by the participants to be possible" Hence, the covariation of these quantities, relative to one another, is of importance and not their absolute magnitudes. The hypothesised variation is directly inferred from the mechanism sketched in Fig 8 (and its elaboration in terms of <u>DB</u> and <u>PB</u> operations).

No situation can be tackled before it is described (and this is believed to be by an application of <u>DB</u> operations). Given a description the <u>PB</u> operations may build $\underline{\mathrm{Con}}_Z(T)$ and, at some point instant, this concept is trivially made up of one procedure (perhaps an imperfet one, since there is doubt about the result of using it, an outcome doubt) . The cycle in Fig 8 produces more procedures which do the same thing; $\underline{\mathrm{Con}}_Z(T)$ is the coherent collection of them all. Since they must work effectively (perhaps, under variable circumstances), doubt about outcomes will decrease and it continues to do so. As the equivalent procedures become more numerous, the doubt about method increases (the participant is no longer aware of <u>how</u> he does what he <u>does</u>).

The argument is different, of course, if circumstances change, so that a procedure which has worked, ceases to work. Either further procedures are created to enlarge the concept, or the situation is respecified as a distinct topic in the representation, with corresponding changes in other uncertainties (doubt about method and doubt about topic description). The latter transformation places the representation (or a part of it) in doubt, which is of type (c) (Section 2.10), and in turn, of type (b) (Section 2.10) thus contravening the underlying premises, namely, that doubt of type (a)(Section 2.10), does not exist; the entailment mesh is fixed and perspectives are fixed.

Setting these complications aside, but not forgetting them, consider a system time, t, and an interval of system time, Δt, starting at an origin of t_0. This interval, Δt, represents the interval within which, for one reason or another, a certain situation must be dealt with by the participants and is determined, in value, by the task environment. It is, as a rule, independent of the participants (for, if they could anticipate such contingencies and represent them adequately, they could usually avoid them).

What is the probability (here, in the traditional sense, not to be confused with doubt)# that the participants can resolve their doubt

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Zadeh's latest work convinces me that the use of probability in this context is improper, and that a possibility measure should be employed.

if Δ t has various lengths and various origins ie. that the participants can, in a realistic sense, decide under these constraints. Further, what mechanisms are involved, excluding, as unacceptable, the "guessing" of Section 2.10.(g).

2.11.1. How to Take Control

The thrust of the enquiry is directed towards the issue of when and under what circumstances control of a decision process should be handed over to a machine (like machine C, in Fig 6) rather than left with a person, or a team, who would otherwise "guess".

There are several aspects to this problem; for example, it is only possible for C to act competently in the decision process if an external and, thus, C-accessible representation exists, to begin with (it has been assumed that the participants do exteriorise their usually hidden conceptual operations, as indicated in Figs 5, 6, and 7; this assumption is realistic for the present empirical base, our Decision System, but it is not universally valid).

Another aspect is the extent to which the representation obtained is open to manipulation. The participants in Figs 1, 2, 5, 6, 7 view the task environment imaged in their representation as "it" referenced, just as the external observer in these figures regards the agreements, resolving A, B, doubt, as "it"-referenced events. Thus, to the participants, the task environment, or most of it, is "objective".

One of the results of our preliminary studies is that participants in a complex decision process do not (as decision theorists often suppose they do or suppose they ought to do) image objective events in a task environment in a way that is susceptible to standard probabilistic inference, and this proclivity is probably a large scale, and exaggerated, form of the non standard heuristics that Tversky and Kahneman have observed in the quite different context of small scale decision. (Recall, also, the comments in Section 2.10.(e)).

A representation which plausibly exteriorises A, B, conceptual operations is usually not amenable to a standard calculus of probabilities. This is trivially true of a mesh, because of its cyclic structure. Non trivially, it applies to prunings of a mesh under one, or several, perspectives. Even selective prunings are rarely tree structures (or sets of trees) as required by the probabilistic calculus, and insofar as there are analogical topics they are not trees. If machine C is to be compatible with the A, B, representation, and if its actions are to be comprehensible to A and B, then its heuristics (for any-but-local application, at least) cannot stem, directly, from standard probabilistic inference rules.

However, the appropriate design paradigms for C, may, to some extent, be characterised in terms of the conceptual styles of the participants which are discussed in the next section.

These points (the existence of an exteriorised representation, and the plausibility to the participants of the representation, and of any operations that C may perform) are prerequisites for handing control to C, whilst retaining the participants as decision makers in the same system.

2.11.2. When to Take Control

The question of when to hand over control to C, supposing these prerequisites are satisfied, can be examined in terms of Δ t

If, in respect of a situation, there is (as yet) no description then the decision maker (or, in general, team) may control the system rationally only if there is long enough to elaborate a coherent description (a DB operation, believed to be fast) and a PB operation (believed to be slower) to discover an appropriate procedure and the execution of this procedure. If a coherent description exists, but no procedure, only the PB operation and procedure execution are needed (however, there is believed to be slower than DB operation).

Suppose that one procedure (a trivial concept, just one method of doing what is described) then the execution of this procedure is like an algorithm or a heuristic and Δt must be fairly long to rely upon its effective execution. It may be better if the decision maker tells a machine what to do.

As the cluster of coherent procedures that make up a concept is enlarged so that there are many coexisting coherent and essentially parallel procedures (as in a well learned skill such as motor car driving) then the position is different. The decision maker may act rapidly, reliably and with great versatility. He cannot, however, tell a machine what to do, for in order to do that, the decision maker would have to apply both a DB operation and a PB operation and to execute it (as, for example, if a driver is suddenly asked to act as a driving instructor and create an algorithm or heuristic for use by the learner). Consequently, if there is an overlearned stable concept ("intellectual skill" or "skill") the action should be automatic, unconscious and robotic if Δt is short.

For decision in general, there is no objection whatever to this state of affairs. But two circumstances deserve attention. On the one hand, if the interval Δt is very short, it may be impossible, because of perceptual motor limitations, to execute the entire cluster of methods in time and with the customary, though unconscious, discrimination of conditions ie. with the artistry typical of a skilled racing driver, or, in the case of recognition skills, the innate expertise of a connoisseur in picking out the geniune Chippendale from amongst the fakes . On the other hand, since action is demanded, there is no way of distinguishing (excepting, perhaps, after the event),whether the real cluster of condition sensitive overlearned methods are being executed or whether an irrelevant algorithm is brought into play (a "guessing" operation) , to select between "alternatives" that happen to be apparent at the moment, but may not stand up as appropriate "alternatives" upon more leisurely and perspicuous examination. Both of the two circumstances arise in the context of high risk command /control/management situations, whether the action is manual or intellectual.

These arguments are founded upon the supposition that however many methods there may be to realise some conceptually described aim or objective, the decision maker (a person or team) has <u>one</u> perspective at once. Real situations are often not of this kind, manifestly so in the case of teams (where the team members have different points of view), but even in the case of one person where several coexisting perspectives are juxtaposed.

This is not a pathological feature of the decision system(whether it be one brain,or many; whether a person or team standing alone,or a man machine complex) . On the contrary,the stability and efficacity of the decision process arises out of potentially coherent,concurrent operation under many heads of control; McCulloch called this stable mode of organisation "Redundancy of Potential Command" and Grey Walter, in an early paper,called it "Abcission" (it appears under different names,and less explicitly,in his later work).

2.12. Decision Habits and Conceptual Style

Previous studies of learning and teaching have uncovered the existence of apparently ubiquitous conceptual styles. Under specific circumstances, dominant styles give rise to definite learning strategies; for example, if arrangements are made to secure the "understanding" of each topic in a subject matter (understanding, in its technical sense, is not unlike "completing a decision process") then learning strategies become polarised as holistic or serialistic.

Tests intended to provide an individual stylistic profile are fairly lengthy and quite unlike the usual psychometric instruments. The tests currently in use involve learning, encoding and recalling, a considerable body of data; they are most effectively computer administered and occupy about two hours or longer. As well as recalling a story like sequence, respondents are required to invent and to extrapolate, or interpolate, on the basis of the given data. At the moment, there are two matched tests with known interest reliability (due to the structured content, test/retest reliability is inapplicable. Due to the interdependency of the recall and predictive questions split half/reliability cannot be derived).

The tests are scored in terms of several variables, of which the following aggregates are of primary importance and immediate relevance.

(a) Comprehension learning
(b) Operation learning
(c) Versatility

Of these, comprehension learning is a direct index of description building,(of an individual's propensity to use DB operations, of the rate at which they are used,or their accessibility in the mental repertoire).

Operation learning is an index of procedure building (of an individual's propensity to use PB operations, of the rate at which they are used or their availability in the mental repertoire) .

Both types of mental act are required, by hypothesis, to learn and stabilise a concept,but people may be more facile in one respect than the other.

Since versatility,the last component,calls for concept production and reproduction the versatility score is only (and, by hypothesis, can only be) non zero if the comprehension learning and the operation learning scores are non zero. But these scores need not be high in order to achieve a creditable versatility score,and there are versatile people with a strong bias to comprehension learning,or to operation learning,or to both. Similarly, some people achieve high comprehension and/or operation scores combined with low versatility. For versatility, over and above concept stabilisation, calls for creative or predictive,activity and for an ability to exteriorise (and in that sense, impose upon the environment) a personal representation and a personal perspective. The variable is believed to be an index of how productive (over and above reproductive) are an individual respondent's DB and PB operations.

These tests are administered to subjects before and after participation in the ongoing experiments which use the Decision System of Section 2.4.

Some data are shown in Appendix 1 .

3. Decision Theory

When people agree to make decisions, they construct a representation of how they see a situation, and an activity or mission.

We have conceived this representation as exteriorised and will continue, for the most part, to discuss exteriorised representations. However, under more usual circumstances, the representation is private; it is a mental representation, in one or more brains. In either case, the representation is a personal theory some parts of which are agreed by other people. As such, the consequences are not usually obvious to the one or more theorists (decision makers).

The consequences only become evident when each theorist (decision maker) takes one or more points of view or perspective (in the exteriorised case, one or more prunings are generated; in general "things are seen from one or more points of view").

Taking a perspective is a prerequisite for converting the representation (theory) into concrete or intellectual action which may either be instigated by events in the task environment, or by an anticipated need. In either case, "a decision is made to act" but the decision process is the whole series of concrete and conceptual events, including the construction and assessment of a representation.

At least two kinds of event can be demarcated as articulating the decision process. The two kinds of event are as follows :

(a) Taking, and acting from, a perspective which does not change the representation.

(b) Changing the representation since it is inadequate or unsuitable. It is crucial to notice that "changing a representation" does not, commonly at any rate, lead the participants to "discard a representation" and replace it by another. The representation may be augmented as it stands or a fresh representation may be set up but the original representation can be retrieved.

In this respect, representations differ from formal scientific hypotheses; they apply to some and only some universes of interpretation, whereas a formal theory may purport to hold in any possible world (at least, this is often asserted of formal entities; it is the kind of myth that is harmless in pure science, and pure mathematics, but is manifestly counterfactual in the context of a decision process).

If, by some means or another, a full representation existed for the task environment, then there would be no need to decide (in the sense of this paper), for control could be relegated to a suitably programmed machine.

3.1. Constructibility of a Decision Theory

Is a full decision theory constructible using the conversation theoretic arguments put forward in this paper? If arrangements are made to exteriorise the decision process then it is possible to test hypotheses about aspects of this process and, within limits, to usefully regulate its conduct. There is, however, a logical peculiarity attached to the sharp valued observations of the external observer, in Fig 1, 2, 5, 6, 7, and as a result, about a nexus of statements in the meta-language, L*, which would make up a Decision Theory.

The peculiarity is as follows.

What is the status of a sharp valued observation of an agreement between A and B? Of course, it is true to say, "A has agreed a stable concept of T with B", as an L* expression, denoting the event in Fig. 11. But what kind of statement is this?

It is an L* analogy, "T(AB)" with similarity T* (which can be described in L*, as it is in L), but with distinction between A and B made by the external observer; namely, $\underline{\text{Dist}}_{OB}(AB)$. The minimal sharp valued factually true L* statement, about a conversation in L.

In other publications, it has been pointed out that L analogies are the static forms of real commands, real questions (ie. questions or commands that are active, that initiate processes, that are personally administered, to "you" and "I", rather than statements about questions and commands). The *reflective* quality *permits* the *extension* of commanding to desiring, wishing, persuading, and the like, the extension of questioning, to probing, enquiring, and the like. But, though relevant to the substance of a command and control process, there is no need to pursue these ramifications, at this juncture.

3.2. Formal Status

The main issue is whether or not Conversation Theory or any Decision Theory that might be derived from it, is a formal theory, and, to be strict about the matter, maybe it is not. Its minimal sharp valued L* statements are not propositions denoting facts but metaphors (strict, not sloppy metaphors), designating strict analogies. From a properly formal point of view, this is unsatisfactory. Although there is no denying the predictive and manipulative power of the "theory" it might be denied "theoryhood", by a purist.

Notice, that the external observer's justification for calling himself an external observer, and using a metalanguage L*, is that his minimal statements are analogical. He has the wit to use, and construct, analogy (his distinction of A from B). But so do A and B, themselves, who could, thus, assume the role of external observers.

Conversation Theory is a relativistic theory and a reflective theory, as a result of which its formal theoryhood (though not its usefulness) could be denied and the same comment applies to a Decision Theory derived from it. If (as I contend) Conversation theory, or its equivalent, is a minimal basis for Decision Theory, the same comment applies to any Decision Theory.

A relativistic and reflective theory has a logic of coherence and distinction; also, its models or semantic interpretations are many sorted # (the sorts distinguished by the Dist predicates). The minimal factual metastatements are about processes of agreement with many (dynamic) universes of interpretation; namely, those distinguished by the Dist predicates.

These matters cause no serious manipulative difficulties if the conversation, or decision process, is exteriorised in a representation that is common to A and B, relative to which the

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As, for example, the interpretation of a Montague intensional logic extended by Gergely, Nemeti, and Andreka, to accommodate actions, or events, in the many interpretation sets. As it stands, this scheme is insufficient, if used to image L expressions (as Montague images natural language expressions) for, in that translation, natural language metaphors are rendered, in an intermediate categorical grammar, as similitudes, and do not designate proper analogies. This defect may be remedied, and the group mentioned above, are pursuing the matter.

theory is formulated; the reflections of A on B and of B on A are well rooted, (the decision process manipulates the environment, in which they exist). One hesitates to generalise the argument to cases where (as usual, in practice), the representation is internal-to-the participants; it is not exteriorised but privately constructed, in their heads and minds.

Truly, I am not too diffident on this score, if only because any theory able to accommodate the kinetics of decision, together with a proper interpretation for value (in the sense of evaluation, utility, or worth) and the genesis (as well as the occurrence) of distinction, would suffer from the same formal difficulties.

Regarding kinetics, the approach through conversation theory is quite satisfactory. Regarding value, and genesis of distinction, its contribution has yet to be indicated, and the following subsections are devoted to these topics.

3.3. Value

The "peculiarity over the metastatements" in L*, could be rephrased to read, "they are value ladened."

This is not at all a limited meaning to give value or worth (whatever else, the quantity is a reflective one). But the constructions cited refer to agreements over stable concepts, that lead to concrete or intellectual actions. On this, we insist. Value does refer to sharp valued entities, stable agreements! So, this connotation of "value" and "worth" seems to have limited scope. At first sight, it does have limited scope, but the reference to a stable agreement (as in Section 2.9. and 2.10.) need not be direct . Fuzzy quantification is perfectly acceptable, so long as it is rooted in a sharp valued event.

The general question of value quantification is dealt with comprehensively in a recent paper by Johnson and Huber, "The Technology of Utility Assessment", IEEE Transactions of Systems, Man and Cybernetics, Vol SMC 7, No 5, 1977. One author (Dr Johnson) is contributing a paper, and M'Pherson (who has a recent report on the logic and methodology of value) is participating at the conference. In this paper, I confine attention only to certain aspects of preference and value reiterating that Fuzzy quantification of sharp valued events (ie. what is usually referred to as value estimation) is entirely legitimate.

Two sophisticated candidates are the multi-attribute utility scaling techniques (Hogarth and others) and the Exchange Grid technique, where a vector of evaluative descriptors is negotiated, as well as the values assigned to each, (due to Thomas and Shaw, based on Kelly's personal construct theory, Bannister and Mair, Bannister, Fransella and Bannister). These two techniques and others, somewhat less powerful, do not depend upon an absolute calibration of preference, which, though often attempted, is, in the following sense, impossible.

I do not know what the numbers <u>mean</u>; I <u>cannot</u> unless I <u>am</u> a participant, and suspect that comparing the numbers <u>involves</u> participation. They are relative preferences, open to negotiation (Bråten's dialogic says more than this; more cogently, and more precisely).

At least, preference calibration (and the derived calibration of value) appears to have limited application. In contrast, an index of preference, worth, or value, is justifiable if the events in relation to which the index is used are both sharp and warrant decision. Similarly, the "rational" manipulation of values, requirements of preference transitivity, and the like, have, it seems, local application only. The important point is that existing and elaborate schemes for fuzzily quantifying value are themselves defensible provided that they refer to a sharp event; in this theory, to an agreement over a stable concept; a process punctuated and maintained by distinction (also, a main point of Braten's scheme).

3.4. Genesis of Distinctions

It is easy to say that distinctions exist; sight is not sound, animals are not planets. These facile comments do not come to grips with the real issue of how distinctions are brought into existence or computed in a decision process.

Whilst writing this section, I had the benefit of discussion with Atkin and Nicolis, and seeing a draft of one paper. Hence, it is possible to approach the question in a, hopefully, mutualistic manner.

In order to address the basic question of distinction genesis, it is necessary to examine some non trivial connotation of the common phrase "self organisation", and to follow through the consequences. I first encountered this phrase, used non trivially, under Von Foerster, who defined a self organising system as one in which the rate of change of redundancy is positive, this notion being developed over the years, chiefly at the BCL. Nicolis, Protonarios and Benrubi have independently arrived at the same definition by an entirely different route (and, again independently, Caianiello provides a similar expression in this thermodynamics of modular systems).

In all cases an hierarchical structure is pre-supposed; specifically, Nicolis and Benrubi define a hierarchy as a set of distinct levels (each characterised by a distinct state space or phase space description, usually stochastic), such that, within each level, events are causally or multiple causally related (as in a Singer-type, producer-product relation) whereas simultaneous events at different levels are correlatively associated. Any given level in the hierarchy contains open non-conservative dynamical systems, generally far from thermodynamic equilibrium, which may, at a given level, be initially random. However, the system becomes structured (canonically, in terms of self sustaining, non linear, oscillators embedded in a background of noise, ie. random phasors) which are the components of the system at this level. However, the phasors are still subject to a form of noise, defined as additive (to the amplitude of oscillation) and thus, even at one level, the static trajectories pass through a series of stable modes. Phrasing it differently, the equations describing the system dynamics have singularities which, combined with a storage, or integrating, term, give rise to modifications in the state trajectories of the ensemble (a kind of trapping, but also adaptive, behaviour). I shall call these adaptive modifications "ordinary" (or "inessential") bifurcations, since this tallies with the usage of astrophysics, cosmology, and other sciences.

The "ordinary" or "inessential" modifications are compatible with the organisation of the hierarchical level at which they occur. As a result, they lead to adaptive behaviours but cannot do what must be done if the system is genuinely self organising, ie. to increase the degrees of freedom at this level by constructing a further level (in Von Foerster's terminology, changing the maximum entropy of the system as the system entropy increases towards that maximum possible value).

It has been shown, independently by each of the schools mentioned, that under specific and commonly prevailing combinations of additive noise input (from an environment), behaviour at a given level, and signals received from levels above and below (which constrain the system under scrutiny), certain essential bifurcations also arise. That is, the modifications are no longer compatible with stable modes of organisation at the given level. A novel representation must be developed and has the effect of increasing the degrees of freedom as required. The "catastrophes" of "Catastrophe" Theory (Thom, Zeeman, and others) are special cases of essential bifurcation points (in cosmology, essential singularities are of this kind).

One aspect of all these formulations is that a price is paid for their mathematical elegance; namely, that they are presented in respect of a fixed notion of temporality, randomness, etc, and that we cannot say, except by inspection of a real system, what the higher level variables, or descriptions, are (their creation, by any means, gives the added degree of freedom). My own formulation lacks elegance (it is in terms of production schemes, which are executed by a priori independent (or asynchronous) processors, which become dependent (or synchronous), insofar as there are stable agreements (induced by information transfer between them). It does not, however, require these assumptions.

To summarise the argument, up to this point, self organising systems, in the sense under discussion, are mathematical-descriptions of reality, either concrete or intellectual, and lead to simulations under the suppositions about temporality, randomness and stochasticity noted in the previous paragraph. These self organising systems are also possible descriptions of ...y "production schemes", and it is important to note that essential bifurcations arise, in simulation (with all its restrictions) and are a description of the "genesis of distinctions". It is also of interest that the conditions under which such bifurcations arise, in a simulation, are descriptions of stable productive, as well as reproductive, systems, information transfer and agreement between them.

Bråten, in several papers, underlines the difference between simulation and reality, since, for many years, he has approached his "dialogical" systems from two directions; on the one hand, computer simulation of society, as done by an external observer (a simulation); on the other, the modelling of one participant by another as the two engage in dialogue (a reality). In somewhat different terms, he comes up with a logico-mathematical necessity for the generation of distinctions; that agreement feeds on distinction making; that the flux of dialogue, between real participants, provides an indefinitely large number of real distinctions.

Atkin (1973, 1977), has a different approach to this matter, though, in the end, he comes up with compatible conclusions. Basically, Atkin regards a representation as being generated by relations between things (set members) rather than things being located in an otherwise empty space (or time), either physical or abstract. His scheme includes an hierarchically arranged series of relational structures or "backcloths" that are relatively invariant in time (another relational structure). Upon the entire structure (representing a complex of topological simplices) there is a "traffic" of events, some of which is stable (in the present sense, of productive and reproductive).

As a thought-experiment, replace the state space description by an ordered class of "backcloths" ; let those "backcloths" also stand for my entailment mesh representation*

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* I fully appreciate that the correspondence is not direct. for example, the hierarchy of backcloths is not exactly the hierarchy of spaces; things (or vertices) are not exactly nodes in an entailment mesh. On the other hand, I am prepared to render the correspondences definite at some appropriate juncture.

Atkin argues that decisions are of at least two types which, under this loose translation, correspond to inessential bifurcations and essential bifurcations; namely, circumstances in which decision makers select amongst patterns and forces exerted in a given backcloth and circumstances under which it is necessary to change the backcloth (or more informatively) for the backcloth to change. Although there is still liberty, the set of possible changes is restricted. These changes of the backcloth create distinctions.

The two categories of decision (bifurcation resolving) characterising both the "self organisational" and the "structural" approach appear, again in loose translation, as :

(1) The occurrence of situations in which the decision making participants can do nothing better than (at most) modulate the consequences of the plans and descriptions implicit in their representation of the task environment; when, in fact, machine C, of Fig. 6, should take control of the situation (it will do better than a "guess"). No information need be provided, the process leading to action is automatic, and no essential distinction is generated.

(2) The occurrence of situations in which the representation must be changed by reaching a fresh stable agreement between the participants and where at least one essential distinction is generated (the Dist predicate of an analogy) .

We are accustomed to thinking of the world as a gigantic serial machine or a gigantic parallel machine. Hence, when speaking of distinctions that are generated or computed we incline either to a Turing image (a finite state machine and an infinite storage medium) or in descriptive equivalence, the class of differential equations. In either case, we have a point-interval-ordering for process and a notion of dependence, independence, and the stochastic. #

My proposal for tackling distinction hinges upon this picture. First, I replace the infinite tape or storage medium by an infinity of stacks of a priori independent processors, bearing similar names in each stack and different names in different stacks, Let some computation start,(it could be on one machine). If an essential bifurcation is encountered in the dynamics of the process undergoing execution, then differently named (defined independent) machines or processors are selected from the stacks, as many of each kind as are required to resolve it, and the juxtaposition of their (different) names is a <u>distinction</u> generated from pure independence. The trick is that,once brought from stack

(a) Similar named machines may communicate if the process they execute permits it, for example, if a production allowed by the production scheme under execution, has argument values provided.

(b) Machines with different names may only communicate if they do similar things (are used to execute similar processes) and if their distinction is announced.

(c) "Communication" means information transfer or "local synchronicity" (defined dependency).

(d) The stochastic is derived from the modes of coupling between these modular machines insofar as that coupling is derived (using a logic like Milne and Milner or Byshovsky) as necessary in order to maintain a stable process.

#Atkin avoids the fixed ordering and offers several notions of probability distribution (from which types of stochasticity emerge).

(e) Independence (as of orthogonal coordinates) means the existence of a distinct machine, or a communicating group of machines that are rendered coherent (even though they started out as independent), because of the process they incarnate.

(f) Distinctions are generated (and announced, here, as addressing statements) in order to maintain stability in the face of agreement (increasing similarity and the requirement of coherence amongst the machines that compute).

(g) Conversely, coupling (the mechanical parallel of agreement), resolves distinctions; if the coupling (alias agreement) depends upon a similarity in states achieved by a process, then it is represented as analogy of form; if it depends upon a similarity in computation, it is an analogy of method.

4. The resolution of disruptive distinctions

We are in a position to reach the main conclusion or to restate and put together conclusions that are latent in the preceding discussion (in Section 2, and the earlier part of Section 3); to comment upon what a decision process *is*.

Distinctions are resolved by stable agreements (external to, or internal to, a brain) that are of various kinds but all of which are *represented* as analogical relations.

A decision process permeates the participants and their task environment. It is the resolution, by stable agreements, (*represented* as similarities in analogies) of distinctions that are disruptive, ie.

they constitute "bifurcations", either in the dynamics of thought or of the task environment.#

This dictum can be regarded as a condensed statement, "most decision processes are a form of analogical reasoning". As such, it is merely a reiteration of dogmas already advanced, in Section 2.10 and 2.11. Thus, all of the types of doubt (a), (b), (c), (d), (f), (h) noted in Section 2.10 are resolved by analogical processes. The doubt resolving operations of (e) in Section 2.10 are directly task specific and procedural, they are, preferably, well learned (these are ingredients of a decision process, but do not <u>constitute</u> a decision process any more than the operation of a computing machine <u>constitutes</u> a decision process). The guessing operations of (g), in Section 2.10 are excluded from "real life" decision on somewhat different grounds, though they have a place in "contrived situation" decision making.

However, a more illuminating and slightly more profound interpretation is possible, namely as follows.

The "disruptive" distinctions, to be "resolved" in a decision process, may be <u>inessential</u> or <u>essential</u> (in the sense of Section 3.4).

In the inessential case, their resolution, by a decision process, implies that one configuration is recognised as similar to another or that the similarity is abduced, or invented. It may also

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The idea of "choice" as essentially "imaginative" creative or inventive action is also mooted by Shackle, CLS, (1969) <u>Decision and Action in Human Affairs</u> Cambridge University Press. However, Shackle's key ideas of possibilities as invented (rather than given as <u>alternatives</u>) and of surprise value refer to one human decision maker.

be the case that some applicable method or plan is recognised as similar to another, or that a novel plan is invented. There is no change in the underlying representation, or, to phrase it differently, the task environment retains an analogous structure and an adequate representation may do so, as a result. Deduction and induction are sufficient for the recognition of analogies, though abduction is not excluded.

So far, we have been chiefly concerned with the resolution of disruptive distinctions (those that are responsible, in a given situation, for instability) as they exist or are created between independent entities (participants A, B, ... in the context of a task environment, or perspectives A, B, ... from which the task environment is viewed). A decision process, implicating A, B, their representation of the task environment and the task environment itself "goes on". There is a precondition that A and B maintain their integrity or autonomy, implicit in this verbal statement (though made explicit in the organisationally closed production schemes of Figs. 9, 10, 11, 12, 17) and this precondition is usefully spelled out in greater detail.

Just as A is not B, so A, at time t_1, is not A at time t_2 (where $t_2 > t_1$). Similarly, B, at time t_1, is not B at time t_2. That is "different instants of time" are held apart by a fundamental, unqualified distinction, like "independence". Loosely, one says A (t_1) is "the same participant (perspective)" as $A(t_2)$; that $B(t_1)$ is "the same participant (perspective)" as $B(t_2)$, meaning that some similarity, or morphism between processes or products, is preserved over a distinction in time; Dist (t_1,t_2). Taken together, the similarity and the temporal difference are represented as an analogy with Dist (t_1,t_2) as the distinction; that is $A(t_1)$ is analogous to

$A(t_2)$ or $B(t_1)$ is analogous to $B(t_2)$. For A and B the required similarity is preserved by the reproductive processes of the production schemes (Figs. 9, 10, 11, 12, 17).

Precisely the same temporal distinction applies to the task environment (which at t_2 is analogous to, but not identical with, the task environment at t_1) and to the representation employed by A, B.

Such an invariance characterises a decision process concerned with inessential disruptions. Changes are open to compensation by action that establishes, or is based upon an analogy between the existing state of affairs and some other state of affairs. Surely, variables in the task environment change value, but the essential structure either of the task environment, or its representation, is not modified. Further, if the decision makers remain individually characterised under the changes of perspective needed in order to recognise analogies and to reach an agreement, there is no essential change in their structure.

For the case of essential disruption, the structure of the task environment does change between t_1 and t_2, in such a way that if the representation is to remain adequate, then the representation must be modified (change in the entailment mesh, the backcloth, or the phase space representation)*. Failing this, the bases for

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* Dr Johnson, in his paper at the last Richmond Conference, expressed preference for a rough and ready (but, all the same, useful) distinction between strategic as against tactical and operational decisions, crediting the distinction to Ward Edwards. It is tempting to identify operational decisions as "no decisions at all", ie. operations carried out automatically or by man, in a robotic mode; tactical decisions with reaction to inessential disruptions and strategic decisions as reaction to essential disruption. It is easy to find cases where this identification does fit and indicates a commonality of approach to command and control systems. How generally the identification can be applied, is undetermined.

decision would be violated (the decision process would refer to an imaginary task environment, rather than the actual one) if, in other words, there is to be an ongoing identity (A (t), A (t + 1); B (t), B (t + 1)) between epochs, then a different structure is required. Such a change involves, whatever its form, the <u>creation</u>, not just the <u>recognition</u>, of an analogy and thus, in turn, abduction or invention.

It is equally valid to contemplate the converse process; where the essential change is due to a bifurcation in the mental dynamics of A, B, ... (even though the structure of the task environment does not change). A decision process involves all of these components: A, B, ... the task environment and their representation of the task environment. It is this process which is subject to inessential or essential disruptions.

The reader may regard this construction of a decision process as plausible, so far as complex systems are concerned, but far fetched, or, at any rate over complicated, in the traditionally cited "simple" cases which surely exist. At least, that is my own immediate reaction. How does all this business about recognising and using and creating analogies fit "throwing a dice", for example, or "responding selectively to multiple choice alternatives that are given?"

The answer to this rhetorical question is "it fits perfectly", though, until Larry Phillips expressed his own disenchantment with the "simplistic" ideas of choice and decision, I scarcely felt justified in developing my own thoughts along similar lines, let alone voicing such outlandish sounding opinions. But Phillips has more experience than most of us with behaviour in the face of "simplistic" questions and selections; his studies of intercultural differences call for massive comparisons between response profiles to large numbers of well structured items and he also concludes that decision, even to these items, is not nearly so simple as we are brought up to believe.

On closer scrutiny, what really does take place if a dice is thrown and someone is asked to (and does) cite the chance that one of 6 numerals come up. What really does happen when teenagers are forced, with multiple choice questions, into one and only one correct response?

Well, for the dice, a respondent is asked to place a concrete situation in correspondence with a mathematical situation, (he may be persuaded to do so by the dubious, if not phoney, argument that the mathematics is just a formal expression of "vast experience with dice" or "repeated trials"). The mathematical situation consists in a set of exclusive (ie distinct) and exhaustive alternatives, conveniently indexed by the integers, and a covert existential statement that (on any trial) one and only one event with occur. Thus, right at the outset, the respondent must accept and recognise an analogy between an abstraction and the concrete dice on the table, which <u>is</u> thrown. Further, the successive trials are seen to be <u>isomorphic</u>, but <u>distinct</u>, the labelling integers being in one to one correspondence with the numerals inscribed upon the faces, though they are all independent (no <u>other</u> analogy, just <u>isomorphism</u> and the <u>distinction</u> imposed by a point internal sequence).

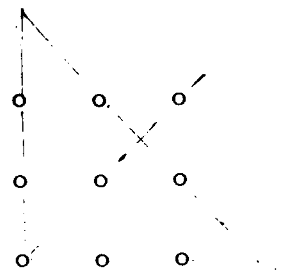
If the dice is not loaded, then, by analogy with an imaginary experiment involving repeated throws, most respondents say "the chance is 1/6 for any face" and "it is so for each trial"; at least, that is the standard analogical resolution (or, if the dice is biassed to favour certain sides, then bias is expressed in terms of probabilities). These, of course, are the standard replies, not necessarily the commonest, for respondents find it difficult to believe in truly unbiassed and truly independent events. But, without the mathematical/concrete analogy of <u>form</u>, the choice situation makes no sense . The standard resolution of the distinction (between the one face at once or between successive trials) depends upon an analogy of method (either with ordinary throwing or, in the case of a "superstitious" resolution, with a "demon" having eccentric behaviour.

For multiple choice questions, in tests or examinations, the situation is more complicated. If the respondent is certain about the correct answer, then the question form is irrelevant (provided that the answers are sensibly devised); a statement is simply recognised. On the other hand, if the respondent is uncertain, the question form is relevant and (if one and only one answer must be given) is recognised as analogous (in fact isomorphically analogous) to the exclusive and exhaustive formalism or (equivalently) to the dice throwing situation, insofar as the respondent believes that one (and only one) answer really is a correct answer.

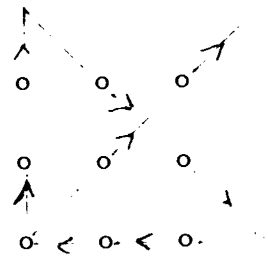
Under these circumstances and given this structural recognition as a prerequisite, resolution typically proceeds by analogical reasoning with respect to all of the alternatives (A, B, C, D, E); for example, that A is known not to be correct, that B and D are similar whether correct or not, so that, since only one alternative is correct, neither B or D are contemplated; that, in complete ignorance of D and E the respondent selects E, because D has the designator of alternatives, in other, adjacent, questions, which were known to be correct and because of a "superstitious" (or, maybe a well founded) belief, that "correct" designator patterns are not repeated in the same block.

The dice and the multiple choice situations are typical of inessential and highly engineered, dispruptive bifurcations. The familiar "9 Dot Problem" presents an essential, (though also engineered) essential disprutive bifurcation, to anyone who does not know the solution.

In the "9 Dot Problem" the respondent is presented with a square 3 x 3 array of dots, about half the size of the page, and is asked to join all the dots by 4 straight lines, without removing the pen from the page. This display is intended to induce a mental set, or frame of reference, such that any lines constructed lie within the 3 x 3 array. If so the problem is analogous to a class of "closed minimisation" problems, and is insolvable until a fresh analogy is created; namely, a set, or a frame of reference, which is the entire page, allowing for the solution



This is completed, by analogy with the "where to start on a tour" class of problems, namely



To summarise, the mechanics of a decision process capable of resolving distinctions of a kind that are liable to impair the stability of organisational closure (in contrast to those which must be computed, to maintain and secure stability), is an agreement or coherence mechanism, <u>represented</u> by means of an analogy. For <u>inessential</u> disruptions an analogy of <u>form</u> and <u>method</u> is <u>recognised</u> and an appropriate process is set in motion. For <u>essential</u> disruptions an analogy (of form, at least, possibly of method) must be created, de novo. # The apparently straightforward conclusion, that decision is primarily a matter of analogical reasoning, has been approached by quite a devious route, but shortcuts would have left the decision process, in some respect, enigmatic.

In fact, this process is complex and fairly subtle. The reasoning may be exact, or it may involve imprecise (fuzzy) descriptions and concurrent operations. Whatever else, it is not simply "guessing" in the usual sense of this word, ie. "weighing up predetermined alternatives on a kind of psychological balance beam", though some very restricted types of decision may be viewed as if they were "informed guesses".

5. Responsibility

For some while it has been intuitively clear that, insofar as decision makers learn and execute a generalisable skill, this skill fits best under the rubric of responsibility. Given equal opportunity to exteriorise their representations, participants who exteriorise complex, far reaching, and coherent representations are placing their foresight in a quasi public domain; as a result they are assuming more responsibility for the task environment than others, who fail to do so.

This section was written last of all. Whilst it was being typed I had the opportunity to read Gaines paper and found him in agreement over the importance of distinction and the role of analogical reasoning. In particular, Gaines category theoretic representation of analogies is compatible with the present representation of analogy creation, though it is independent and expressed in rather different terms.

Maturana and Varela call the quasi public domain a "consensual domain", I use "conversational domain". The differences are unimportant, since, in Maturana's and in Varela's formulations, "consensus of opinion" is a mutual sharing of beliefs; that is, an agreement or coherence (not simply a matter of vote casting). The arena in which the beliefs encoded in this representation, are exposed to debate may be a colloquium of peers, of fellow managers, or of authorities in government, or military superiors. But it could be society at large, or even an entire civilisation . There are no special restrictions.

The equality of opportunity is important. If a manager is called upon to decide about an investment as soon as he occupies his post, little can be said, on that evidence, about his responsibility; even provided with means for exteriorising his representation, he has no opportunity to do so, and any judgement of his action must be based upon his record in other posts. However, insofar as managers A and B both have the opportunity to exhibit their skills over several years, (say) a relative assessment of responsibility can be based upon the relative complexity and coherence (in some, rather inadequate, sense) of the representations they produce and use.

Attempts to quantify responsibility have been hampered by our "rather inadequate" sense of a measure on the representation. The issue is not simple. It involves not only the representative entailment mesh but its prunings and the processes they determine or permit (which as noted in Section 2.11, and Section 4.6) cannot, usually, be analysed by the standard rules of probability calculus and linear temporal succession.

In contending that a "general art of decision" is equivalent to "the art of taking responsibility" I had the hunch that an index of this quantity lies in the "size" of events (in the representation) imaging the "size" of concept entertained by the decision maker. A further hunch was inspired by the empirical work of Elliot Jacques, on time span measurement and the relations between the desired occupational time span (unsupervised interval before remedial feedback) and degree of responsibility . In the Glacier Project, where most of Elliot Jacques' studies were carried out, the responsibility component is equated to the salaried value of the post. In the present case, the length of process that might be derived from the mesh, the time overlooked, the time spanned, back and forth should provide a first order index of responsibility.

Here, the draft of Atkin's conference paper, and some earlier work, which he showed to me about a year ago, came to the rescue. Suppose it is possible to map a representation onto an appropriate hierarchy of Atkin's relational structures (which it is, though the most desirable method is undetermined). If so, Atkin provides a first order measure (namely, the projection of an ordering of patterns over vertices, which corresponds to the maximal system time) onto a point interval Newtonian time (namely, a 1 simplex). It does not end at that, for a family of indices are derivable, and appear to be precisely what is needed to convert a "rather inadequate" quantification scheme for "responsibility" into an adequate one.

A proper specification of responsibility, combined with an operational realisation of the laboratory decision system, (and several kinds of realisation are possible, as indicated in Appendix 2), appears to have consequences far reaching and pragmatic enough to be taken seriously ; at any rate, to justify this slightly unorthodox essay upon the theory of what a decision process is.

Appendix 1

The data contained in this Appendix is intended to illustrate all of the often non standard types of result in the paper.

First of all the records typical of the team decision task are shown in Ap 1.1 and in Ap 1.2; they are extracted from the final technical report of Grant DAERO 76G069 where detailed decoding is provided; here it would be gratuitous to say which numbers mean, for example, number of "Klingons" or "Spacecraft Energies" or other identifications needed for decoding the recordings. The Behavioural or State variables appear on the left, the interrogation session responses (referring to that interval) on the right and, in a typical experiment there may be a dozen summaries of which two are shown.

Next Ap 1.3 and Ap 1.4 are two typical but quite different planning meshes. Prunings and selective prunings of the first one are shown in Ap 1.5.

Finally Ap 1.6 indicates the characteristics of the test for conceptual style as used in the context of decision making.

```
SESSION N7.
 0        0        71821    19898    1500
 0        1        64566    19745    0
 0        0        76110    19742    0
 0        1        77797    19814    0
 5        13       2145     0        6500
 2        2        13765    0        0

COMMANDS
MGIMGIMGMI
  2  3   7
CRACKS
 0   0   0   0   0   0   0

TACTICS

 0        2        3        0        0        0
 0        1000     0        0        0        0
 0        8        0        0        0        0
 0        1        9        0        0        0

 0        2        3        0        0        0
 0        1000     0        0        0        0
 0        8        0        0        0        0
 0        1        9        0        0        0


 0        1        71821    47460    500
 0        1        64566    48994    8000
 0        1        76110    74255    0
 0        1        77797    52474    0
 2        6        8640     0        4500
 8        0        11456    0        0

COMMANDS
4MDI4MII4M
       9
CRACKS
 0   0   0   0   0   0   0

TACTICS

 0        2        3        0        0        0
 0        500      0        0        0        0
 0        2        0        0        0        0
 0        3        0        0        0        0

 0        2        3        0        0        0
 0        500      0        0        0        0
 0        2        0        0        0        0
 0        3        0        0        0        0
```

```
SESSION N7.

QUESTION  1   PART  1
29
5
QUESTION  1   PART  1
3
10
QUESTION  2   PART  1
8000
7
QUESTION  2   PART  2
10000
7
QUESTION  2   PART  1
9490
10
QUESTION  2   PART  2
11450
10
QUESTION  3   PART  1
3
3
QUESTION  3   PART  1
NA
10
QUESTION  4   PART  1
Y
6
QUESTION  4   PART  2
Y
5
QUESTION  4   PART  3
N
7
QUESTION  4   PART  4
N
9
QUESTION  4   PART  5
N
8
QUESTION  4   PART  1
NO
10
QUESTION  4   PART  2
NO
10
QUESTION  4   PART  3
YES
10
QUESTION  4   PART  4
YES
10
QUESTION  4   PART  5
NO
10
```

```
QUESTION  5   PART  1
A OR CO RO D
9
QUESTION  5   PART  1
NA
10
QUESTION  6   PART  1
NA
1
QUESTION  6   PART  2
NA
1
QUESTION  6   PART  1
STARBASE C
10
QUESTION  6   PART  2
TOTALLY
10
QUESTION  7   PART  1
2
9
QUESTION  7   PART  1
2
10
QUESTION  8   PART  1
I FORGOT TO LOOK
9
QUESTION  8   PART  1
48000
10
```

Appendix 1.2.

SESSION N0.

0	0	90536	19898	1500
1	1	62093	19745	0
0	0	88573	19742	0
0	1	93879	19814	500
5	13	21200	0	7500
2	2	13765	0	0

COMMANDS
MGIIGMIMGI
28 6 3
CRACKS
0 0 0 0 0 0 0

TACTICS

0	2	3	0	0	0
0	1000	0	0	0	0
0	8	0	0	0	0
0	2	11	0	0	0
0	2	3	0	0	0
0	1000	0	0	0	0
0	8	0	0	0	0
0	2	11	0	0	0

0	0	90536	47460	500
0	1	82093	48994	8000
0	1	88573	74255	0
0	0	93879	52474	0
2	6	3440	0	5500
11	0	11206	0	0

COMMANDS
DIIIMDMIMD
24 8
CRACKS
0 0 0 0 0 0 0

TACTICS

0	2	3	0	0	0
0	500	0	0	0	0
0	2	0	0	0	0
0	3	0	0	0	0
0	2	3	0	0	0
0	500	0	0	0	0
0	2	0	0	0	0
0	3	0	0	0	0

SESSION N0.

QUESTION 1 PART 1
N
9
QUESTION 1 PART 2
N
9
QUESTION 1 PART 1
NO
10
QUESTION 1 PART 2
NO
10
QUESTION 2 PART 1
NA
9
QUESTION 2 PART 1
NO
10
QUESTION 3 PART 1
N
9
QUESTION 3 PART 2
Y
9
QUESTION 3 PART 1
NO
10
QUESTION 3 PART 2
NO
10
QUESTION 4 PART 1
Y
7
QUESTION 4 PART 2
N
3
QUESTION 4 PART 1
YES
10
QUESTION 4 PART 2
NO
10

QUESTION 5 PART 1
NA
9
QUESTION 5 PART 2
NA
9
QUESTION 5 PART 1
STARBASE A
10
QUESTION 5 PART 2
COMPLETELY
10
QUESTION 6 PART 1
INCREASE
10
QUESTION 6 PART 1
INCREASE
10
QUESTION 7 PART 1
NA
10
QUESTION 7 PART 1
NA
10
QUESTION 8 PART 1
NA
10
QUESTION 8 PART 1
3
10

Appendix 1.3.

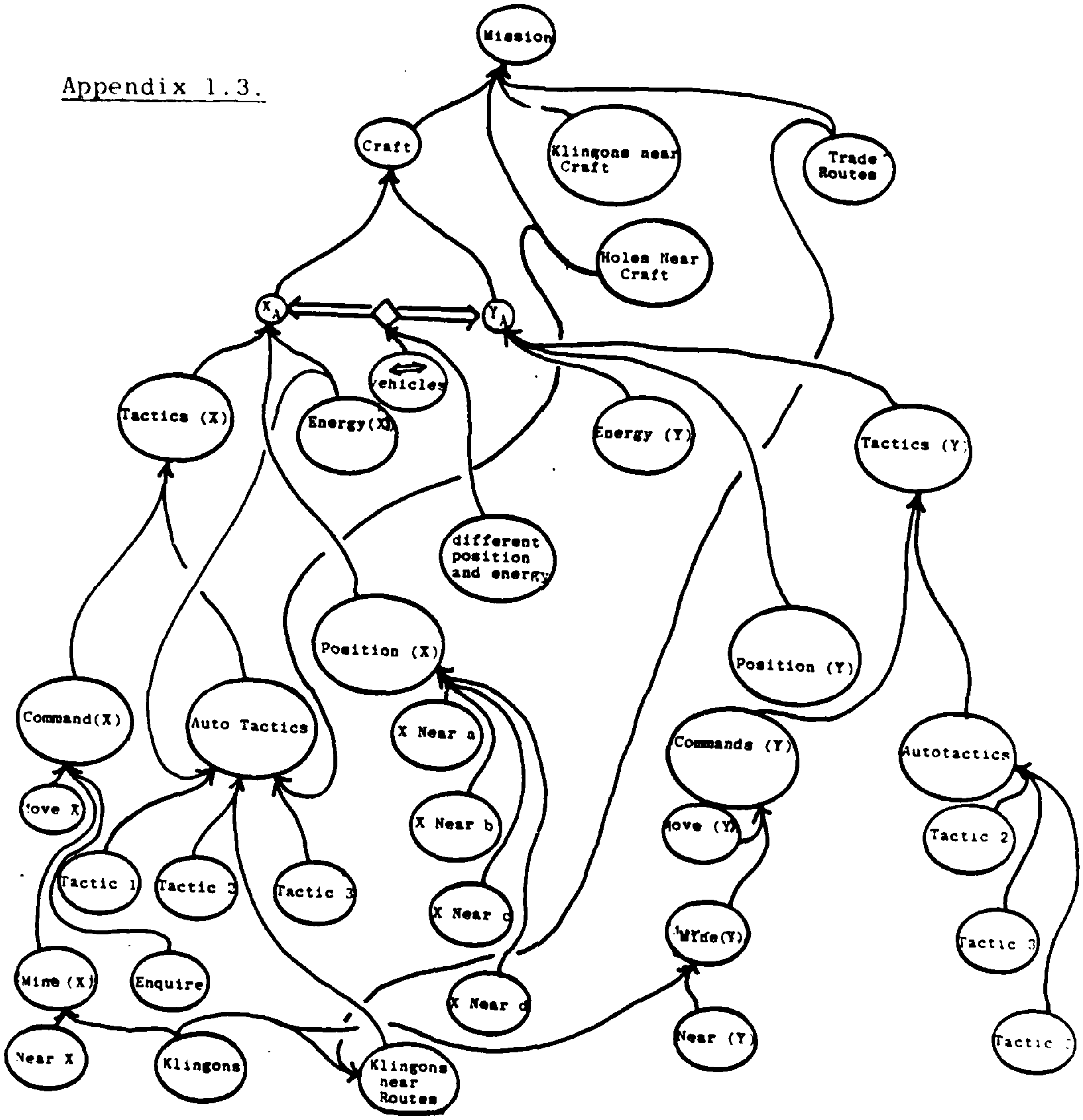
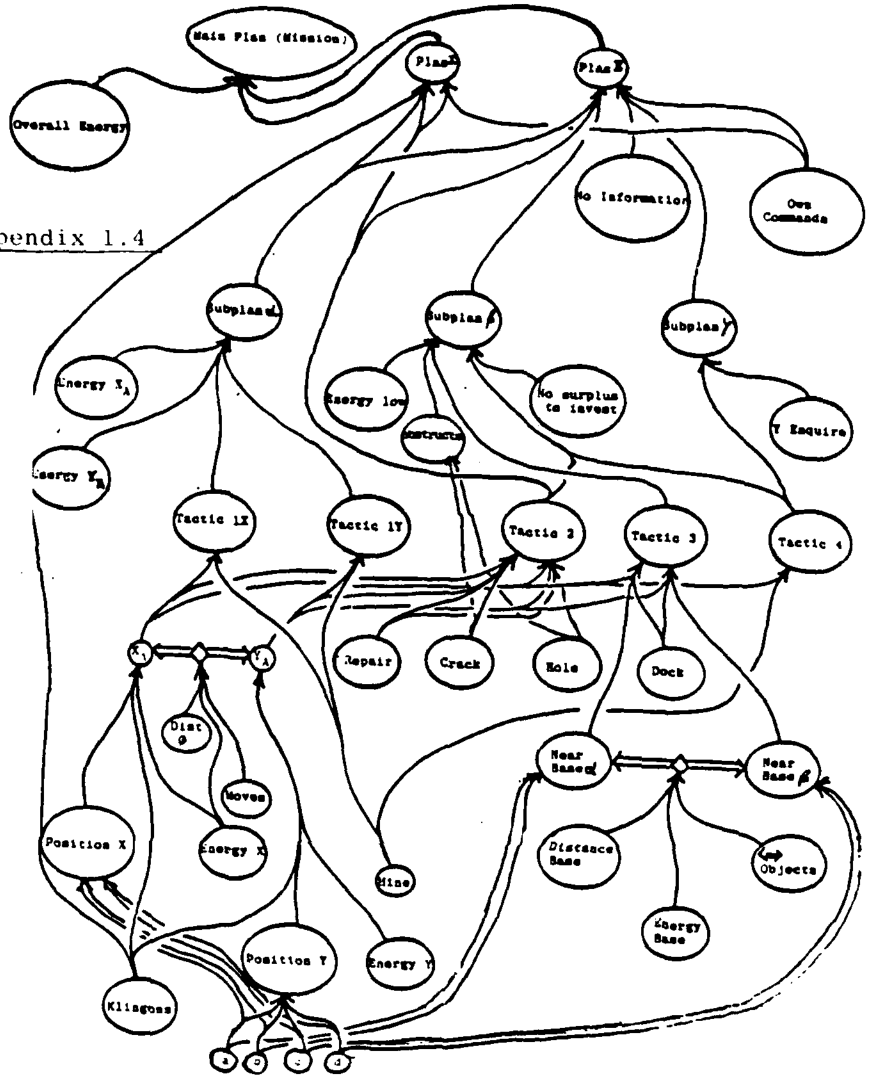
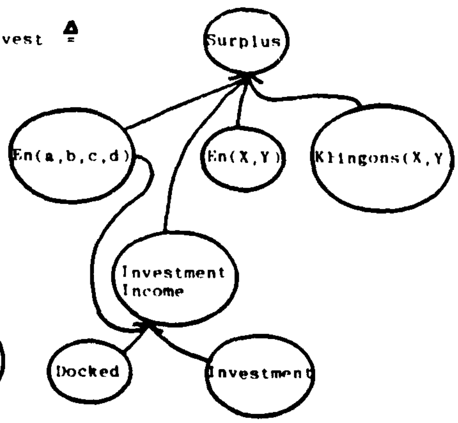


Fig 11(A). Planning Entailment Mesh Constructed by a subject regarding Spacecraft and mission as having tactics that are used by the Spacecraft in the mission.

Appendix 1.4



Notes: Surplus to Invest $\triangleq$



Overall Energy $\triangleq$

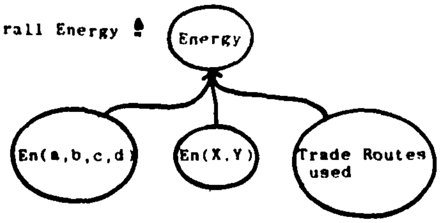
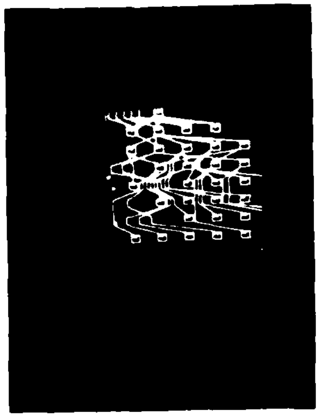
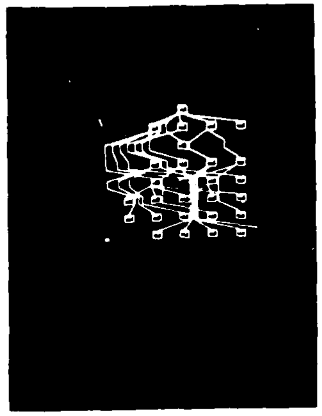


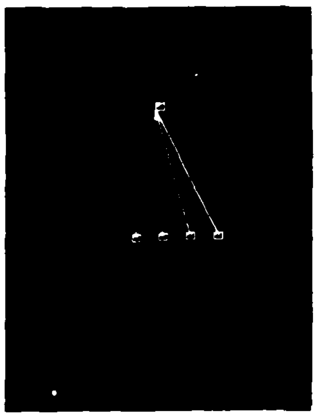
Fig 11(3) Planning Entailment Mesh Constructed by subject seeing plans and tactics as using Spacecraft for achieving goals and other objectives



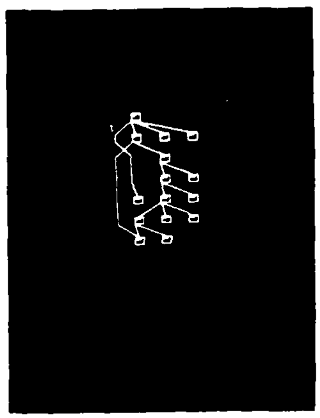
I Pruning From Node 19



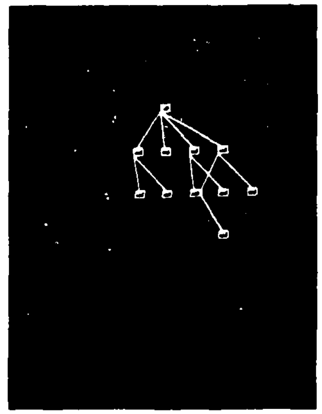
II Pruning From Node 11 (Spacecraft)



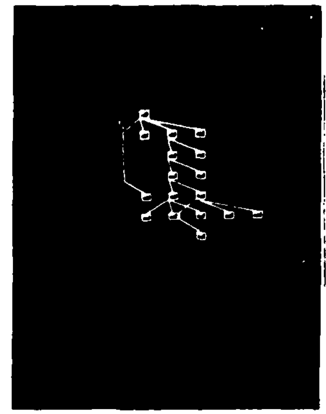
III Alternatives that are presented for selective pruning decision at Node 5 (mission)



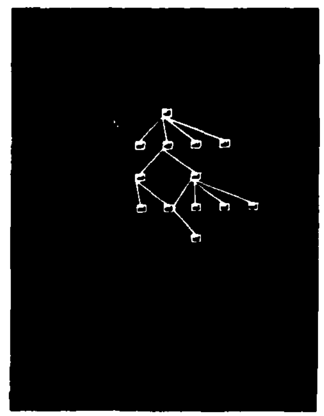
IV One derivation (plan) exhibited under Node 5 (mission)



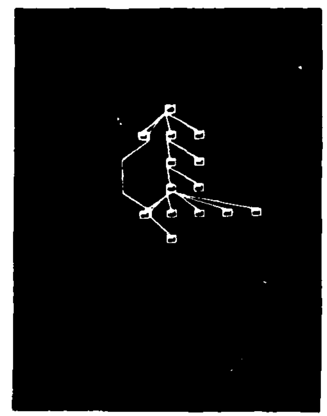
V Another derivation (plan) exhibited (selectively) under Node 5 (mission)



VI A derivation (plan) under Node 33 (autotactics and tactic set)



VII A derivation (plan) under Node 14 (Trade Routes)



VIII A derivation (plan) under Node 44 (Klingons near Trading Routes)

Prunings and Selective Prunings carried out on an Entailment Mesh of Fig 11A.

Appendix 1.6

Subject Number	N	O	C	V	a	b	c	d	e	f	g	h
1	81	78	77	65	80	66	61	50	90	17	53	80
2	22	12	53	0	6	57	5	4	11	18	46	63
3	85	53	63	47	64	74	18	27	70	24	60	85
4	64	64	33	8	24	20	58	6	5	26	61	50
5	88	65	76	35	60	80	54	22	67	32	70	84
6	76	50	63	34	60	81	47	20	63	40	66	90
7	50	79	55	46	70	79	78	35	64	15	48	85
8	85	60	56	33	49	75	60	26	58	18	69	57
9	83	52	45	46	75	34	59	33	57	17	61	86
10	53	12	24	0	6	12	2	2	12	27	57	49
11	55	54	32	21	30	11	44	16	29	22	58	60
12	60	84	83	67	90	78	81	51	88	43	49	92
13	83	53	48	41	52	65	19	45	60	61	91	75
14	88	78	77	0	8	60	61	3	5	15	82	47
15	55	32	75	80	88	90	40	55	95	50	58	89

	N	O	C	V	a	b	c	d	e	f	g	h
Means	68.53	55.07	57.33	34.87	50.67	58.8	45.8	25.67	51.6	28.33	60.4	72.87
SDs	19.16	22.28	18.51	25.32	29.38	26.58	24.56	18.24	31.34	14.17	19.39	16.54

Table 2: Data from 15 subjects under revised conditions and scores of the subjects' neutral, operation, comprehension and versatiloty scores are recorded. As these aggregates are, so far, the most reliable estimates when judged against a large sample test and retest population (Appendix 4). The Spearman's rank correlation coefficients are shown together with the scores on the tests and various properties, both of behaviour and answers to questions. Critical values of Spearmans Rank Correlation for n = 15 are 0.456 (for $0.05 > p$) and 0.645 (for $0.01 > p$)

<u>Key</u>

Stylistic Tests

N = Neutral score "Spy Ring History Test"
O = Operation Learning Score "Spy Ring Hisotry Test"
C = Comprehension Learning Score "Spy Ring History Test
V = Versatility Score "Spy Ring History Test"

Questions correctly answered. These scores have a necessarily evaluational component in determining if an answer is correct or not.

a. = Plans or hypotheses stated that are subsequently followed up, successfully or not, (see h)

b. = % Correct Klingon position predictions.

c. = % Correct density or energy estimates

d. = % Correct response to where are ships and objects in space.

e. = % Time operating ships in parallel

Behaviours

f. = % Time operations concurrently by mean number of operations per hour

g. = % Correct density or energy estimates

h. = Number of plans or hypotheses stated

Appendix 2 Some correspondences between conversation theory and the theories of R Atkin and other participants in the meeting

In order to perform mathematical operations, it is necessary (in a sense) to freeze events or processes and regard them, for manipulative purposes as static objects (for example, members of a set). There is no objection whatsoever, of course, to a set of events but, upon manipulation, the set members must act like simple elements. Nor, of course, is there any objection to depicting process in mathematics, provided there are elementary components (so that, for example, events can be represented as a relation belonging to the Cartesian product of several sets, or any set with itself)*. This, however, is beside the point. The choice of set members (in Atkin's case the criteria of hard data and observability) are not, in themselves, mathematical, in *the ordinary sense of this word*; in this context, of course, Atkin is speaking as a philosopher of science or a practitioner.

In contrast, "choice criteria" and "process type" are very much in the province of conversation theory, which is not a mathematical theory (I have even, in the paper, conjectured that the purist might deny it theoryhood). Yet much could be gained if conversation (theory or not), could import Atkins theoretical structure in an other than arbitrary manner, and it is conceivable that the benefit would prove to be reciprocal. Hence, I shall attempt a translation.

* The point is that Atkin is primarily talking of relations and the sets [illegible] of elements or observations are coordinates of the Cartesian Product [illegible] a relation [illegible]. The legitimacy of [illegible]ing process is given by the familiar observation that for [illegible] or [illegible] as a result [illegible]. Hence, processes are reproduced in an hierarchy of cover sets or types. The legitima[illegible] does not, however, say what a process is.

It seems that the first question which must be asked in trying to place a formulation such as Atkin's into correspondence with conversation theoretic entitities is to stipulate a unit which satisfies Atkin's own criteria for set membership, and the augmenting criteria that are introduced in that important part of his paper where it is proposed that man "is a relation upon a set of events".

For these two criteria, there are two obvious candidates; namely, *for the part of the theory that* deals with hard data observables (and, surely, observation is an event), the sets in question should be sets of "L agreements over understandings" as seen in the metalanguage, L*, by an impartial external observer. On the other hand, in the part of the theory which deals with man (acting as a *reflective observer or participant*), the appropriate choice is of "stable concepts", specified in L, as organisationally closed (which gives them integrity), but informationally open.

In either case, the entity in question is a process. *However, within the conversation theoretic framework it is* possible to obtain static inscriptions representing dynamic processes. These are the kernels (not the nodes) of an entailment mesh and static inscriptions may be preferred; that is, it may be preferable to deal with tokens that stand for stable concepts and/or/understandings rather than the processes in question.

There is no harm in doing so, provided that the character of the static inscription is kept in mind and, in particular, that the rules governing mesh construction and manipulation give a specific meaning (as mathematical axioms would do) *to set membership "$\in$" and to Cartesian Product "X"* and the difference between either "[illegible]" or "[illegible]" equivalences and isomorphisms " $\Longleftrightarrow$ " . Similarly, a special meaning is assigned to the idea of independence (which is only represented by , for example, orthoganality) .

It should be note ! that the identification satisfies Atkin's criterion for set membership. Membership is agreed (and in this case, is an understanding) or has been agreed as a norm (stable concept which exists in a participant's repertoire).Thus, the truth value of agreement is a consensual truth or (as I prefer it) a coherence truth value. The tricky point is that the metalinguistic veridicial truth value of an agreement is an L* metaphor designating a valid analogy rather than a proposition denoting a fact. However, it is "sharp valued" (or "hard") enough.

With this identification, others follow and are tabulated.

(a) Set members agreed between observers in H +1,H or participants in H and at level N in H.	Understandings observed or concepts exteriorised (notice, each involves a "built in" traffic); or kernels of mesh are objects in "backcloth" at level N.
(b) View from some vertex in S(N), S(...of H (backclost . of H)	Pruning under a perspective determined (as system specification) in H + 1 or by intended action in H which appears as term in H + 1 . Any S(N) is a pruning or union of prunings
(c) Hierarchy H+1,M and Hierarchy H,N	L* compared to L
(d) N+1 → N+2 , in H	Expansion operation
(e) N+1 → N , in H	Condensing operation
(f) Coordinates in E^{2n+1} (R_1 of Report 10)	Independent processes
(g) Pattern in H, on S(N)	Fuzzy quantities in respect of sharp valued understandings (or the set of stable concepts or understandings, unique to participants)
(h) Q vector,or obstruction vector, Q	Imported unaltered
(i) Change of pattern and traffic	Is Zadeh - Fuzzy Equivalent of concept-execution process and could be concept execution process itself. Is movement or communication if independence and analogy are introduced, as in (k) below
(j) Time (Report 10 p116 to p118)	Imported unchanged. Note only in a strict conversation are understandings ordered in Newtonian time or [illegible]

(k) Relations between $\mathcal{U}$ and $\mathcal{V}$ (understanding a stable concept) if $\mathcal{U}$ and $\mathcal{V}$ are sets of different sorts and the sorts are given by Dist predicates

Analogy relations (this is the only questionable point; do we need category theoretic representation as in Gaines). Equivalently use a many sorted (non-classical) model theory and an action-valued logic.

(ℓ) Potentially reflective quality of Atkin's (already relativistic) theory exploited in discussion of man and of meta levels of strategy and strategic changes in $\mathcal{R}(N)$ induced by forces in $\mathcal{R}(N)$, $\mathcal{R}(N+1)$ of H. (if the pattern on $\mathcal{R}(N)$ gives rise to any change in $\mathcal{R}(N)$)

Observer may be participant, and vice versa. One way of phrasing the matter is that though structures H, N+1, and N, N+1, are (minimally) required, the series of hierarchical systems H, N+1, ... may be represented (given the criteria of (a)) as a process in H, for some collection of systems $\mathcal{R}(N)$, $\mathcal{R}(N+1)$ in H. As noted in the paper (k) is crucial at this point. A specific interpretation of independence is needed to furnish the distinction needed for the representation.

If these identifications are accepted it is possible to muster Atkin's theory to predict quantitative differences of a kind that are qualitative, in conversation theory. Conversely, this formulation removes the need to introduce traffic as a postulate in Atkin's theory since, under this interpretation, its existence is a consequence of the argument. This possibility arises because of a subtle point. Atkin is dealing with relations (not functions) and the set theoretic formulation, though of value in securing clarity is not essential.

In conclusion, it seems to me that the correspondences, which may be refined as desired and spelled out in detail, establish a possibly valuable rapprochement between the points of view adopted by Bråten, Gaines and Nicolis (also those, as well, of Hogarth and M'Pherson). A measure of unification and logical coherence is introduced, some corners of arbitrariness are smoothed out and an elegant, precise, mathematical system is put to work in the domain of processes, albeit regarded as things, or objects.

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VOLUME 2

DECISION-MAKING AS AN EVENT-SEARCH:

TRAFFIC ON A MULTIDIMENSIONAL STRUCTURE

Decision-making as an Event-search : traffic on a Multidimensional Structure

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Contents

[0.1] Introduction

This paper is an attempt to show how the Methodology of Q-Analysis, which has been developed by the author over recent years, is relevant to the general question of decision-making. This "general question" arises in many circumstances - such as in an organisation of committees (which is a kind of social structure) or in the organisation of Self (involving the structure of the individual psyche) or in the Man-Machine context (whose structure exhibits the rigidity of the machine with the flexibility of the man). In any of these circumstances I would claim that we are dealing with a structure which is capable of being defined in mathematical terms - and that when that is done it becomes a simplicial complex (which has a representation in a multidimensional space). This particular kind of structure, in any specific study, acts the part of (what I have previously called) a static backcloth - denoted by S - and that this replaces the conventional 3-space of the physical scientist. In that event the backcloth must carry the "traffic" of various kinds of action - such traffic becoming the dynamics of the study. Furthermore, the topology of that backcloth, S, must determine how the dynamics is to be manifest (how the traffic is allowed to change) : the point about making decisions is then the question of "where do we go from here ?".

So in a fairly obvious sense "decision-making" will fall into two broad categories, which are not mutually exclusive, viz.,

C1 : decisions which are associated with changes in traffic over a given (unchanging) backcloth structure, S ;

and C2 : decisions associated with changes in the backcloth itself.

Whichever is the case, and sometimes both will apply at once, the notion of "structure" must be well defined and the backcloth topology (the basic connectivities of its geometry) must be discovered. The

Methodology of Q-Analysis rests on the assumption that such structures are those simplicial complexes which arise via mathematical relations between finite sets (of data). If Y and X are two such sets and λ is a relation between them (being a subset of their cartesian product) there will be two (conjugate) complexes, normally denoted by $K_Y(X)$ and $K_X(Y)$ - the first being defined by λ and the second by its inverse λ^{-1} . Then the simplest topological properties of any such complex are those defined by the sharing of subsets of the vertex set - what I have introduced as <u>q-connectivity</u> .

Illustrations of this approach are given in the following pages and are taken from specific research papers and projects completed over the past few years. They are meant chiefly to outline the ideas and methods and to lay a foundation for discussion ; greater details can be found in the references, particularly my books

<u>Mathematical Structure in Human Affairs</u> , Heinemann, 1974 and <u>Combinatorial Connectivities in Social Systems</u>, Birkhauser, 1977 .

Here I would like to stress that it is a <u>methodology</u> we are talking about, and not just a model . Q-Analysis does not provide us with any model (in the ordinary sense of that word) - it provides us with a method of building a model out of the basic data sets. So, if you like, it is a meta-model (or even a meta-meta-model etc.?) . Because of this there is no simple "package" of tricks which somehow give the answer - although there are one or two basic computer programs which can be used in the process of building the structures . Perhaps it is better to describe it as a <u>language</u> - a language for the discussion of structure.

[1.0] Q-Analysis - a language of structure

The following sections, [1.n], contain some of the basic definitions and concepts associated with the structural analysis of data. The theory has been developed from a consideration of what constitutes hard data , in social and other systems. In effect this consists of data which is associated with set membership - all other data is to be regarded as "soft" . In its turn this means that the sets (of data) are to be well-defined (and being finite this can always be done by listing the members) and also that they are to be naturally organised into a hierarchy (or hierarchies) of sets. This hierarchical ordering is compatible with Russell's Theory of Types and consists of relations between successive levels which are defined via the notion of cover sets (which are not usually partitions - as in conventional hierarchies).

Each such relation, and similar relations between sets at the same hierarchical level, defines a simplicial complex K, and the union of appropriate complexes constitutes the static backlcoth S of the data. Since the hierarchical levels can be named as N-level, (N+1)-level, (N+2)-level, etc., it is more appropriate to refer to the backcloth S(N), S(N+1), etc. since the structures are significant at the different levels.

This approach provides us with a language which allows us to distinguish between the kinematics and the kinetics in any system - a distinction which in pre-Newtonian dynamics was a pre-requisite for creating a science in that field. This language must therefore contain in it suitable words and constructions which can enable us to discuss both the basic topological properties of the backcloth as well as the way in which changes in traffic (on that backcloth) are expressible in a way which is comptaible with that topology.

[1.1] The hierarchy of data sets

Data (hard data) is provided only via set-membership (finite sets).

This requires (i) the sets must be well-defined (agreed by all)

(ii) we must not confuse types of sets (Russell).

Type 0: the elements (members) of a set $X = \{X_1, X_2, \ldots X_n\}$

Type 1: the elements of the power set $P(X) = \{\text{subsets of } X\}$

Type 2: the elements of the power set $P^2(X) = \{\text{subsets of } P(X)\}$, etc.

We therefore set up a hierarchy of levels for data, as follows.

H : N-level, (N+1)-level, (N+2)-level, etc.

where (e.g.) if the set X is at N-level then a corresponding set Z at (N+1)-level will be $Z = \{Z_1, Z_2, \ldots Z_k\}$ where Z is a cover of X. This means

that (i) $Z \subset P(X)$, each Z_i is a subset of X

and (ii) $X = \bigcup_i Z_i$, each X_j appears in at least one Z_i.

Note (i) The N in N-level is arbitrary and relative. We could equally well think of H as ... (N-2), (N-1), N, (N+1), (N+2), ...

(ii) When Z_i and Z_j are disjointed (for all $i \neq j$) we call the set Z a partition of X (old-fashioned hierarchy).

H is defined by a set of relations, λ, μ, ...

For example, if

$$\left.\begin{array}{lll} (N+1) & \longrightarrow & \{Z_1, Z_2, \ldots Z_k\} \\ \text{and } N & \longrightarrow & \{X_1, X_2, \ldots X_n\} \end{array}\right\} \updownarrow \lambda$$

then λ relates X and Z by:

"X_i is λ-related to Z_j if $X_i \in Z_j$".

Such a λ is represented by a binary (0/1) matrix $\Lambda = (\lambda_{ij})$ with n rows and k cols, and $\lambda_{ij} = 1$ if $X_i \, \lambda \, Z_j$

$= 0$ otherwise.

Examples of hierarchies via cover sets.

(I) In a study of the University of Essex (as a Community Study), published in 1974 as Research Report IV (this has since been published in "Combinatorial Connectivities in Social Systems, Birkjauser, 1977) it was found that five hierarchical levels were needed, and these were named (N-2), (N-1), N, (N+1), (N+2). The visual structure of the campus was described by relations between sets of visual features identified at the levels as follows ;

Visual features	Set	Hierarchical level
Wide views	W	N+1
Views	V	N
Facade strips	F	N-1
Details	D	N-2

The set W contained 4 views of the campus, roughly from the four points of the compass : the set V contained 20 elements, whilst F and D contained 36 and 84 elements respectively. No one of these sets was a partition of any other, the whole hierarchy of data sets was therefore defined by relations as follows :

$$H : \quad D \quad \overset{\lambda}{} \quad F \quad \overset{\mu}{} \quad V \quad \overset{\beta}{} \quad W$$

and these relations contained within them the essential structure of this hierarchy of cover sets.

In the same study the Committees of the university found natural places in this hierarchy - subcommittees being at a level one down from the committee itself. So with Senate at (N+2) we naturally find the School Boards at (N+1) and the Departments at the N-level. Similarly with Amenities at (N+1) we get Housing, Catering, Student Societies at N, whilst at (N-1) we find specific housing accomodation, specific restuarants etc., and specific student socities . Then with Degree Courses (about 50 of them) at the N-level we naturally find Lecture Courses at (N-1) and Lecture Topics at (N-2). Clearly these are not partitions either, since Degree courses can share lectures etc..

(2) In a Regional Study of East Anglia (during 1974-77) we actually used three hierarchical levels as follows :

Level	Local Authorities	Sets	Number of elements
N+2	Names of Counties	A(N+2)	4
N+1	Local Authority Names (Districts)	A(N+1)	72
N	Parish Names	A(N)	1184

and with these were took the Land Use data whose sets fitted in :

N+2	Land Uses	L(N+2)	8
N+1	Land Uses	L(N+1)	46
N	Land Uses	L(N)	966

In this case, with the sets A(h), we do have a partition at successive levels - since different authorities do not share the same piece of geographical area. But with Land Use (which includes all kinds of human economic and social activities) the sets were not partitioned.

Most of the conclusions about the hierarchical data structure in that study can be found in Research Report X (attached) .

(3) In the application of this Methodology to the study of the Game of Chess (v. Fred CHAMP, positional-chess analyst) it was found necessary to use a hierarchy H containing three levels as follows :

N-level : A set of squares on the board, S

The set of white pieces, W, and of black pieces, B.

(N+1)-level : A set consisting of sets of squares (like files, diagonals, centre, etc).

A set of sets of pieces (like pawns on the King-side, the Knights, the Bishops, the Rooks)

(N+2)-level : A set consisting of sets of sets of squares (like "the King-side squares" etc.).

A set of sets of sets of pieces (like "the pawns" and "the minor pieces" etc.).

Furthermore, in that study, it was necessary to take cognisance of the presence of a meta-hierarchy (denoted by H+1) any one level in which referred to (controlled) the whole of H. This was an important notion since it was only in the meta-hierarchy that playing "strategy" could be sited - that is to say, where decisions could be made about evaluation of the structure (the "positional" features) of the game at any stage.

In the hierarchy H itself we could identify the tactical and positional play as follows :

N-level : Here the players make the tactical moves ; where each move requires a decision about a particular piece and a particular square.

(N+1) : Here we get the (1st order) positional play ; where sets of pieces (if only two or three) combine to affect sets of squares (like "the centre").

(N+2) : Here we find the 2nd order positional play ; where (eg) the whole of the King-side is attacked.

[1.2]. Example of relation $\lambda \subset Y \times X$ (relating Y to X).

λ	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10
Y1	1	1	1	1	0	0	0	0	0	0
Y2	0	1	1	0	1	0	0	0	0	0
Y3	0	1	0	0	1	1	0	0	0	0
Y4	0	0	0	0	1	1	1	1	0	0
Y5	0	0	0	0	0	0	1	1	1	0
Y6	0	0	0	0	0	0	0	1	1	1
Y7	0	0	0	1	0	0	0	1	0	1
Y8	0	0	1	1	0	0	0	0	0	1

Each Y_i is called a SIMPLEX and written (e.g.) as

$$Y_1 = \langle X1, X2, X3, X4 \rangle \text{ , a 3-simplex.}$$

If Y_i is defined by (p+1) X's then it is called a p-simplex (written σ_p). A p-simplex can be represented by a convex polyhedron (with (p+1) vertices) in p-dimensional euclidean space, E^p, and not in any E^q with $q < p$.
All the Y's, in the above relation λ, combine to give a SIMPLICIAL COMPLEX denoted by $KY(X,\lambda)$ or $KY(X)$ or K. The X provides the vertex set and Y is a set of names of simplices.
Associated with this λ is the inverse relation λ^{-1} (incidence matrix as Λ^T, the transpose of Λ). This gives the conjugate complex $KX(Y)$ or $\tilde{K}$.

Each complex K, $\tilde{K}$ possesses a geometrical representation in E^k, for suitable k. In fact there exists a theorem: k can be taken as (2n+1), where n = max p in K. This max p is called the dimension of K, dim K.
KY(X) and KX(Y) are shown in Figs. 1 and 2, whilst a list of practical examples of how relations can arise is given in Table 1.

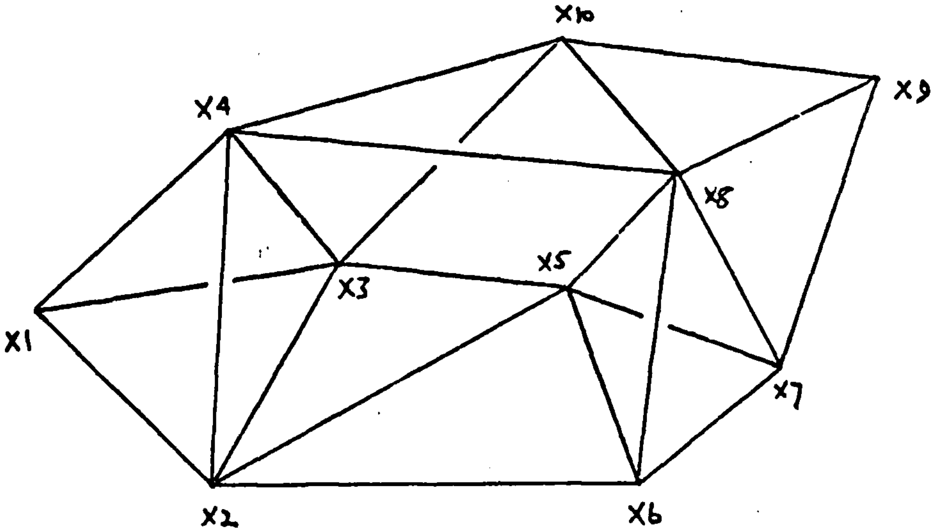


Fig. 1 KY(X) Structure Vector $\underline{Q} = \left\{ \begin{array}{cccc} 3 & 2 & 1 & 0 \\ 2 & 8 & 1 & 1 \end{array} \right\}$

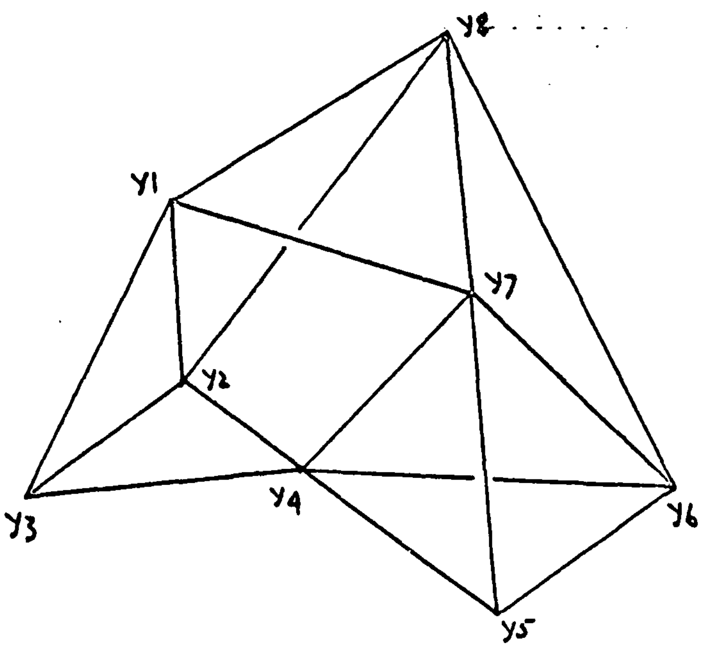


Fig. 2 KX(Y) Structure Vector $\underline{Q} = \left\{ \begin{array}{cccc} 3 & 2 & 1 & 0 \\ 1 & 6 & 1 & 1 \end{array} \right\}$

	Set Y	Set X	Interpretation of λ_{ij}
1)	Individual People	Named Committees	$\lambda_{ij}=1$ means that person Y_i is on committee X_j.
2)	Kinds of people (ethnic groups, etc.)	Committees in a wide sense (assemblies etc.)	$\lambda_{ij}=1$ means that group Y_i is represented in X_j.
3)	Kinds of people	Business enterprises	$\lambda_{ij}=1$ means that type Y_i invests cash in X_j.
4)	Traffic routes	Types of vehicle	$\lambda_{ij}=1$ means that route Y_i carries type X_j.
5)	Streets in a town	Types of retail business	$\lambda_{ij}=1$ means that in street Y_i we find business X_j.
6)	Educational courses	Named colleges	$\lambda_{ij}=1$ means that college X_j offers course Y_i.
7)	Individual people	Leisure interests	$\lambda_{ij}=1$ means that person Y_i devotes leisure time to X_j.
8)	Medical illnesses	Pathological symptoms	$\lambda_{ij}=1$ means that diagnosis Y_i involves the symptom X_j.
9)	Manufacturing processes	Industrial techniques	$\lambda_{ij}=1$ means that process Y_i requires technique X_j.
10)	Political groups	Social reforms	$\lambda_{ij}=1$ means that group Y_i advocates reform X_j.

Table 1 Examples of relations λ between sets Y and X

[1.3] The idea of q-connectivity (topology) in KY(X)

If Y_i and Y_j share (q+1) vertices (the X's) they share a q-simplex as a common "face". We then say that they are q-near (or are q-connected) If (eg) Y_1 is q_1-near Y_2 and Y_2 is q_2-near Y_3 (etc.) we say that Y_1 is q-connected to Y_3 , where $q = \min(q_1, q_2, \ldots)$.

This defines an equivalence relation on all the simplices of KY(X) - those which are (at least) q-simplices. Hence we obtain equivalence classes of q-components in KY(X). If Q_q = the number of distinct q-components in K, and q = 0,1,2,... dim K , we introduce the structure vector , $\underline{Q}$, which gives a first idea of the global topological properties of the complex. This is simply the integers :

$$\underline{Q} = \{ Q_n, Q_{n-1}, \ldots Q_2, Q_1, Q_o \}$$

If (eg) $Q_o > 1$ the complex K is the union of disjoint complexes (Q_o of them) . In the case that $Q_o = 1$ (and if it does not then we consider the separate components of the complex, in each of which $Q_o = 1$) we introduce the Obstruction vector , as follows :

$$\hat{\underline{Q}} = \{ Q_n - 1, Q_{n-1} - 1, \ldots Q_1 - 1, Q_o - 1 \ (=0) \}$$

and we shall see that this is a better measure of the topological obstruction to change in traffic on the structure (than is $\underline{Q}$ itself).

In addition to this global idea of the way in which the complex K is connected, we have a measure of the local role played by any one of the simplices (any one Y_i). This measure is what has been called the eccentricity of a simplex, Y_i . We define it as follows :

If $\hat{q}$ = dimension of Y_i = (what we shall call) its top-q, and

$\check{q}$ = the q-value at which this Y_i is first connected to some distinct Y_j,

= (what we shall call) its bottom-q, then we define

$$\mathrm{Ecc}(Y_i) = (\hat{q} - \check{q}) \div (\check{q} + 1) .$$

Examples of q-connectivities, structure vectors, etc..

(1) In an urban study of Westcliff-on-Sea (Essex) in 1972-3 the data at a suitable N-level consisted of a set L(N) of locations (streets) and its relation to another set V(N) of commercial activities. The first set contained 54 elements (names of streets) whilst the second contained 152 elements (names of various activities under the broader headings (at the (N+1)-level) of Retail-I, Retail-II, Private Services, Public Services, Community Amenities, Catering, Light Industry, and Residence). It was possible to compare the structures KL(V) for two years separated by 60 years, viz., for the year 1910 and theyear 1972. In each case the complex was analysed (by our computer programs) - the so-called Q-Analysis - to give the components at various q-values. These were also separated under the (N+1) headings so that we could obtain a feeling for how the character of the area had changed in various respects. [Incidentally the difference between Retail-I and Retail-II was that between "hardware" and "food" etc..]. Since the street pattern had not changed over these 60 years the comparison between KL(V)-1910 and KL(V)-1972 seemed to make intuitive sense. The set V was appropriate to include those activities which were to be found in the area either in 1910 or in 1972 or in both.

The following table shows how Q changed over this period of time :

Subcomplex heading	Q in 1910	Q in 1972
Retail-I	$\overset{9}{1}\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ \overset{0}{1}$	$\overset{4}{2}\ 2\ 2\ 2\ \overset{0}{2}$
Retail-II	$\overset{24}{1} \ldots \overset{10}{1}\ 3\ 3\ 3\ 5\ \overset{5}{4}\ 2\ 1$ $1\ \overset{0}{1}$	$\overset{17}{1} \ldots \overset{11}{1}\ 2\ 2\ 3\ 3\ 2\ \overset{5}{2}$ $3\ \overset{3}{3}\ 3\ 4\ \overset{0}{2}$
Private Service	$\overset{30}{1} \ldots \overset{17}{1}\ 2\ 3\ 4\ \overset{13}{5}\ 5\ 3$ $3\ 4\ \overset{8}{4}\ 2\ 1 \ldots 3\ 5\ \overset{0}{2}$	$\overset{11}{1}\ 1\ 1\ 4\ \overset{7}{5}\ 5\ 6\ 4\ 3\ \overset{2}{1}$ $3\ \overset{0}{1}$
Public Service	$\overset{8}{1}\ 1\ 1\ 1\ \overset{4}{2}\ 7\ \overset{2}{10}\ 3\ \overset{0}{1}$	$\overset{2}{1}\ 5\ \overset{0}{9}$

and so on - where the superscripts denote q-values corresponding to the Q_q numbers in $\underline{Q}$.

There is a noticeable contraction in dim K over this period (the structure vectors become shorter in each subcomplex) : a typical list of contracting top-q values, with names of streets attached, is the following.

Element	Street name	1910	1972
L7	High Street	$\hat{q} = 24$	$\hat{q} = 17$
L11	Alexandra Street	$\hat{q} = 13$	$\hat{q} = 11$
L9	Weston Road	$\hat{q} = 10$	$\hat{q} = 8$

these all referring to the subcomplex headed Retail-II.

The way in which (eg) the High Street, L7, changed its dimensions under the various headings is shown in the following table ;

N+1 heading	1910		1972	
	$(\check{q},\hat{q})$	Ecc (L7)	$(\check{q},\hat{q})$	Ecc (L7)
Retail-I	(7,9)	1/4	(1,4)	3/2
Retail-II	(8,24)	16/9	(7,17)	5/4
Private Service	(11,30)	19/2	(4,7)	3/5
Public Service	(2,8)	2	(0,1)	1
Amenities	(1,4)	3/2	(0,0)	0
Catering	(1,3)	1	(1,1)	0
Light Industry	(0,5)	5	(-1,-1)	infinity

We can see that in general High Street has become more isolated (drop in bottom-q values) and that it contains less shopping/service facilities (drop in top-q values) : also Light Industry has vanished from L7 altogether. It is also less eccentric than it was in 1910 - showing that the connectivities of the whole area (as between the streets and via those vertices in V(N)) have resulted in a shrinking of the commercial features into a few streets - which results in their being less eccentric - so shopping is less interesting and more concentrated.

(2) In the University Study already referred to, one relation between two N-level sets was that between Degree Schemes and Departments ; this relation was analysed for each of the academic years 1970/1 to 1974/5. If we let Y = set of Departments and X = set of Degree Schemes, we obtain the following comparison of structure vectors (for both KY(X) and KX(Y)).

Year	Q for KY(X)	Q for KX(Y)	Set Y	Set X
1970/1	1.1.6 6 6	5 6	11	20
1971/2	3 4 6 5 6	6 6	11	23
1972/3	2 3 4 8 8 5	9 5	12	28
1973/4	1 2 3 3 6 9 9 4	12 4	12	33
1974/5	1 3 3 3 3 7 7 10 9 3	17 3	14	43

From this alone we can see that the number of disjoint pieces of the university (department-wise), as counted by Q_o , steadily decreased from 6 in 1970/1 to 3 in 1974/5 - the same applying to the pieces (fields of study possible) as measured via the Degree Schemes (KX(Y)) . This indicates the rise of Joint Degree Schemes which cross the departmental barriers. But we notice that the obstruction vector has increased dramatically at the q=2 level, through the values of 0, 0, 3, 5, 7 indicating that there is a price to be paid for this increasing academic unity (as measured simply by Q_o). We shall refer to this in another section when dealing with traffic and patterns.

(3) In the Regional Study of East Anglia (1974-77) data was obtained from the Office of Censuses and Population which gave the numbers of employed persons throughout the region in all the classes of jobs which are normally listed under that Office's Standard Industrial Classification. This latter list is really published in 3 hierarchical levels - our (N+1), N, and (N-1) . The numbers of headings (elements)

in this list at these respective levels are 27, 223, and c. 12000 . We therefore found that we had a matrix (containing integers) which related (at (eg) the (N+1)-level) the 72 vertices in the list of Local Authority Areas (Districts), already referred to as A(N+1), to these 27 elements in the (N+1)-list of SIC-jobs. Such a matrix contains many binary matrices (out of which we can build our complexes) and one way of deriving such a binary matrix is to "slice" the numbers in the data matrix. This is done by choosing an interval (possibly a different one for each row of data) and if $[a_r, b_r]$ is the choice for Row-r, creating a binary matrix λ by setting $\lambda_{rs} = 1$ of the data element D_{rs} falls within this interval, and setting $\lambda_{rs} = 0$ otherwise. So if we let each b_r become infinite we effectively replace each D_{rs} by a 1 if it is not less than a_r and by a 0 otherwise.

In this particular analysis the slicing was done by taking the number a_r to be the <u>mean</u> of all the data numbers D_{rs} (fixed r) in the rth Row, and letting b_r be effectively infinite. By this device we obtain a complex (from the resulting binary matrix) which represents the structure of the data (the employed classes distributed over the county district areas) insofar as the distribution is not less than the average throughout that District (this is the average of all jobs distributed over the 27 possible kinds of jobs in the SIC).

The resulting structure vectors for KY(X) - where Y is the set of Authority Areas and X is the set of SIC-jobs.- are listed below for the 1971 Census Data.

<u>County</u>	<u>structure vector Q</u>
Cambridgeshire	$\overset{7}{3}\ 4\ 5\ 2\ \overset{3}{1}\ 1\ 1\ \overset{0}{1}$
Norfolk	$\overset{8}{2}\ 8\ 8\ 5\ \overset{4}{2}\ 1\ 1\ 1\ \overset{0}{1}$
East Suffolk	$\overset{9}{1}\ 4\ 5\ 4\ \overset{5}{4}\ 3\ 1\ 1\ 1\ \overset{0}{1}$
West Suffolk	$\overset{8}{1}\ 3\ 6\ 5\ \overset{4}{3}\ 1\ 1\ 1\ \overset{0}{1}$

Graphs of these are shown in the accompanying figures.

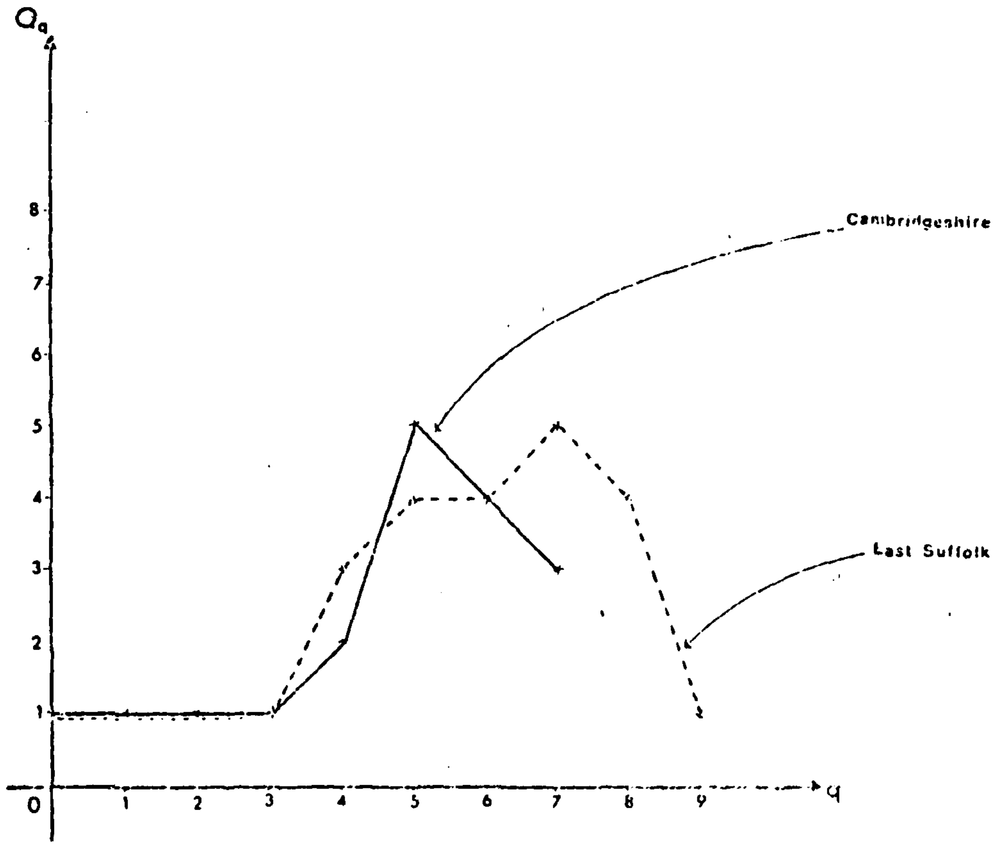


Fig. 3 Structure Vectors - 1971 Data

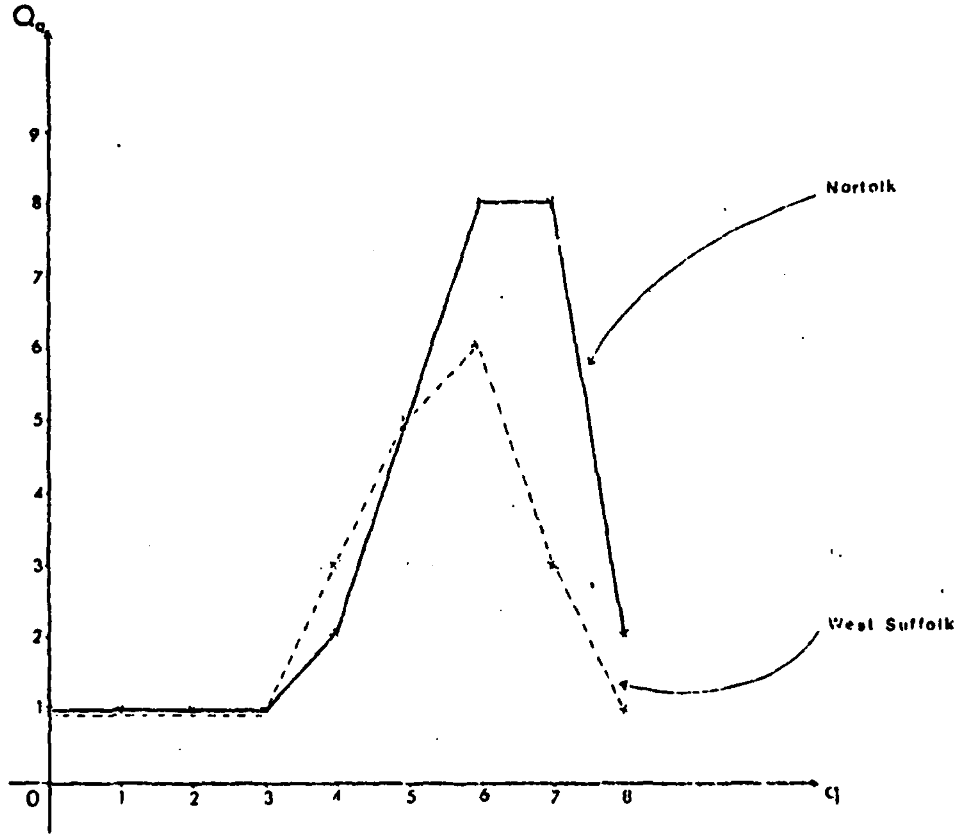


Fig. 4 Structure Vectors - 1971 Data

[1.4] Traffic and patterns on a complex $KY(X)$

By "traffic" we mean anything associated with a complex $KY(X)$ which

(i) is defined by subsets of the vertex set X ; that is to say, it is to be defined on each of the simplices, the Y_i , and

(ii) can be described by a graded set function, called its pattern,

$$\pi \; : \{\text{simplices of } K\} \longrightarrow J \qquad \text{(ring of values)}$$

We can write the value of π on a simplex σ_p as $\pi(\sigma_p)$, or as (σ_p, π) ; each is a value if J (usually the integers).

It is clear that a pattern can be written (indeed it must be written) as a graded set function, in the form

$$\pi = \pi^o \oplus \pi^1 \oplus \ldots \oplus \pi^t \oplus \ldots \oplus \pi^n$$

where $n = \dim K$, and where π^t means π restricted to the t-simplices of K. The direct sum is required because we do not normally expect to be able to add these values together (naturally, $\oplus$ is defined in terms of the + in the ring J of values, in the usual way).

Examples of traffic are listed in Table 2 below ; each is a plausible example on a static backcloth, $KY(X)$, defined at some particular hierarchical level, N . This is essential to enable us to distinguish between the dynamics on the structure and the structure which is to "carry" it - just like "velocity of a particle" is a dynamic pattern on a physicist's backcloth, E^3 (euclidean 3-space). Indeed it is clear that "velocity of a particle" is an example of a π^1 (since it is defined on a 1-simplex in E^3, viz., the pair of points needed to measure the thing called "velocity").

It is because of the above analogy with physics that it seems desirable to interpret change in π , denoted by $\delta\pi$, as a "force" experieneced by the traffic in this backcloth structure $KY(X)$ - just as δv is associated with Newtonian "force" in E^3 and is experienced by the traffic - which in this case is "the particle" itself.

Set X	Set Y	Example of traffic on KY(X)
Individual people	Named committees	Business discussed/ranked in the committees
Sets of people with specific skills	Business enterprises	Capital investment in business activities
Traffic routes through towns	Types of vehicles	Goods and people carried by different vehicles
Streets in town	Retail businesses	Weekly shopping turnover in the shops
Medical illnesses	Pathological symptoms	Drug treatment of the symptoms
Educational Courses	Colleges	Student applications to colleges
Groups of people in community	Leisure interests	Current expenditure on leisure activities
Political groups	Social reform measures	Costs of social reforms in specific community
Manufacturing processes	Industrial techniques	Research costs per technique
Company executives	Responsibilities in company	Share of advertising "cake" per area of responsibility

Table 2 : Examples of traffic on a complex KY(X)

In the Regional Study of East Anglia, backcloth given by the structure KY(X), where Y is the set of (N+1)-level Local Authority Areas (Districts) and X is the set of SIC-jobs at the same level. Listing some typical graded traffic :

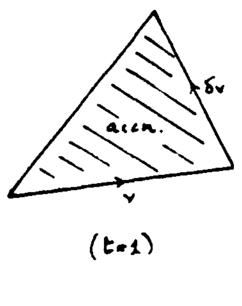
Traffic	Meaning of specific grading in π^t
[1] Numbers of employment-seeking males per area.	2-traffic (t = 2) means people who are eligible for 3 kinds of jobs.
[2] Private houses built in each area	4-traffic means houses readily accessible to 5 kinds of work
[3] Commercial buildings	5-traffic means buildings which house 6 job categories
[4] Volume of traffic flow through each area	0-traffic means vehicles which are concerned with only one kind of job
[5] Goods exported per area	3-traffic means goods which have been produced by 4 kinds of job categories
[6] Capital investment per area	1-traffic means capital available for 2 kinds of work
[7] Taxation levied per area	7-traffic means tax revenue derived from businesses which involve 8 kinds of jobs
[8] Investment in education per area	1-traffic means investment in education which is concerned with 2 kinds of industrial training
[9] Investment in health clinics	3-traffic means investment in clinic which deal with 4 kinds of job-workers

Table 3 : Examples of π^t, graded traffic

We know that, in orthodox physics, the notion of incremental velocity δv requires, for its representation, the presence of the triangle - as shown in Fig 3. This δv, or acceleration, is therefore closely associated with the 2-simplex (the triangle - not just the three edges) just as the velocity (which is a π^1) is associated with the 1-simplex (edge). This is in spite of the fact that we can apparently represent the v along one side of the triangle - because that side cannot be drawn unless it is possible to draw the whole triangle. This acceleration is an example of what we shall call a t-force - a force which is manifest in the structure (on the appropriate traffic) whenever $\delta\pi^t$ occurs . We see also that this will involve, in some way, the presence in the structure (of the backcloth) of some (t+1)-simplex. Just as the presence of δv ($= \delta\pi^1$) manifest itself as a 1-force (acceleration) which is closely associated with the 2-simplices of the structure.

A similar parallel exists with the physicist's notion of a torque, which is the manifestation of a change in angular momentum δh . Since h is associated with an area it is an example of a π^2, and so δh requires the presence of a 3-simplex (a piece of volume) in the underlying backcloth before it can be observed (or experienced by the traffic).

Both of these parallels are shown below.



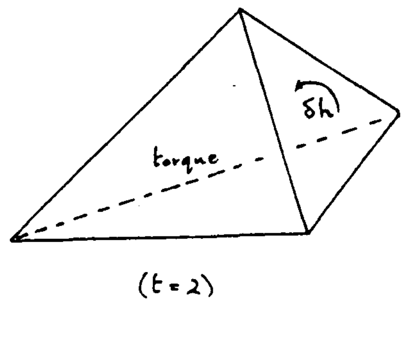


Fig. 3 Examples in physics of $\delta\pi^1$ and of $\delta\pi^2$.

[1.5] Mathematical operators on a complex

The following operators are designed to help us to describe patterns π and possible increments π in the context of the connectivities of the backcloth (a typical complex KY(X)).

(i) The face operator, f, defined by

$$f\sigma_p = f\langle X_1X_2\ldots X_{p+1}\rangle = \bigcup_i \langle X_1 \ldots \hat{X}_i \ldots X_{p+1}\rangle$$

where $\hat{X}_i$ means that the vertex X_i has been clipped out.

Thus $f\sigma_p$ = the set of all the (p-1) simplices which are faces of σ_p

$$= \{ \sigma^i_{p-1} : \sigma^i_{p-1} < \sigma_p \}$$

(ii) The coface operator, Δ, (dual to f) which is applied to patterns and definded in the usual way via:

$$(f\sigma_{p+1}, \pi^p) = (\sigma_{p+1}, \Delta\pi^p) = \sum_i (\langle X_1\ldots \hat{X}_i \ldots X_{p+2}\rangle, \pi^p)$$

We notice that $\Delta\pi^p$ is a π^{p+1}, so Δ increases the grading by +1, and $\Delta\pi^p$ takes a value on each σ_{p+1} equal to the sum of the values of π^p on all the p-faces of that (p+1)-simplex.

We also notice that if we are given some π^o (defined on the vertices only of the complex K) then we can find an associated π^p on all the p-simplices by using this operator Δ as follows:

when $\sigma_p = \langle X_1X_2 \ldots X_{p+1}\rangle$ then

$$(\sigma_p, \frac{\Delta^p}{p!}\pi^o) = \sum_{i=1}^{p+1} (\langle X_i\rangle, \pi^o)$$

It follows that, for any p-simplex σ_p, the terms of the series $e^{\Delta}(\pi^o)$ give the successive graded patterns π^q on all the faces of σ_p. If now we prune the set Y so that each Y_i has non-zero eccentricity (is not merely the face of some other Y_j) then we can identify an operator e^{Δ}_i, for each such i, and so obtain a pattern generator, Θ, for the whole complex KY(X) - given by

$$\theta = \sum_i e_i^\Delta - \sum_{i<j} e_{ij}^\Delta + \sum_{i<j<k} e_{ijk} - \text{etc.}$$

where (eg) e_{ij}^Δ denotes the operator common to the interface of Y_i and Y_j .

(iii) The <u>supraface operator</u> , f_{+1}, which identifies the simplices of which a given simplex is a face (at one dimension down). It is given by

$$f_{+1}\,\sigma_p = \bigcup_i \left\{\sigma_{p+1}^i \text{ such that } \sigma_p \in f\,\sigma_{p+1}^i\right\}$$

(iv) The <u>inverse coface operator</u> , Δ^{-1} ,

$$(f\,\sigma_p,\ \Delta^{-1}\pi^p) = \sum_i (\langle X_1 \ldots \hat{X}_i \ldots X_{p+1}\rangle,\ \Delta^{-1}\pi^p)$$

$$\overset{\text{Def}}{=} (\sigma_p,\ \pi^p)$$

and which incidentally = $(\sigma_p,\ \Delta\Delta^{-1}\pi^p)$ by (ii) .
So Δ^{-1} decreases the grading of π^p by 1, and gives a partition (into equal parts) of the value of this π^p for the values of $\Delta^{-1}\pi^p$ on the (p-1)-faces of any σ_p .

(Further properties of Δ^{-1} are discussed in Research Report X, attached).

(v) Now we can discuss the <u>incremental changes</u> $\delta\pi$, in any pattern π, in terms of the topology of the complex K. For we have noticed in the previous section that the t-force (manifest via some $\delta\pi^t$) is associated with the presence in the structure of suitable (t+1)-simplices. So we make this explicit by identifying the $\delta\pi^t$ with some pattern $\Delta\mu^t$ (which requires the presence of a σ_{t+1} for its definition), by making the following diagram commutative, that is to say, the mapping α must be such that $\underline{\underline{\Delta\alpha\,\pi^t = \delta\pi^t \equiv \Delta\mu^t}}$

$$\begin{tikzcd} \mu^t \arrow[r, "\Delta"] & \Delta\mu^t \\ \pi^t \arrow[u, "\alpha"] \arrow[r, "\delta"] & \delta\pi^t \arrow[u, "\equiv"'] \end{tikzcd}$$

This means that a t-force can only be manifest in the structure if there are (t+1)-simplices to be found there, even though thevalue of $\delta\pi^t$ can be observed on the t-simplices on which π^t is measured. *Any sensible measure of the "strength" or "intensity" of the* t-force probably depends on the data context, but one possibility is to measure it as the ratio

$$(\sigma_t, \delta\pi^t) \div (\sigma_t, \pi^t)$$

when that exists.

This does not exclude a measure like

$$(\sigma_{t+1}, \Delta\mu^t) \div (\sigma_t, \pi^t)$$

where $\sigma_{t+1} \in f_{+1}\,\sigma_t$, when the association with the geometry needs to be made more pointed.

[1.6] The role of the Obstruction Vector $\hat{Q}$

The obstruction we are talking about is an obstruction to changes in traffic on the structure. This means that it must be an obstruction to π, when π is the pattern which represents that traffic. And in intuitive way we would not expect there to be any obstruction if the geometry is adequately connected. Thus if the complex K consists of a single simplex (σ_n) with all its faces then we would expect there to be zero obstruction to changes $\delta\pi^t$ for all values of t from 0 to (n-1) . But in this case the structure vector Q (as distinct from $\hat{Q}$) consists of a string of 1's, viz.,

$$Q = \{\overset{\sim}{1}\ 1\ 1\ 1 \ldots 1\ 1\ \overset{\circ}{1}\}$$

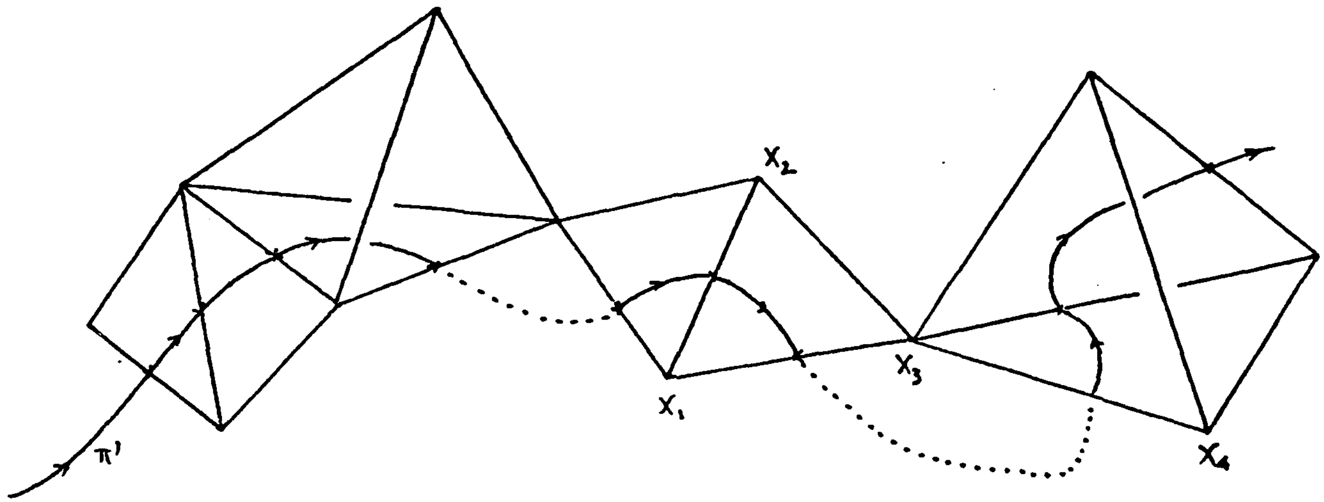
and that is why our definition of $\hat{Q}$ involves taking away 1 from each Q_q to give the *obstruction vector*

$$\hat{Q} = \{\hat{0}\ 0\ 0\ 0 \ldots 0\ 0\ \dot{0}\}$$

(though possibly we should not subtract 1 from the value of Q_n ?)

If we refer to Fig. 6 , as an illustration, we can suppose that π^1 is a given 1-pattern on the 1-simplices (the edges) of K. Then consider the state of affairs in which, at some time t_1, π^1 is non-zero on the simplex $\langle X_1X_2\rangle$ and zero elsewhere (so the 1-traffic is located on this particular edge). Now suppose that at time t_2 we wish to have the traffic located on the edge $\langle X_1X_3\rangle$. This change in the traffic condition corresponds to an incremental change $\delta\pi^1$ which involves a decrease on $\langle X_1X_2\rangle$ and an increase on $\langle X_1X_3\rangle$. Since $\langle X_1X_2\rangle$ is a face of $\langle X_1X_2X_3\rangle$ this change can occur via our $\Delta\mu^1$ (which requires the 2-simplex as its domain). So the geometry (the topology) allows this flow from one edge to the other. In contrast we see that a flow of this 1-traffic from $\langle X_1X_3\rangle$ to $\langle X_3X_4\rangle$ would not occur, because these two edges are in distinct 1-components ($Q_1 = 3$, which is > 1). This illustrates that q-traffic experiences zero obstruction to flow (to changes in its representative pattern π) in any one q-component of the structure, but is absolutely obstructed between distinct q-components. Hence the effective obstruction to forming $\delta\pi^t$ (or $\Delta\mu^t$) is measured by the number $Q_t - 1$, and this is the component $\hat{Q}_t$ of $\hat{\underline{Q}}$.

Expressed otherwise, we can say that if $Q_t > 1$ then there exist at least two t-simplices, say, σ_t^1 and σ_t^2 (in separate components) such that $f_{+1}\sigma_t^1$ and $f_{+1}\sigma_t^2$ are disjoint (do not share any common σ_{t+1}) and so $\Delta\mu^t$ cannot be defined for any change in a π^t which corresponds to a traffic flow between them.



$$\underline{Q} = \left\{ \begin{array}{cccc} 3 & 2 & 1 & 0 \\ 2 & 6 & 3 & 1 \end{array} \right\}$$

$$\hat{\underline{Q}} = \left\{ \begin{array}{cccc} 3 & 2 & 1 & 0 \\ 1 & \bar{5} & 2 & 0 \end{array} \right\}$$

$$\pi : Y \longrightarrow J$$

$$\underline{\pi} = \pi^0 \oplus \pi^1 \oplus \pi^2 \oplus \ldots \oplus \pi^n \qquad n = \dim K.$$

Fig. 6 Traffic on a complex $KY(x)$

[1.7] Examples of induced t-forces in the structure

Changes in the structural backcloth, say $S(N)$, (over some time interval) automatically induce changes $\delta\pi$ in any pattern π which is defined on that complex. The traffic which is measured by this π therefore experiences these changes as t-forces, and these are manifest in appropriate places in the geometry of $S(N)$.

In a simple way we can see this as follows. Let Y_1 be the name of some 3-simplex $\langle X_1X_2X_3X_4\rangle$ in a complex $KY(X)$ and suppose that π has a component π^3 which takes a value on this Y_1 (which we would naturally write as $\pi^3(Y_1)$). Then suppose that the structure changes in such a way that the vertex X_4 is removed from it. Then we see that

(i) the 3-simplex $\langle X_1X_2X_3X_4\rangle$ collapses to the 2-simplex $\langle X_1X_2X_3\rangle$,

(ii) $\pi^3(Y_1)$ must change to zero, so $\delta\pi^3 = -\pi^3(Y_1)$, because the 3-simplex has disappeared : this means that the traffic which π^3 represents must do one of two things,

either (a) change its character and become 2-traffic, so that it can now exist on 2-simplices,

or (b) move through the structure until it finds a new home in some 3-simplex ; this of course is only possible if our Y_1 is itself 3-connected to some other 3-simplex (so the topology is involved in this possibility).

(iii) these changes are experienced by the original traffic as forces of repulsion (3-force) in the structure, over this time period.

Illustrations of the calculation of t-forces which are consequent upon changes in backcloth, S, are given in the following diagrams.

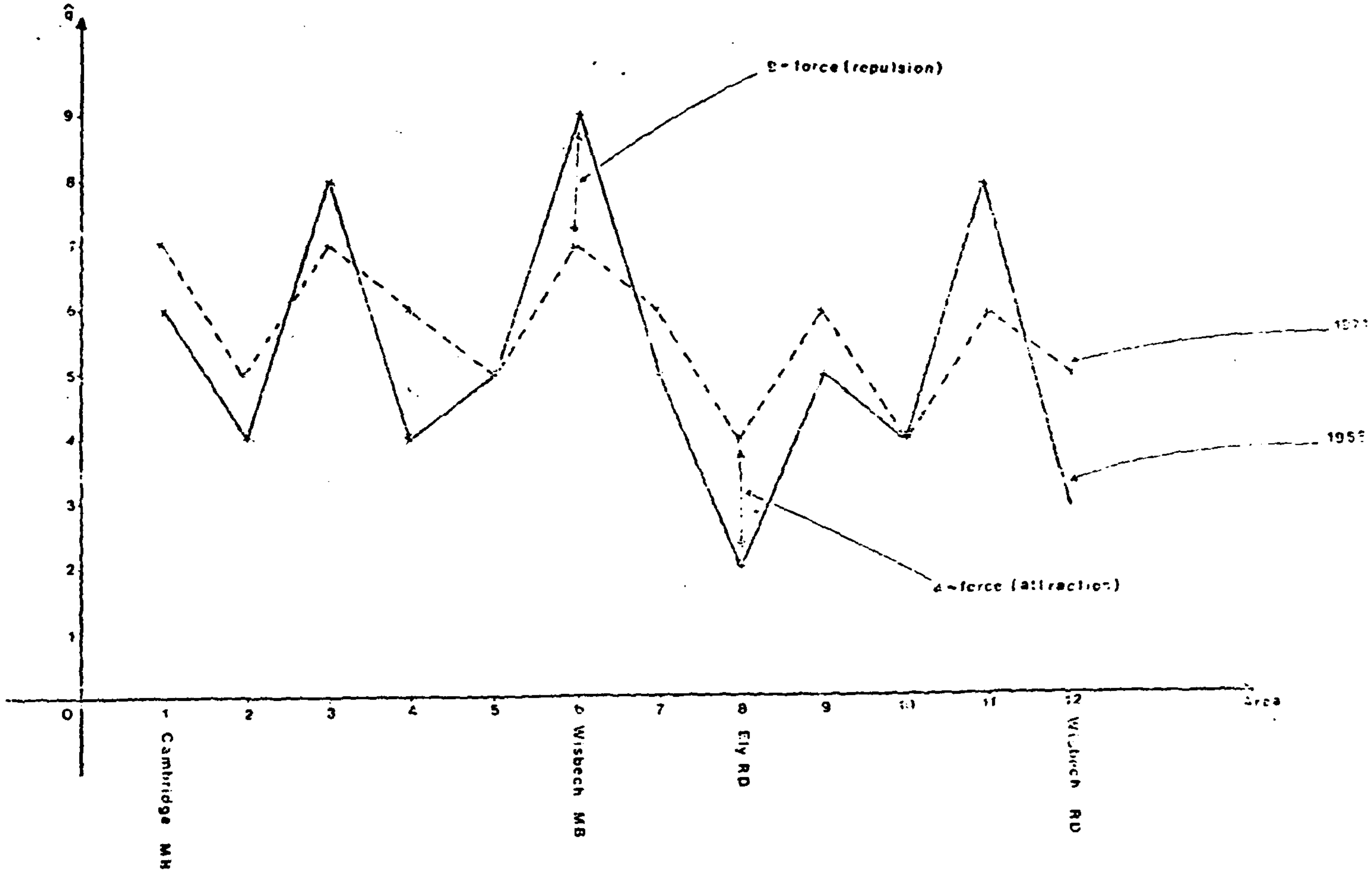


Fig.7 Cambridgeshire — t-forces due to ΔS(N+1)

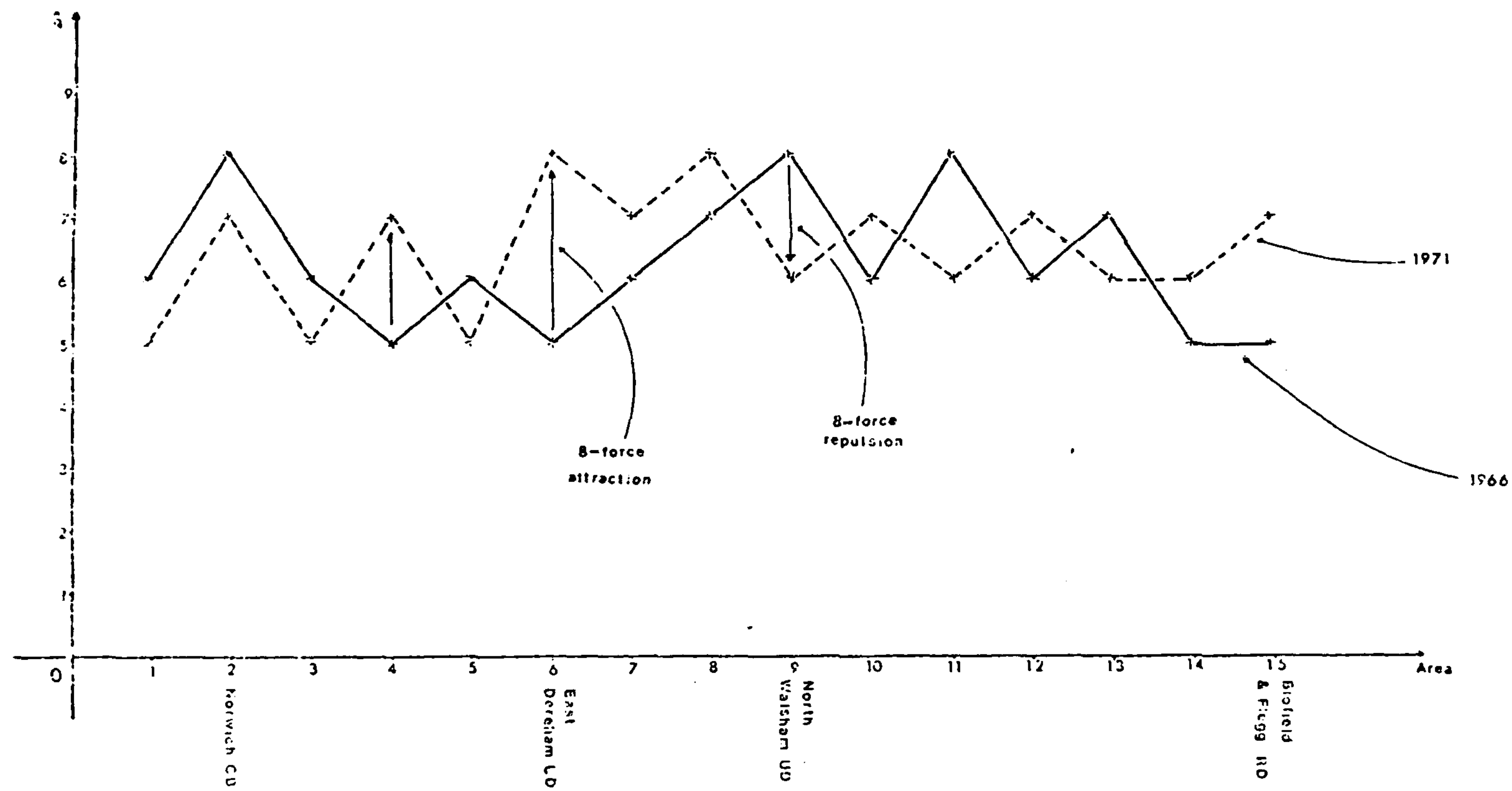


Fig. 8 Norfolk — t-forces due to $\Delta S(N+1)$

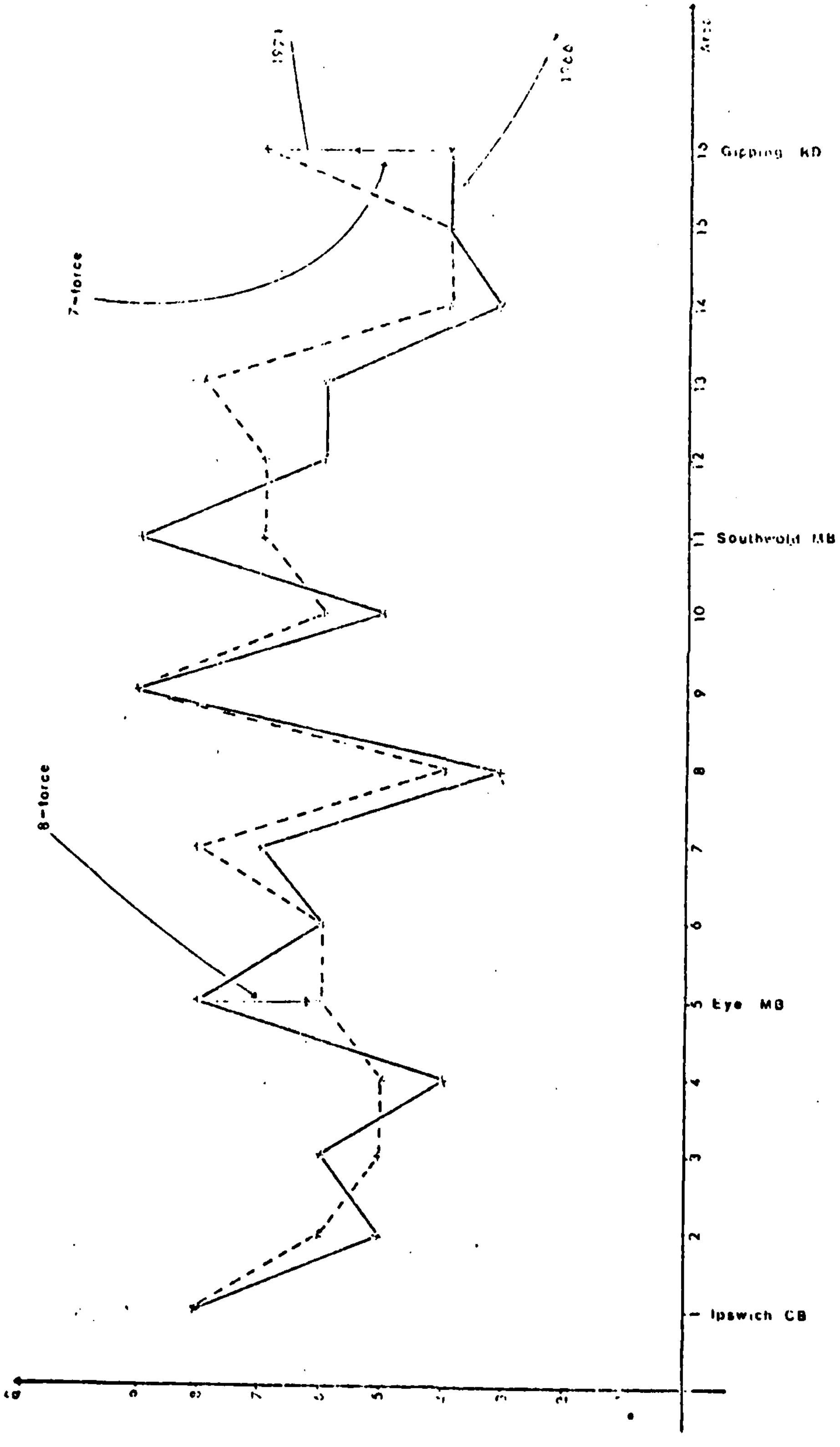


Fig.9 East Suffolk t-forces due to ΔS(N+1)

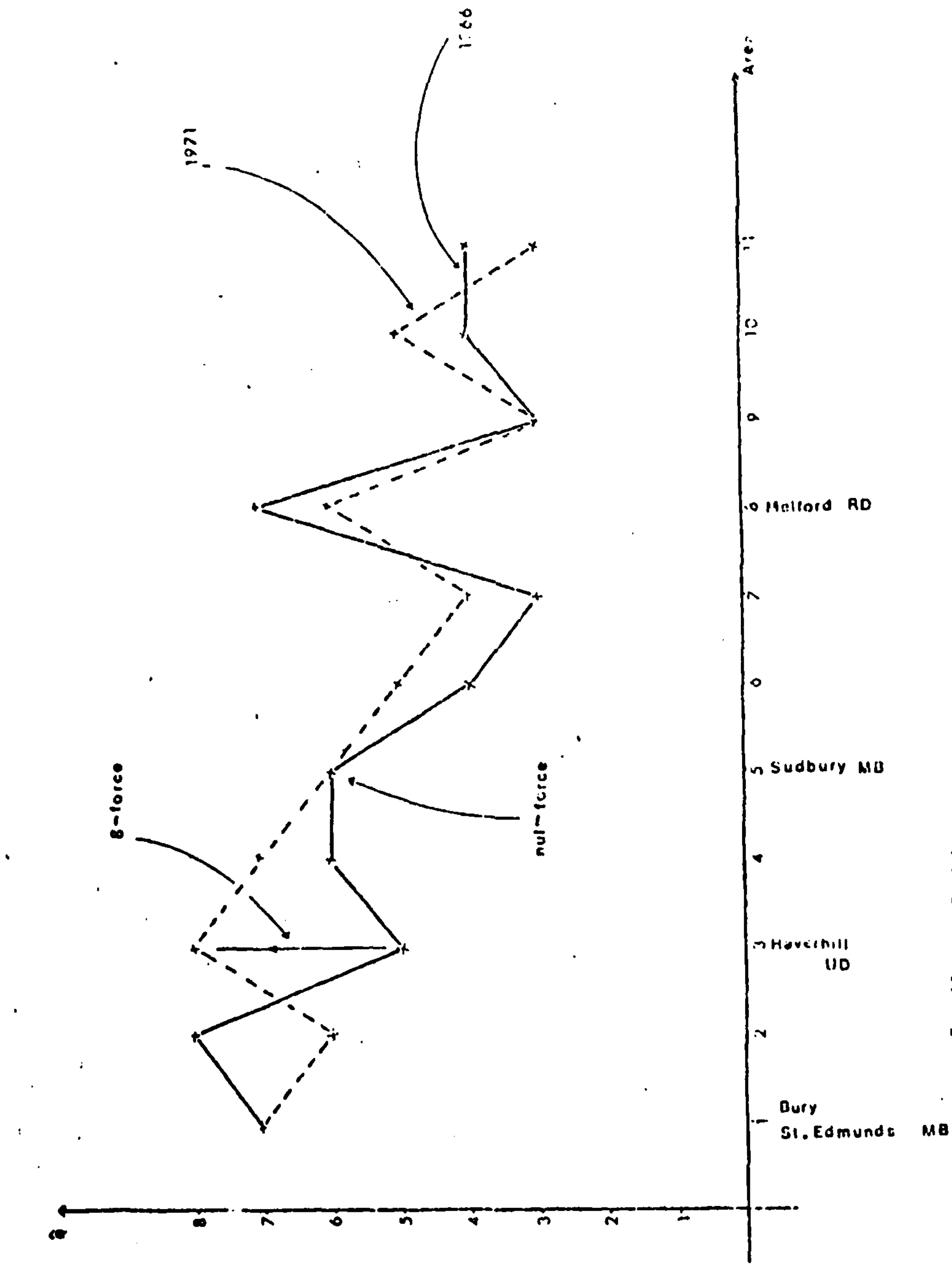


Fig. 10 West Suffolk t-forces due to ΔS (N+1)

They are taken from the author's Regional Study (Research Report V) and refer to the structures obtained from the Census Data (for 1966 and 1971) for the four counties of East Anglia - the relation being that between the District Authorities and the Industrial Classification (Employment) at the (N+1) -level , and the data being sliced at the mean values over each County (as described earlier). The separate changes for the four counties are represented in Figs. 7,8,9,10, and the t-forces are identified in selected Districts in each county - consequent upon the changes which occurred during the period from 1966 to 1971 . For example, in Cambridgeshire and Wisbech MB (which means Wisbech Municipal Borough) we notice that the top-q value of that District dropped from a value of 9 in 1966 to one of 7 in 1971. This meant that traffic on the structure must have experienced a 9-force of repulsion during that period (quite apart from other changes at lower q-values) and that this repulsion was associated, in the geometry, with Wisbech itself. This represents a shrinking of the "employment horizon" (which t measures) in Wisbech - so it would be experienced by (eg) the traffic of "employment seeking males" or the traffic of "capital available for investment in industry which involves 10 of the job classifications in the SIC list" , etc..

In Figs. 11 and 12 we show similar results for the structures for the single county of Cambridgeshire, but separated for the male and female employment data. Thus we are seeing the differences (for 1966 and then for 1971) between the geometry which carries male-oriented traffic and that which carries female-oriented traffic throughout the county.

In Fig.11 we see that throughout the greater part of the county there is a strong bias in favour of male-oriented traffic. For example, in

Wisbech MB, we see that the experience of shifting from the male structure to the female structure would require the traffic to overcome t-forces for which t = 5,6,7,8,9. The only forces of attraction on female-oriented traffic occur in March UD, Whittlesley UD and Ely UD. In Wisbech RD (that is, Wisbech Rural District) there is no apparent difference in thetwo structures.

In Fig. 12 we see that the position has noticeably deteriorated as far as the female-oriented traffic is concerned. Only in S.Cams.RD is there any bias in favour of this traffic. In all other areas there is a total absence of positive (attractive) t-forces acting on the female-oriented traffic.

The following list shows the occurrence of t-forces in the female structure (only) over the time period 1966-1971 - for the county of Cambridgeshire.

Area	t-force
Cambridge MB	nil
Chatteris UD	4-force (attraction)
Ely UD	nil
March UD	nil
Whittlesley UD	6-force (repulsion)
Wisbech MB	nil
Chesterton RD	nil
Ely RD	nil
Newmarket RD	nil
N. Witchford RD	nil
S. Cams. RD	7-force (attraction)
Wisbech RD	5-force (attraction)

This shows that the overal effect on female-oriented traffic as resulting in a repulsion from the urban district of Whittlesley, attraction to the urban district of Chatteris, attraction to the rural district of South Cambridgeshire, and attraction to the rural district of Wisbech.

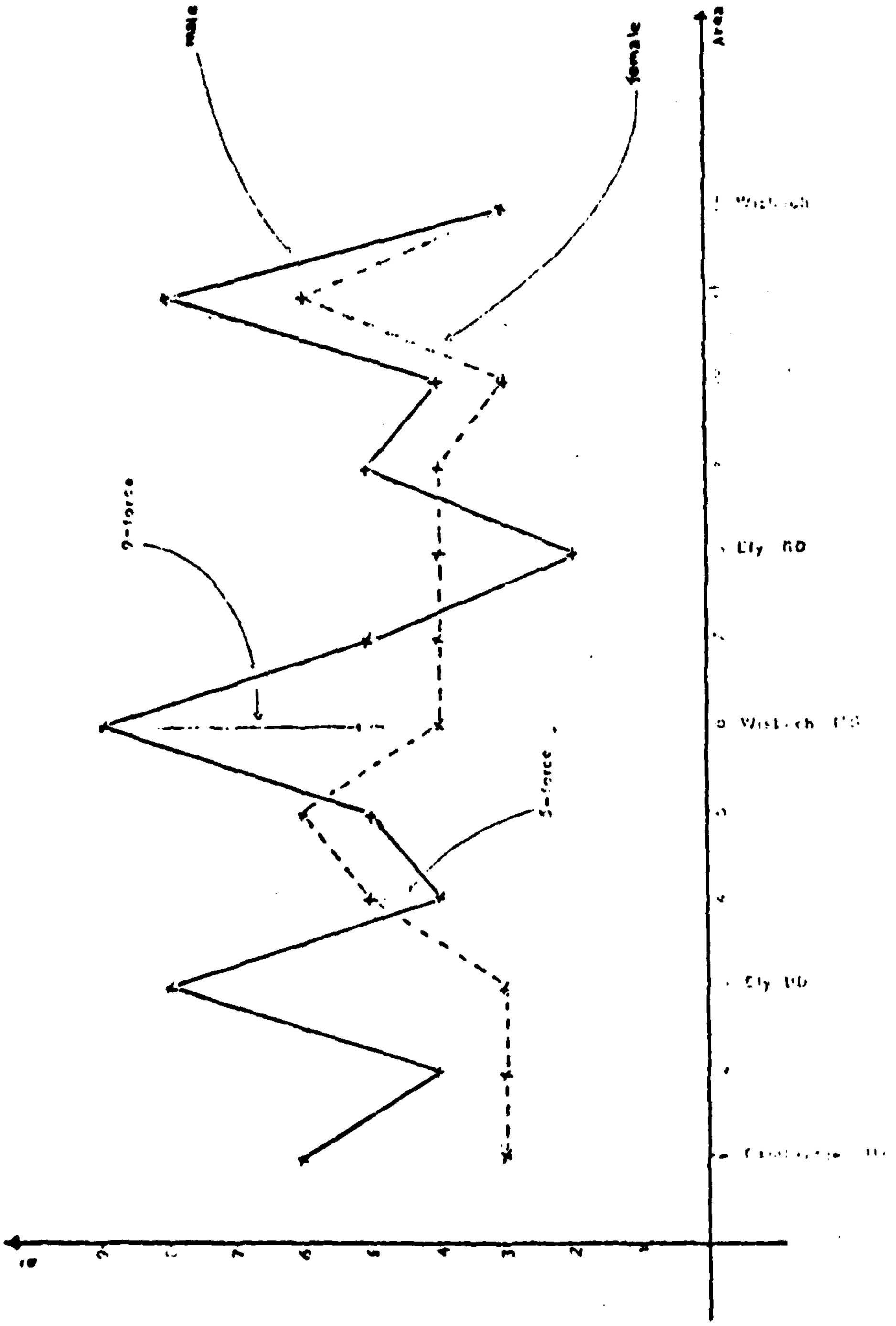


Fig.11 Female — Male E-forces 1966 D[illegible]

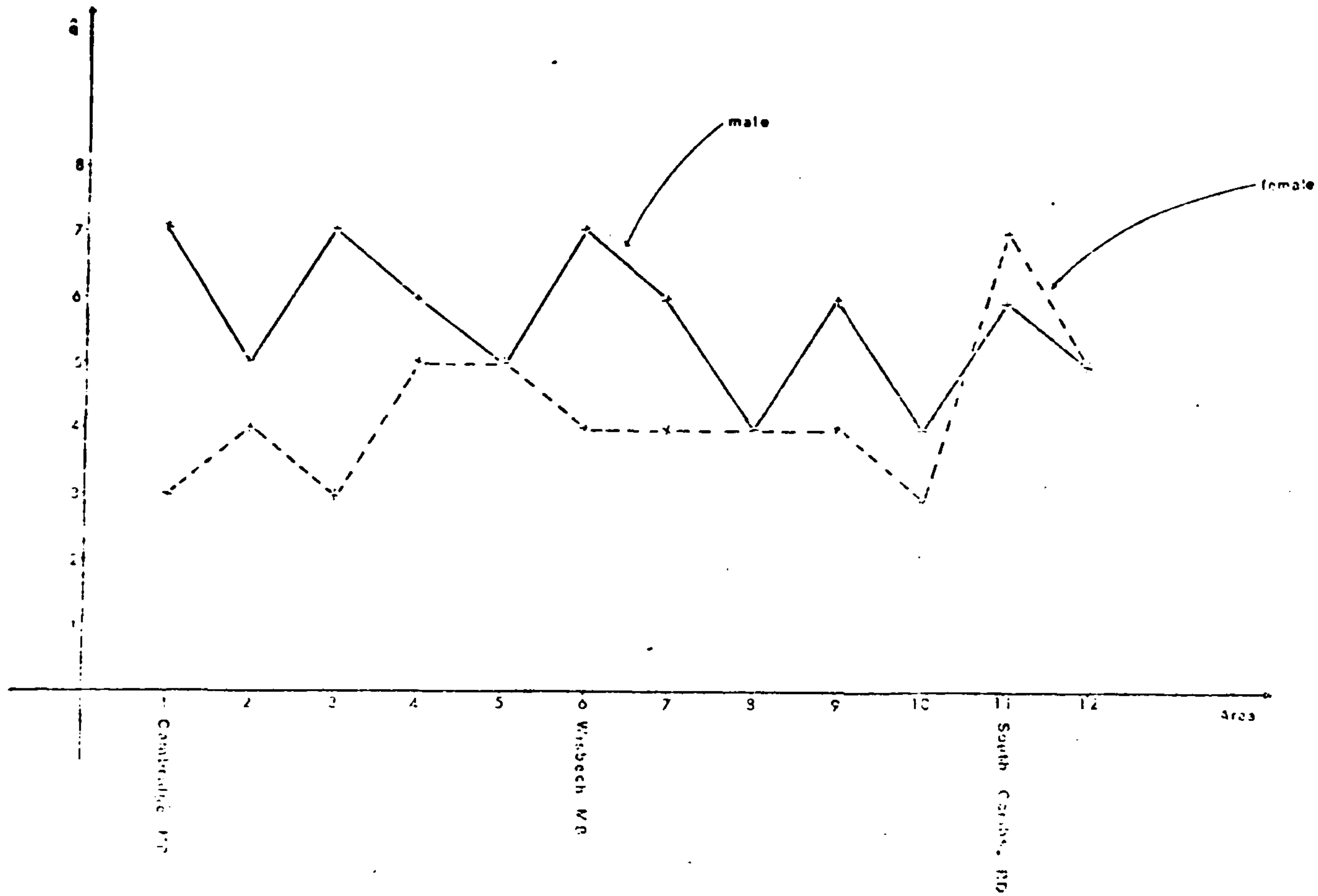


Fig.12 Female — Male t-forces 1271 Data

In general we can discuss these induced t-forces by the following schematic diagram, which is meant to represent a computer-based algorithm for finding the $\delta\pi$ when we are given (or contemplate) a change $\overset{\bullet}{\delta}S$ in the backcloth. If that backcloth is regarded as a complex KY(X) then we shall expect the effect to be experienced on the union $K \cup \overset{\circ}{\delta}K$, where K is the initial complex structure.

We imagine that we are given some pattern π^0 on the set Y (so that it is a 0-graded pattern on KX(Y) - because in that structure the Y's are the vertices). Then θ is the pattern generator (already defined above) which provides us with a graded pattern π over the whole of the complex KX(Y) - and θ' does the same for the structure $K' = K \cup \overset{\bullet}{\delta} K$. Now the change $\overset{\circ}{\delta}$ takes this KX(Y) up to K'X(Y) and this induces the change $\bar{\delta}\pi$ associated with K'X(Y) : so we have the equations

$$\bar{\delta}\pi(K') = \pi(\overset{\circ}{\delta} K) \quad \text{and} \quad \bar{\delta}(\overset{\circ}{\delta})\cdot\theta = \theta'\circ\delta$$

since $\bar{\delta}$ is a function of $\overset{\bullet}{\delta}$.

The above discussion takes us up the left-hand side of the diagram. For the right-hand side we consider that we are given some pattern $\tilde{\pi}$ on the conjugate complex KY(X) (with values on the Y's, as simplices) and then the mapping η identifies these numbers as an equivalent π^0 on the vertex set Y in KX(Y) . Then the map β tells us how to associate π, on KX(Y), with this $\tilde{\pi}$ on KY(X) : it must be such as to make

$$\beta = \theta\cdot\eta \quad , \text{ and of course } \quad \beta' = \theta'\cdot\eta',$$

then, as before, we have $\tilde{\delta}\tilde{\pi}(\tilde{K}') = \tilde{\pi}(\overset{\circ}{\delta}\tilde{K})$.

These are shown in Fig. 13 , and the ideas are discussed in greater detail in the attached Research Report X .

$$
\begin{tikzcd}
 & \bar{\delta}\pi[K'X(Y)] & \\
\delta\pi^{\circ}[Y] \arrow[ur, "\theta'"] & & \tilde{\delta}\tilde{\pi}[K'Y(X)] \arrow[ul, "\beta'"'] \arrow[ll, "\eta'"] \\
 & \pi[KX(Y)] \arrow[uu, "\mathring{\delta}"] & \\
\pi^{\circ}[Y] \arrow[uu, "\delta"] \arrow[ur, "\theta"] & & \tilde{\pi}[KY(X)] \arrow[uu, "\mathring{\delta}"'] \arrow[ul, "\beta"'] \arrow[ll, "\eta"]
\end{tikzcd}
$$

Fig. 13 Effect of structural change $\mathring{\delta}$ on patterns π.

[1.8] Some local geometry in the structure : q-holes

There seems to be the possibility of the local geometry, in a complex K, exhibiting what I have called q-holes (they are discussed at greater length in Combinatorial Connectivities in Social Systems (Birkhauser, 1977)). These will consist of at least four simplices which are q-connected in such a way as to form a loop with consequent "hole" in the middle. But this "hole" is weaker than the corresponding hole in homology theory (because it can be a cylinder with open ends) - rather it shows parallels with the generators of $H_1(K_q)$ - where H_1 denotes the 1st. order Homology Group and K_q is the network we get from the simplices of K which are at least q-connected.

In the author's University Study (Research Report IV and reproduced in the Birkhauser book) q-holes were found in many of the components of the complex KC(P), derived from the relation between the set C of university committees and the set P of people who sat on them. An example was the 4-hole with members

Academic Planning Committee	:	AC/PL
Committees on Chairs	:	CHAIRS
Board of Maths Studies	:	MA/ST
Committee on Computing	:	CPTG
Senate	:	SENATE

in that order, looping round to AC/PL again. This meant that (eg) AC/PL and CHAIRS shared a 4-face (5 people in common as members of their separate committees). Similarly there were at least 5 people common to the other neighbouring pairs in this 4-hole. Now this hole must provide a certain kind of boundary to the traffic which can move on the backcloth KC(P). Thus an item of business which interests (not less than) 5 people on two committees on opposite sides of the loop (eg. AC/PL and MA/ST) cannot find a single home (committee) in which it can be discussed (for decision-making purposes) - because the local geometry does not offer a place for it. So this item of business must exist in at least two places round the loop, and the class of such items of business can therefore only find the "place" in the geometry which consists of the whole loop. Thus this class of business becomes identified with traffic which circulates around the 4-hole. So this kind of traffic must "bounce off" any such q-hole it encounters in the structure - because it cannot go through it - and so it "sees" the q-hole as an opaque "solid object". This is why it is also legitimate to describe a q-hole as a q-object: it is a q-hole when viewed as a property of the local geometry of the backcloth, but a q-object when encountered by dynamic traffic which is moving on that backcloth.

In this particular context one would suppose that the q-objects for higher q-values would be relevant to the more significant traffic in the community - insofar as it refers to business items which are of interest and concern to a larger number of people. So the filling of a q-hole (if that is possible) is likely to be appreciated by more people if q is large than if q is small. And this "filling" of the q-hole, in this context, would seem to be highly relevant to the question of making decisions on university business items. For such a process involves a considerable re-arrangement of priorities/rankings over many matters of policy (what is often unkindly known as horse-trading) and this requires all the relevant traffic to be accomodated in one place in the geometry (that is to say, in one committee). If the traffic is constantly having to move around a q-hole then decision-making becomes impossible in that geometry. For example, in this particular example, the 4-hole was located in S(N+1) and so any (N+2)-traffic (referring to (N+2)-business items) will make a contribution to this category of business which is circulating around. After all, the so-called "policy" matters are usually at a higher hierarchical level, and so (N+1)-committees discussing (N+1)-business items will often find themselves invoking (N+2)-policy matters in their quest for decision. But these (N+2)-matters usually <u>cover</u> many items of (N+1)-business : they therefore appear as (disguised) matters in more than one of the members of our 4-hole. This is why there is a great deal of "circulating traffic" around the q-holes in a structure such as the one of this context. Because of this I have called such traffic merely <u>noise</u> - since it gets in the way of the "signal", or decision-making possibilities. Noise can be recognised as traffic on the structure which is (often) obviously trying to avoid the decision-making net : what is called "passing the buck" is what noise is all about.

Of course, all noise is not going to be consciously of the "buck-passing" nature. Some of it will arise because of the innocence of the local geometrical structure (accidental rather than design) and then there is the problem of how decisions are made in practice - when they relate to traffic which is circulating because of the mere presence of various q-holes in the backcloth. This is when it is necessary to "fill" in the q-hole. In our committee context this will require either an official or an unofficial "committee" with suitable q-connections to join up the sides of the 4-hole, say. If it cannot be done officially (which requires a meta-hierarchy from which the structure of the hierarchy can be modified) then it is done unofficially by creating a <u>pseudocommittee</u>. This often turns out to be the Vice-chancellor, in an English university, or the President, in an American university, ... and so on for other kinds of organisations. Of course the pseudocommittee is naturally blamed for pulling strings and weilding undemocratic power - and I think this charge must be true.

However it requires a re-examination of what is democratic (or what is regarded as virtuous in that context) in this light - because we can hardly "blame" the traffic for circulating around q-holes if that is what the backcloth contains. Democracy goes out of the window when the structure of the backcloth gets punched full of q-holes !

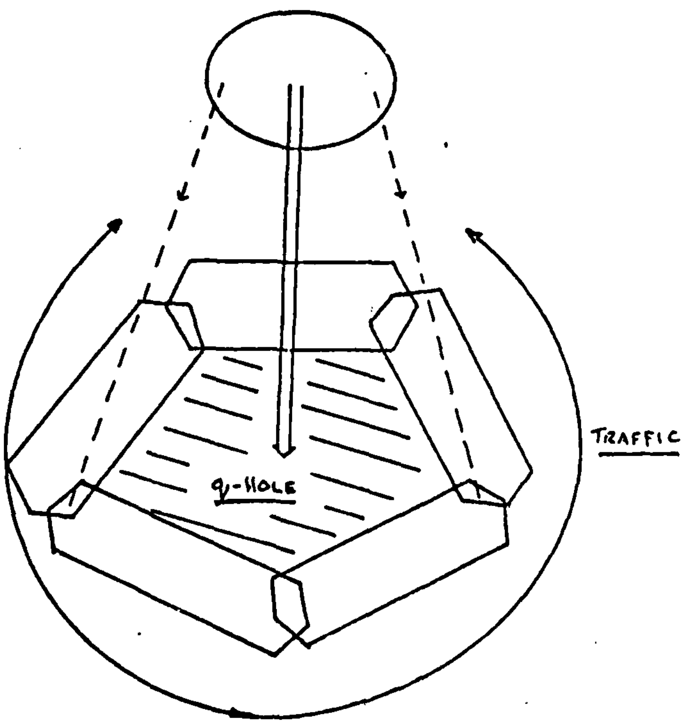


Fig: 14 q-hole and pseudo-committee

[2.0] Making decisions by trying to choose events

In a very broad sense we can regard "decision-making" as the choice of (possibly) a sequence of events (in some suitably defined space of such things) out of a set of possibilities. And this approach is behind the conventional ideas which appeal to probabilistic and/or statistical techniques. But there are hidden problems in this attitude which stem from the difficulties inherent in the following confusions.

(i) The "space of events" (which is to include all the conceivable things to be called "events") might be, on the one hand, some fixed set (like when we gamble with the roulette wheel) or, on the other hand, some variable set which is changing during our experience of it. When are we to understand the one or the other and how are we to build this understanding into our formal (mathematical) discussions of the process of decision-making ?

(ii) When we try to "control" events by making decisions of choice in a standard cybernetic way (where we might use ideas like feedback) then how are we to "model" the space in which we are operating ? For in a real sense we should be operating in a meta-space (?) - since the space of events (in the model) must be subordinate to the "space" in which we make the "decisions" ? So how do we model a space of events which are themselves models of the space of events ? This deep self-reference problem means that we cannot use conventional "modelling" ideas - now we need a meta-modelling methodology, wherein we can see the relations between models and whereby we construct models which are now so flexible that conventional representations - such as via systems of differential equations - seem inappropriate.

(iii) Our sense of control, by defining a sequence of events, is intimately mixed up with our sense of <u>time</u> . If we adopt the orthodox (Newtonian) notions then "time" is some a priori sequence and "events" somehow fit into this by being associated with the "now-moments" (member of the a priori sequence). This is essentially a <u>linear</u> concept and so it dominates (that is to say, the ideas of linearity dominate) the mathematical techniques. Furthermore it gives us no facility for the notion of "meta-time" (where the meta-decisions are to be made) and this emphasises the rigidity of the modelling which is conventionally used. There is no doubt that this rigidity is appropriate in many physics-type science situations - where the "laws" determine a rigid "sequence of events" which (rather like that generated by the roulette wheel) remains constant. But in the sphere of social science, where human beings make decisions, where politics/economics/sociology all interact, we can hardly expect the same circumstances . Rather we we would expect the "natural laws" appropriate to these areas to be "meta-laws" (relative to those of physics) - that is to say, laws about "how ordinary laws change" - or even meta-meta-laws ? So we need a different appreciation of what we mean by "time" - and consequently of "events" associated with that time ?

The following two sections give a discussion (a critique) of conventional attitudes to (a) simple-minded "laws" in social science, and (b) basic ideas of probability (which is regarded by many scientists as genuinely representing our intuitive notion of "likelihood"). The third section is devoted to a discussion of our accepted scientific use of "time" and indicates how it can be changed.

[2.1] Regression analysis destroys the structure in the data

When we consciously or unconsciously reject the idea of the need for a "meta-law" in dealing with social scientific problems, then naturally we fall back onto the hard science of physics and try to find "laws" in that sense. One widespread idea is that of a functional relation between sets - the sets, say X and Y, being a numerial representation of some relevant traffic (to use the word "traffic" in the sense of this paper). Linear regression analysis (the linearity is not essential to the validity of this criticism) is a good illustration of the technique.

The idea is that, suppose we are given a scatter diagram which relates the set X with the set Y and that, using some agreed numerical scaling, we can fit co-ordinates to each of the members of these sets. So then we find that the points (which represent this relation) form a subset of the cartesian product $Y \times X$.

We then try to replace this relation bya function (linear regression looks for a linear function of course) and to make it as reasonable an ambition as possible we find a "best-fit" curve (a straight line) between the points with co-ordinates (X_i, Y_j) - after we have identified the co-ordinates to be used on the diagram. As an illustration we take the points (the relation, λ) as shown in Fig. 15. X contains 10 and Y contains 8 members ; there being a total of 26 points in all .

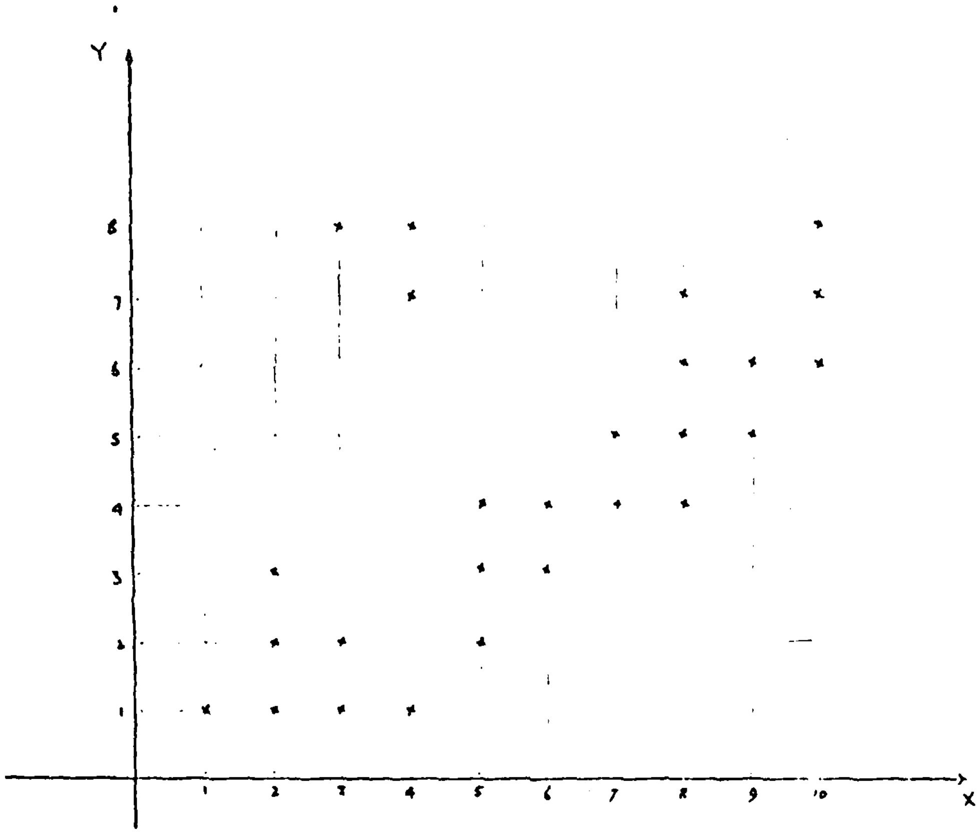


Figure 15

For computational purposes we suppose that the pair (X_i, Y_j) in the relation λ possesses the co-ordinates (i, j). Then standard theory considers the two regression lines

$$Y = a + bX, \quad \text{of } Y \text{ on } X$$

and

$$X = c + dY, \quad \text{of } X \text{ on } Y$$

obtained by minimising (e.g.) the expression

$$\sum_{i=1}^{N} (a + bX_i - Y_j)^2$$

with $N = 26$.

These regression lines are shown in figure 16 and have equations

$$Y = 1.5 + 0.49\,X$$
$$X = 2.1 + 0.71Y$$

The correlation coefficient is given by r where $r^2 = 0.49 \times 0.71 = 0.35$, so that $r = 0.59$.

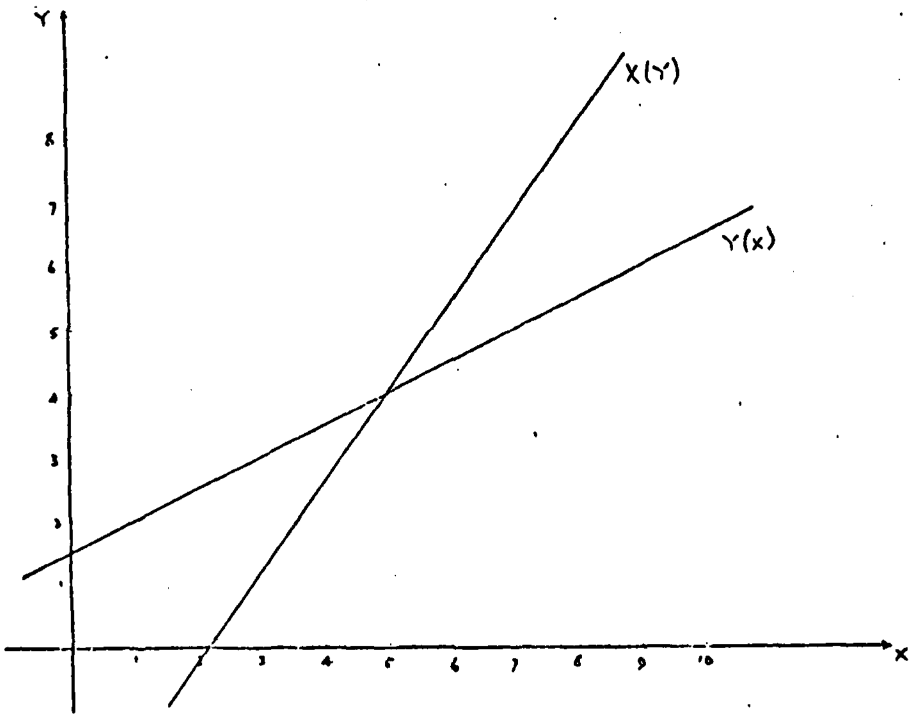


Figure 16

When the two regression lines are coincident ($r = \pm 1$) there is a perfect linear relation between X and Y which means that the relation λ is actually a function (single-valued). Otherwise r is taken as a measure of how nearly the relation λ is a linear map.

But these two regression lines fail to represent the structure of the complexes KY(X) and KX(Y), although they are an attempt to replace them as follows:

regression line Y on X, Y(X) replaces the structure KY(X)

and regression line X on Y, X(Y) replaces the structure KX(Y).

In either event we lose the multidimensional geometry which is inherent in the relation λ. In fact, when we accept one of these regression lines, say Y(X), as a substitute for KY(X) we are regarding the structure of λ as that of a collection of disconnected 0-simplices. This follows because Y(X) is now a functional relation between X and Y and its incidence matrix (compare that for λ in Section 4.2) reflects this, containing only a single 1 in any one row or column. If X contains n elements the complex KY(X) is replaced by a regression-line complex whose structure vector is the simple one with $Q_o = n$ and $Q_q = 0$ for all $q > 0$. Similarly, if Y contains m elements, the complex KX(Y) is replaced by the simple structure with $Q_0 = m$ and $Q_q = 0$, for $q > 0$.

The regression analysis therefore loses all the connectivity structure which is contained in the multidimensional geometry of the relation λ.

In comparison we can attempt to compare the two structures KY(X) and KX(Y) directly by examining the structure vectors $\underline{Q}$ and $\underline{Q}'$. By embedding these vectors in a vector space over the reals, V(R), we regard this Q-space as a euclidean metric space, say E^{k+1}, where k = max {dim KY(X), dim KX(Y)}. If we denote an orthonormal basis for V(R) by the set $\{e_o, e_1, \ldots, e_k\}$ then we have

$$\underline{Q} = Q_o e_o + Q_1 e_1 + \ldots + Q_n e_n \qquad n = \dim KY(X)$$

and

$$\underline{Q}' = Q'_o e_o + Q'_1 e_1 + \ldots + Q'_m e_m \qquad n = \dim KX(Y)\ .$$

It then seems natural to introduce a <u>first structure coefficient</u>, h, for these complexes and define it as cos θ, where θ is the angle between the vectors $\underline{Q}$, $\underline{Q'}$. This will be given by the standard formula,

$$|\underline{Q}| \, . \, |\underline{Q'}| \, . \, h = (\underline{Q}, \underline{Q'})$$

where $|\underline{Q}|^2 = Q_o^2 + Q_1^2 + \ldots + Q_n^2$

and the scalar product $(\underline{Q}, \underline{Q'}) = Q_oQ'_o + Q_1Q'_1 + \ldots + Q_nQ_n'$
it being understood that $n \geqslant m$ and that $Q'_r = 0$ if $r > m$.

For the numerical example of this Section we have

$$\underline{Q} = e_o + e_1 + 8e_2 + 2e_3$$

$$\underline{Q}' = e_o + 2e_1 + 6e_2 + e_3$$

and so $|\underline{Q}|^2 = 72$, $|\underline{Q}'|^2 = 42$ and $(\underline{Q}, \underline{Q}') = 53$

This gives

$$\underline{h = 0.96}$$

suggesting a much higher structural correlation than regression-line. correlation.

Another interesting example of the difference between h and r occurs in the relation of figure 17.

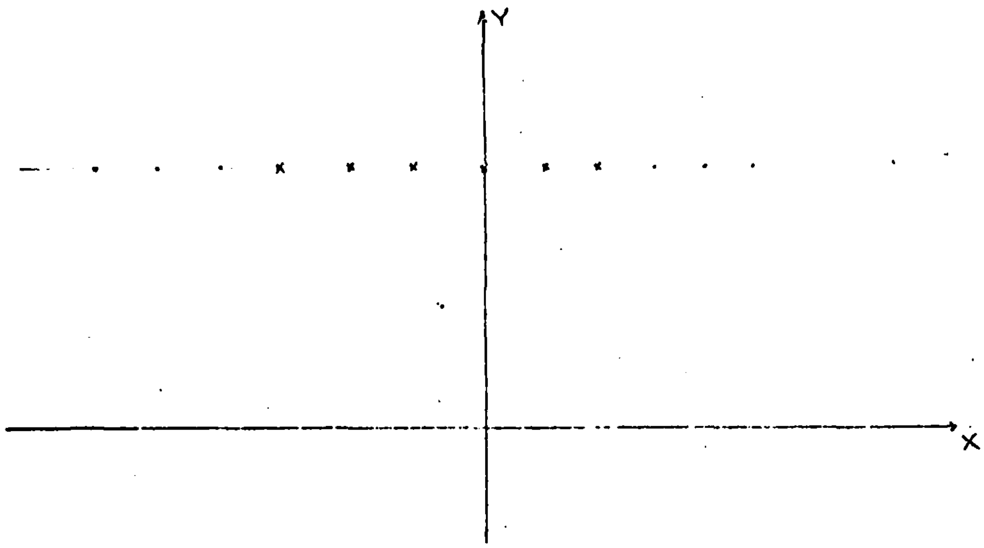


Figure 17

Here we suppose that Y contains one element which is related to the N elements in X. Since the relation is already linear we can take $r = 1$. But from the point of the structures we have:

$$KY(X) = \{\sigma_{N-1}, \text{ only}\}, \quad 1 = Q_{N-1} = \ldots = Q_0$$

$$KX(Y) = \{\sigma_0, \text{ only}\}, \; Q_0 = 1, \; Q_i = 0 \text{ for } i > 0 .$$

We therefore get $|\underline{Q}|^2 = N$, $|\underline{Q}'|^2 = 1$ and $(\underline{Q}, \underline{Q}') = 1$

so that $$h = {}^1/_{N^{\frac{1}{2}}}$$

illustrating that h is a measure of structural dependence (in a multidimensional geometry). Its value corresponds closely to the intuitive idea that KX(Y) is only "$^1/N$ th." as structured as is KY(X).

On the other hand we would expect the structure coefficient h to give us a value of unity when the structures are identical. This will occur when the incidence matrix λ is symmetrical, so that it equals its transpose. This certainly occurs when λ defines a finite projective plane or represents a symmetric block design. From this point of view we have the intriguing idea that in the family of all simplicial complexes the finite projective plane corresponds to the same thing as does the linear relation in the family of all statistical scatter diagrams.

[13] Connectivity of events - likelihood and probability

In this section I want to raise the question of the role of probabilistic statistics in our social sciences. To what extent is the use of mathematical probabilities justified as an expression of our intuitive sense of likelihood ? After all, that is what the use of distribution theories is really all about ? By taking refuge behind a standard statistical formulation are we too readily abandoning the ancient scientific rationale which searches for a determinism in our universe ?

To look at this problem we examine the basic notions of the probabilist's "space of events" (sample space) and search for its intrinsic structure (in our terms). This leads us to a specific conjecture, viz.,

> probabilistic interpretation of events is justified if and only if the corresponding structure of those events is described by a zero obstruction vector.

Interestingly enough we shall see that the concept of the two conjugate complexes KY(X) and KX(Y) play an interpretable role in the elementary algebra of probabilities.

Whether or not it is reasonable to apply probability theory (its analytic/algebraic results) to social problems via, for example, distribution theories (c.f. the entropy-maximising arguments discussed in Res. Report X) is an important methodological question. The mathematical distribution theories which purports to describe how people do their shopping, choose their homes or marriage partners, are subject to the spread of disease, etc., leave many of us with a sense of lilliputian insignificance. But perhaps these theories are relevant only to an event structure KY(P) where the people, P, become the vertices. The conjugate structure KP(Y) naturally represents each P_i as a significant structure in its own right and how does probability theory deal with that situation ?

1. Structure of the Space of Events

In the first place we consider the set of events, E, as the finite set generated by the n elementary events $\{X_1, X_2, \ldots, X_n\} = X$, and suppose that $n > 2$. The sample space is then regarded as the power set X, viz. P(X),

$$E = P(X)$$

and this means that, omitting the empty set $\emptyset$, the contemplated set of events number $N = 2^n - 1$. If we write

$$X = \{X_i \mid i = 1, \ldots, n\}$$

and

$$E = \{E_j \mid j = 1, \ldots, N\}$$

the conventional view of this probabilistic sample space is equivalent to the postulate of a mathematical relation

$$\lambda \subset E \times X$$

where $(E_j, X_i) \in \lambda$ means that X_i occurs as an element in the event E_j.

Since $E = P(X)$ every subset of X is an event in E and this means that the simplicial complex

$$KE(X)$$

must be a single simplex σ_{n-1}, together with all its faces. Thus any particular event E_k denotes a face of σ_{n-1}, and in the geometrical representation of KE(X) (in the euclidean space $\mathcal{E}^{n-1}$) this means it is a subpolyhedron of the (n-1)-dimensional polyhedron σ_{n-1}. This is why, in the Binomial Distribution, the number of p-dimensional events must be $\binom{n}{p+1}$, this being the number of combinations of n vertices taken (p+1) at a time.

<u>Example 1</u> When X has 4 members $\{X_1\ X_2\ X_3\ X_4\}$ they become the vertices of a tetrahedron (σ_3) in E^3 and the sample space E is the set of $15 = 2^4 - 1$ events represented by the faces of σ_3, viz.,

4 vertices, 6 edges, 4 triangles, 1 tetrahedron.

These events (members of E) can be denoted by the following notation, in which (e.g.) E_{123} means the event represented by the "triangle" $\langle X_1\ X_2\ X_3 \rangle$.

	X_1	X_2	X_3	X_4
E_1	1			
E_2		1		
E_3			1	
E_4				1
E_{12}	1	1		
E_{13}	1		1	
E_{14}	1			1
E_{23}		1	1	
E_{24}		1		1
E_{34}			1	1
E_{123}	1	1	1	
E_{124}	1	1		1
E_{134}	1		1	1
E_{234}		1	1	1
E_{1234}		1	1	1

By inspection of this matrix we obtain the following structure vectors.

In KE(X), $\quad \underset{\sim}{Q} = \{\overset{3}{1}\ \ 1\ \ 1\ \ \overset{0}{1}\}$

In KX(E) $\quad \underset{\sim}{Q}^{-1} = \{\overset{7}{4}\ \ 4\ \ 4\ \ \overset{4}{4}\ \ \overset{3}{1}\ \ 1\ \ 1\ \ \overset{0}{1}\}$

The sample space, E, from this structural point of view is therefore a <u>graded space of events</u>, conveniently written as

$$E = E^1 \cup E^2 \cup E^3 \cup \ldots \cup E^n .$$

when (e.g.) event $E_{\alpha_1\alpha_2 \ldots \alpha_p}$ is in the space E^p and the conventional binomial distribution of probabilities must then correspond to the <u>pattern</u>

$$\pi = \pi^0 \oplus \pi^1 \oplus \ldots \oplus \pi^p \oplus \ldots \oplus \pi^{n-1}$$

where $\pi^p : E^{p+1} \to \mathbb{R}$ and $\pi^p(E^{p+1}) = \frac{1}{N}\cdot\binom{n}{p+1}$. This means that π takes the <u>constant value</u> N^{-1} on each event in E.

Furthermore, the Q-Analysis of KE(X) immediately gives the <u>structure vector</u> as

$$\underline{Q} = \{\overset{n-1}{1} \; 1 \; \ldots \; 1 \; \overset{0}{1}\}$$

with a corresponding <u>obstruction vector</u>

$$\hat{\underline{Q}} = \{\overset{n-1}{1} \; 0 \; \ldots \; 0 \; \overset{0}{0}\}$$

Thus, between $q = 0$ and $q = (n-2)$ there is <u>zero obstruction</u>, in KE(X), to changes $\delta\pi$.

Since the Poisson and Normal distributions may be derived from the Binomial they too are based on the case that $E = P(X)$ so that we see that this simplicial complex structure plays a fundamental role in that general statistical theory which relies on probability distribution functions.

The conjugate complex KX(E), defined by λ^{-1}, is such that each X_i is λ^{-1}-related to m events, E_j, where

$$m = \binom{n-1}{0} + \binom{n-1}{1} + \binom{n-1}{2} + \ldots + \binom{n-1}{r-1} + \ldots + \binom{n-1}{n-1}$$

Since (e.g.) X_i appears in events like $\langle X_1 \ldots X_i \ldots X_p \rangle$ $\binom{n-1}{p-1}$ times (fixing X_i and selecting the remaining (p-1) elements), this gives us $m = (1+1)^{n-1} = 2^{n-1}$, and so each X_i in KX(E) is a $(2^{n-1} - 1)$-simplex.

How are these simplices connected?
Well, if we consider X_1 and X_2 as a typical pair we can easily evaluate the q-values of their shared faces, in the following way. X_1 and X_2 share a vertex when that vertex is any event with a name like $E_{12\ldots}$. This means that

in E^2 they share 1 vertex (the event E_{12}),
in E^3 they share $\binom{n-2}{1}$ vertices, because they share E_{12i} for all i,
and in E^p they will share $\binom{n-2}{p-2}$ vertices - every event $E_{12\ldots}$ which possesses (p-2) subscripts (other than 1, 2).

It follows that X_1 and X_2 (and similarly every pair X_i, X_j, where $i \neq j$), share a q-simplex (are q-connected) where

$$q + 1 = \binom{n-2}{0} + \binom{n-2}{1} + \ldots + \binom{n-2}{n-2} = 2^{n-2} \qquad (n > 2)$$

The structure vector $\underline{Q}^{-1}$ for KX(E) must therefore be

$$\begin{array}{lllllllll} & \underline{Q}^{-1} = \{n & n & \ldots & n & 1 & \ldots & 1\} \\ & & \downarrow & & & \downarrow & & & \downarrow \\ \text{where} & q = & 2^{n-1}-1 & & & 2^{n-2} & & & 0 \end{array}$$

This vector contains an equal number of 1's and n's, viz.,

2^{n-2} entries of 1

and $2^{n-1} - 2^{n-2} = 2^{n-2}$ entries of n.

The obstruction vector $\hat{Q}^{-1}$ contains zero's only for $q = 0$ to $q = 2^{n-2}-1$. inclusive.

If $|Q|$ denotes the standard norm of Q we get, for KE(X):

$$|Q|^2 = 1^2 + 1^2 + \ldots + 1^2 \qquad (2^n \text{ terms}).$$
$$= 2^n$$

whilst KX(E): $|Q^{-1}|^2 = 2^{n-2}(n^2) + 2^{n-2}(1^2)$
$$= 2^{n-2}(n^2 + 1)$$

The <u>first structure coefficient</u> h is now given by

$$|Q|^2.|Q^{-1}|^2.h^2 = Q \,.\, Q^{-1} = 2^{n-2}(n) + 2^{n-2}(1)$$

giving

$$h^2 = \frac{2^{n-2}(n+1)}{2^{n-1}\sqrt{n^2+1}} \quad \text{and} \quad h = \frac{1}{\sqrt{2}}\left\{\frac{n+1}{\sqrt{n^2+1}}\right\}^{1/2}$$

If we now regard the space of events E as the power set, P(X), of a <u>countable</u> set X then we must regard the structure vectors as corresponding to the case $n\to\infty$. This means that Q^{-1} tends towards Q but that the angle between these vectors approaches from above the limiting value of $\pi/4$, since $\lim_{n\to\infty} h = \frac{1}{\sqrt{2}}$.

2. <u>Difficulties in Interpreting probability as likelihood</u>

If we consider the probability distribution as a pattern π on the events E we are actually dealing with a graded pattern on the structure KE(X). When we write down the orthodox condition, viz.,

$$\sum_{E} \text{prob } (e \in E) = 1$$

where the summation is over all the events in E, we are in effect ignoring the grading of the pattern π. For, in fact,

$$\text{prob } (e \in E) = \pi^t(e) \quad \text{whenever } e \in E^{t+1}$$

But if we overlook this feature, for the moment, we can move on to the notion that prob (e) = 1 means that the event "e" is certain to be observed. The assumed connection with the intuitive idea of likelihood (the "chance" of an event being observed, on the occasion of a measurement or trial) is that the values of this probability set function constitute a legitimate and plausible measure of that idea.

In terms of the methodology of Q-Analysis this needs to be formulated in the following way.

(i) The notion of likelihood is an example of traffic on the structure KE(X) of the space-of-events. This is legitimate because it meets the requirements that traffic must be defined in terms of the vertex set X (the set of elementary events).

(ii) The representation of this traffic-of-likelihood by a suitable pattern π on KE(X) can also be always arranged so that

$$0 \leq \pi(e) \leq 1 \quad \text{and} \quad \sum_E \pi(e) = 1, \text{ for all } e \in E$$

since these are trivial arithmetical constraints.

(iii) But π must behave in such a way that it represents the traffic both before and after the observation of an event. So whatever the initial distribution (over KE(X)) we assume for π, the process of observation which results in an event e_0 must be manifest by a change $\delta\pi$ via which the new pattern π' satisfies

$$\pi' = \pi + \delta\pi$$

and $\pi'(e_o) = 1$ whilst $\pi'(e) = 0$, $e \neq e_o$

(iv) The above discussion means that the traffic (sense of likelihood) experiences a whole set of t-forces characterised by $\delta\pi$, forces which evacuate some simplices (events) by repulsion and which attract towards other simplices (in fact, e_o). This would be the standard Newtonian interpretation on a rigid static backcloth S(N), viz., the complex KE(X) of events. But it is therefore important to stress that the geometry of KE(X) must be capable of carrying the t-forces represented by $\delta\pi$. In the usual case which we are now considering this geometry is adequate because it has a zero obstruction vector $\hat{\underline{Q}}$.

This suggests that we have already found, via consideration of the geometrical structure of KE(X), a criterion for interpreting a probability function as a pattern appropriate to the traffic of likelihood (expectation, chance).

<u>If KE(X) is the structure of the event-space then likelihood (as traffic) can be represented by a probability function in which the grading is ignored if and only if the obstruction vector $\hat{\underline{Q}}$ is zero.</u>

We see that this condition is certainly fulfilled whenever the set of events is of the form E = P(X). But this situation is actually uncommon. Whereas it is plausible to guess at a zero obstruction vector in traditional gambling activities (throwing dice, drawing cards) the study of data in many other areas (such as social communities, urban studies, psychology, medical diagnosis, design, etc.) points strongly to the breakdown of this criterion. A zero obstruction vector for a complex KE(X) seems to be the exception rather than the rule. It is of course confused by a confusion of hierarchical levels of data — because if we increase N sufficiently (in considering $KE(X)$ as the backcloth $S(N)$) then we can usually simplify the structure so much as to make it dimensionally trivial and so arrange for $\hat{\underline{Q}}$ to approach zero. But this process inevitably trivialises (in the colloquial sense) the data so that its content evaporates. Making N large enough is like ending up, in the extreme case, with $S(N+k)$, where the data set at the $(N+k)$-level contains one element and this will be a cover of all lower level data sets. Seen from this $(N+k)$-level the data at any lower level will be one simplex and $\hat{\underline{Q}} = \underline{0}$, but the price to be paid for this will be that there will be only one event - the "whole" data set.

3. <u>What is the role of the conjugate complex $KX(E)$?</u>

The structure of $KX(E)$ is what affects changes in traffic on $KX(E)$, traffic which is defined by the events E and which is manifest on the set X (the elementary events). If this traffic is represented by a pattern

$$\Theta : KX(E) \to R$$

then Θ could be (e.g.) a sort of inverse probability distribution. The values of Θ (regarding it, for the moment, as ungraded) would be probabilities of vertices $\{X_i\}$ which are found in events $\{E_j\}$. This situation is well known in applications of probability theory; it is particularly manifest in the appeal to Bayes' Theorem, and illustrated by some such pattern as the following.

There are m people in a set P and n social groups in a set G; given a probability function π = prob (person P_i is in group G_j), how can we find a probability function Θ = prob (group G_r contains a person P_s)?

Here we are appealing to the events in two conjugate structures. If π is a pattern on $KY(X)$ then Θ is a pattern on $KX(Y)$. A theorem such as Bayes' Theorem is a means of relating the (ungraded) patterns π and Θ , in special circumstances, These circumstances involve a partitioning of the set (the group G), which is equivalent to identifying G as the formal set X of elementary events. Thus the "problem" quoted corresponds to identifying $G \equiv X$ and $P \equiv E$.

But we have seen that even when KE(X) possesses zero obstruction it does not follow that KX(E) possesses zero obstruction. This means that even in situations in which $\underline{Q} = \underline{0}$, and ungraded probability is the legitimate pattern for likelihood traffic, it does not follow that $\underline{Q}^{-1} = \underline{0}$. So even if the pattern π represents likelihood for KE(X) the pattern Θ cannot represent likelihood for KX(E).

This leaves us with a new research problem, viz., how are we to find a new pattern on a "structure of events KE(X)" which can more faithfully represent the traffic of likelihood and which, in the special case in which $Q = 0$, coincides with the standard probabilistic theory ?

[2.3] Time as a pattern on a multidimensional structure

We cannot form any sensible idea of theory-predictive matters without, in the last analysis, "taking on" the notion of time. In this paper (and in associated published work) the idea of "space" has been developed via the notion of relation between observed "points" : this point of view is Aristotelian/Leibnitzian rather than Newtonian/Cartesian. The latter view (which is commonly held in hard science) says that "space" is a priori and "objects" just sit in it. But the relational view says that our awareness of "space" comes through our awareness of the relation between "objects" (these are the first things, space comes afterwards).

But now if we propose that space is not just an old euclidean wastepaper basket for holding objects, then how can we avoid saying the same sort of thing about time ? The Newtonian view of time is that of a temporal wastepaper basket waiting for the events to drop into place.

Now we shall look at the consequences of supposing that time is the manifestation of relations between events .

This must mean that time will correspond to some sort of traffic on a structure of events, the backcloth S (and of course this backcloth will be dependent on the hierarchical level of events - so it should be written as S(N)). In this case, however we decide on how the traffic can be identified, time will be graded into multidimensional features, dependent on the multidimensional grading inherent in the structure of S . We can in fact see that this is a natural extension of that view of time which even the physical scientist has found to be mutable - as we compare the radical change from Newton to Einstein, in this context.

Time in Physics

The idea of time which is current in scientific circles is almost unchanged since the days of Isaac Newton. This is probably due to the fact that it has hardly been questioned by the practising scientist. All he has really required of the notion is that it shall fit into his theoretical discussions as easily as other quantities do. By "fitting in" he means only that there shall be, in some standard manner, a mathematical way of representing the thing called time and that this mathematical way shall be no more complicated than the way which represents his orthodox view of space. This has meant that, for the theoretician, it has been quite sufficient for him to be able to represent his time by a time axis. This means that when he is drawing his typical cartesian picture by using standard geometrical representations then he is quite content to have, as a representation of his time, a line in space - one which starts as a convenient origin and which extends outwards vaguely forever, marked with an arrow at the far end and with a small letter t printed beside the arrow head. Thus the scientist begins to use an algebraic symbol, suoh as the letter t, and as with his discussion of space and other quantities, this letter t is to stand for a real number. His vague idea that time is an ever-rolling stream is adequately represented in his mind by some imaginary variable progressing through the continuum of the real numbers. The idea of the progression through these values becomes modelled as an imaginary point which is allowed to move along this particular time-axis in his geometrical representation. In this respect his use of the variable t, and the assumption about time on which it is based, are still anchored in the description of time which was given by Isaac Newton in his Principia.

> " Absolute, true and mathematical time, of itself, and by its own nature, flows uniformly on, without regard to anything external. It is also called duration."

Since these early days of Newtonian mechanics, which require a variable for the time t which could be comparable with similar variables for distance, velocity, etc., the practical scientist has regarded his prime function in this context as one of refining mechanical measurements of time. This has resulted in a quite fascinating technological development throughout the centuries of devising new chronometers and other horological devices. But we must not let the scientist get away with it completely, chiefly because we can hardly be expected to accept in such a glib way the idea that, for example,a grandfather-clock is measuring"the time". This belief that it actually is measuring the time is a clear expression of the above philosophical position expressed by Isaac Newton. This position is one which asserts the existence of some "absolute time". It is as absolute and as rigid and as given as similar ideas about "absolute space". Indeed the whole of the Newtonian mechanics, and sunsequent theoretical physics, is peculiarly attached to this sense of absolutism. It has all the overtones of the Platonic philosophy and bears witness to the belief that somewhere there is "a great pendulum in the sky" which is beating out a thing called time.

But we cannot have an absolute time in this sense, if we are to be practical scientists. This is because we can only accept as universal facts things which we can observe. We cannot measure the absolute time unless we have some means of knowing that the thing which we call absolute time exists independently of the way we can observe it. This is really an impossible feat and so we are faced with the need to keep on measuring something which we intuitively believe, or hope, to be time. Thus a grandfather-clock certainly seems to have a kind of regularity about it, as it cycles through its repetitive conditions, and this intuitive sense of its regularity is what we must rely on as an expression of our inbuilt sense of time. It is quite idle at this stage to ask "how regular is the grandfather-clock". This cannot be answered without referring to another kind of clock. In the end, we find ourselves in a vicious circle chasing the idea of an absolute time. Hence we are destined, as practical scientists to press on with various devices for measuring this sense of sequential ordering and to rely on our intuitive senses in the last analysis. We have of course a great deal of personal experience of sequential ordering and this ranges over the experience of day-followed-by-night to the seasons of the year and the simple repetitive motion of a pendulum. It is in fact commonly believed among scientific historians that the great Galileo first observed the isochronous behaviour of the pendulum by watching a chandelier swinging to and fro in the cathedral at Pisa. How on earth he knew that it was isochronous is speculative, although it has been suggested that he timed it by his pulse rate. But presumably he was willing to believe that it was worth while using the words "of equal duration" to describe what he was observing and that if he followed this plan he would not come up against any serious conflict with his intuitive sense of what equal time intervals consisted. No one of us would presumably wish to question this kind of simple scientific faith, and had we been present at the time and been as inquisitive as Galileo each of us would presumably have thought about the chandelier as he did.

This Newtonian idea of time can be quite adequately represented by the following diagram, in which the numbers denote successive moments of time measuring.

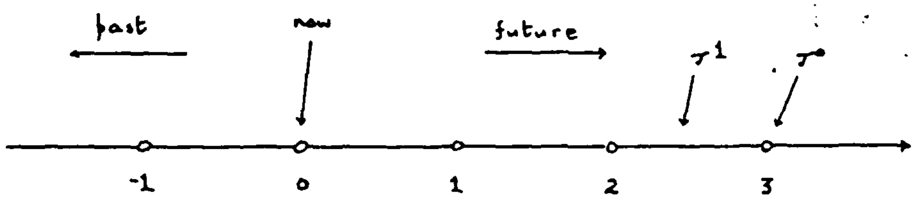


Figure 18: Newtonian time structure

These time measurements might be, for example, the completion of the swing of a simple pendulum or the successive jumps of a modern electric clock. They are denoted in the Figure by points, or vertices, whilst the lines which join them up are indicative of our sense of time duration. What we have, of course, is a particularly simple complex. This complex has an unending set of vertices and the numbers which are attributed to the vertices form a pattern τ on the vertices, that is to say on the zero-simplices. If we are traditional physicists who use this Newtonian representation of time then we usually take the next step of subtracting the successive pattern values and thus obtaining a new set of numbers which we attribute to the edges which join them. These numbers we refer to as the time intervals between the successive moments of measurement. Thus we get along scientifically with 1) a simplical complex K consisting of a continuous zero-connectivity as shown in Figure 18 and

2) a pattern on this complex which represents both "moments of time" and "time intervals".

This pattern of time is therefore a graded pattern as shown.

$$\tau = \tau^0 + \tau^1$$

It is because of this kind of representation that we can refer to the Newtonian view of time as a peculiarly <u>linear</u> concept. The linearity of the complex K is precisely the one-dimensional property which characterises it. Even here we see that the pattern which the scientist has had to use to represent his idea of time is one which has certain dimensional characteristics. Thus the Newtonian use of the time variable is a peculiarly one-dimensional idea. When the Newtonian time-axis is used to represent a set of observed events the idea behind it is to somehow produce a kind of "clock" whose time-moments (the vertices in Figure 18) can be matched up into a one-to-one correspondence with the set of events. The idea of a time interval between two events is automatically provided by appealing to the corresponding term in the time-pattern, τ^1. An important feature of the Newtonian notion of time, and how it could be used mathematically in a discussion of physics, was the idea of complete <u>independence</u> between this parameter t and all other parameters - such as those which were to represent the geometry of space or other properties of matter. In terms of our structural analysis this is equivalent to saying that the complex K represented in Figure 18 above is not connected to any other complex of operations - such as the complex of points obtained by observing 3-dimensional space.

<u>Relativity Theories</u>

This technical and philosophical position remained adequate for over

200 years until it had to be modified at the beginning of this century. This modification came about through measurements of the velocity of light and via the subsequent theoretical contributions of Albert Einstein. In his Theory of Relativity in 1905 and 1915, he found it necessary to break this idea of independence between the time structure and the space structure. This was expressed by way of the so-called space-time continuum, a concept which was essentially mathematical in its nature and which combined the traditional 3-dimensions of geometrical space with an additional dimension which was naturally associated with the observer's sense of time This combination of geometrical and time axes, particularly in the General Theory of Relativity, had to be arranged in such a way that although for one possible observer in the universe the time and the space might be quite separate and independent, yet for any other observer this would be quite impossible. Hence the time-pattern which any particular observer (a pseudonym for an experimental physicist) will find is itself dependent upon, that is to say a function of, space-like properties of the system wherein the observer is situated. Thus the time intervals will be a function of velocity and of position. This strange mix-up cannot be unravelled for all observers at all positions and all moments of time. Indeed it becomes impossible to speak of a "moment of time" as if it were something that could be simultaneously observed and understood by different observers. Thus the idea of Newtonian absolutism as well as the idea of independence had to be eliminated from the theoretical structure. But it is perhaps ironic to notice that this was only achieved by replacing one kind of absolutism by another. In place of the idea of an absolute time and an absolute space the theory of relativity introduced the idea of an absolute <u>signal velocity</u>. This refers to the velocity of light usually denoted by the letter c, which plays a fundamental physical and mathematical role in the formulation of the Special Theory of Relativity. This is because, in a practical situation, it had to be assumed that all the information about the geometry of a system is carried to the observer by the light signal. If this signal has a finite

velocity, which it has, then it builds into all the observations its own sense of delay. The absolutism of this philosophy is to be found in the observation that however it is measured it turns out to have a constant value. Thus it does not matter whether the observer is moving relative to the source of light or not. The value which he observes as the velocity of the light he receives always remains constant. It is clear that this situation is bound to affect the time-pattern τ which we have introduced above.

In order to illustrate how this happens we need to describe, if only briefly, the simplest results in Einstein's Special Theory of Relativity. To do this we consider the basic situation which is illustrated in Figure 19.

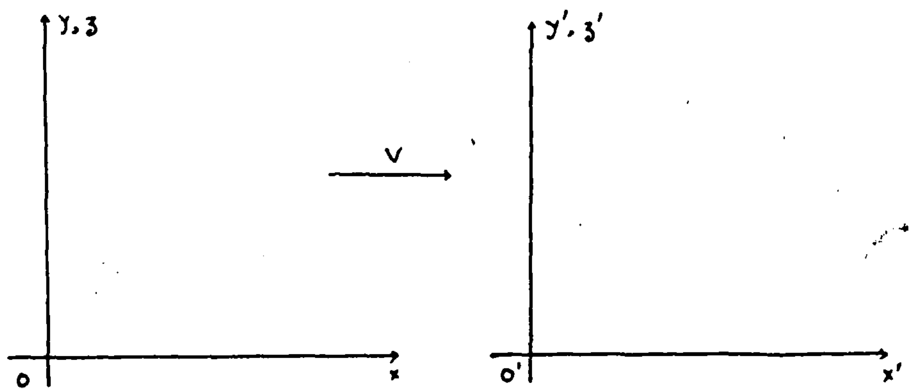


Figure 19 : Relativity frames of reference

This is a standard diagram which represents the possibility of two observers who are busy trying to observe the geometrical position and time coordinate of some moving particle. The one observer O' is supposed to be moving with a uniform steady velocity of V relative to the other observer O. The direction of this relative V is taken as the common direction of the X-axis of both observers. Then, in this simplified case, the other two geometrical axes - the Y-axis and the Z-axis - do not enter into the description in any significant way. But each observer needs to use his own frame of reference. Observer O uses coordinates $\{X, Y, Z, t\}$ whilst observer O' uses coordinates $\{X', Y', Z', t'\}$. The basic theory of this relative notion, taking into account the fact that only the velocity of the light signal itself can be regarded as constant for the two observers, gives us the standard relation between these two sets of coordinates .

$$x' = \beta(x - Vt) \qquad y' = y,\ z' = z \qquad t' = \beta\left(t - \frac{Vx}{c^2}\right)$$

where $\beta^2 = \left(1 - \frac{V^2}{c^2}\right)^{-1}$

The number denoted by β is a constant of proportionality and since it involves taking the square root of a certain expression it is clear that if V is greater than c then this expression is negative and so β does not exist as a real number. Herein lies a simple mathematical explanation of why in this theory it is not possible to observe anything moving with a velocity greater than that of light.

Now we can see that it is possible to represent a time-pattern which is compatible with the structure inherent in a relativistic system, by the following considerations.

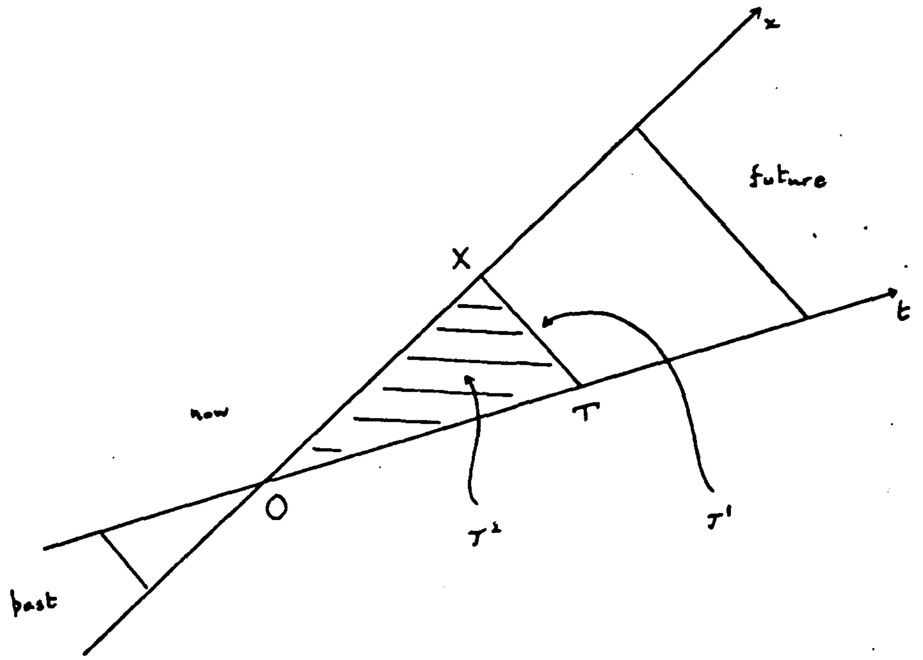


Figure 20 : Simplified Relativity $\tau = \tau^1 + \tau^2$

In Fig 20, which is based on the simplified version of Special Relativity, the Newton time-pattern $\tau = \tau^0 + \tau^1$ has given way to the pattern

$$\tau = \tau^1 + \tau^2$$

Here the "time moments" need to be represented by the 1-simplices (such as) $\langle XT \rangle$, whilst the "time intervals" become the corresponding 2-simplices (such as) $\langle OTX \rangle$. This is because, to a general observer - such as S' it is not possible to keep separate the two measures of the classical notions of x and t.

But in the more general discussion of Special Relativity, in which no one of the apparent space axes, x,y, or z is allowed any privileged role, the time-pattern must clearly be of the form

$$\tau = \tau^3 + \tau^4$$

This is illustrated below in Figure 21. Students of Relativity will appreciate that this structure is also what underlies the General Theory.

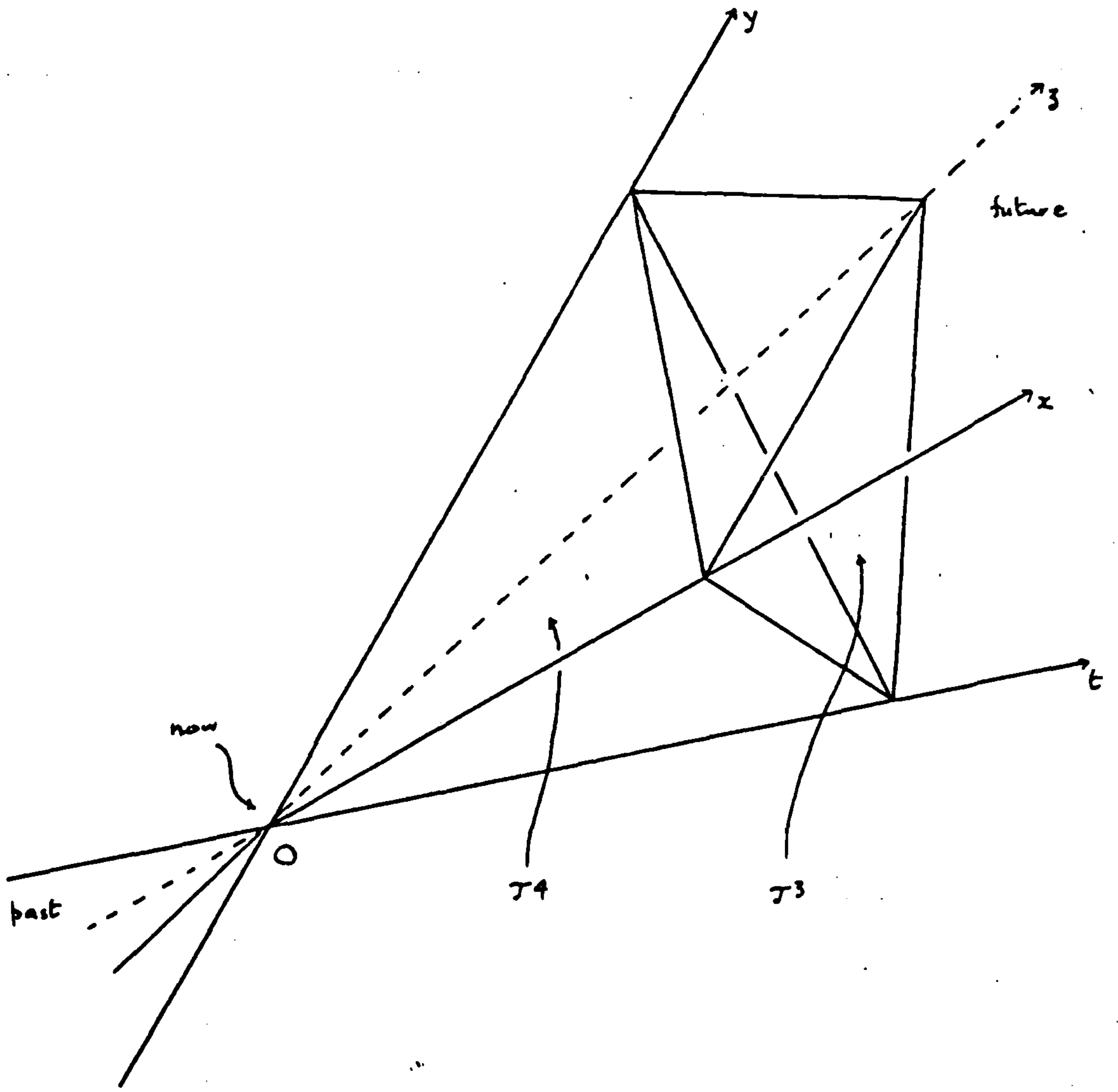


Figure 21 : Relativity $\tau = \tau^3 + \tau^4$

It is because of this shift in the dimensions attributed to time-measured that Einstein (following Minkowski) asserted the predominance of "proper-time" (given by the 4-dimensional interval ds, where $ds^2 = c^2dt^2 - dx^2 - dy^2 - dx^2$). This proper-time is eligible to be a thing like or τ^4 above, being a mapping on the typical 4-simplex $\langle O\ T\ X\ Y\ Z \rangle$ of Figure 21.

Time as a t-force

It is not an exaggeration to say that Relativity Theory is essentially a theory of time. It demanded that the classical Newtonian time-pattern be replaced by a new and different one. This difference arose because of the requirement for a new structure of physical observations (of the point-particle kind). This new structure was itself a consequence of the physical role played by the light signal. It is reasonable therefore to see this scientific advance as strong evidence for time being a multi-dimensional pattern on some structure. If that structure is the relatively simple one required for the point-particle dynamics of conventional physics then the time-pattern is the simple

$$\tau = \tau^3 + \tau^4$$

But when we move into the backcloth S(N) associated with the social and structures discussed in this research we would expect the dimensions of the graded time-patterns to be varied as a consequence.

All this would suggest that it is compatible with the physical sciences to introduce the following structural definition of time.

Definition : On any given backcloth S(N) time is a specific traffic, viz.,

(i) the traffic which consists of a total ordering of all p-simplices in S(N)

(ii) the time-pattern of this traffic is of the form

$$\tau = \sum_{p=0}^{n} (\tau^p + \tau^{p+1}) = \sum_{p} T^p$$

(iii) each T^p contains the pattern τ^p and τ^{p+1}, where

τ^p is the now-pattern

and τ^{p+1} is the interval-pattern.

It is because of this shift in the dimensions attributed to time-measures that Einstein (following Minkowski) asserted the predominance of "proper-time" (given by the 4-dimensional interval ds, where $ds^2 = c^2dt^2 - dx^2 - dy^2 - dx^2$). This proper-time is eligible to be a thing like our τ^4 above, being a mapping on the typical 4-simplex $\langle O\ T\ X\ Y\ Z\rangle$ of Figure 21.

In the Newtonian case we are supposed to accept that S(N) is a 1-dimensional structure, and so the time-pattern can only be $\tau = \tau^0 + \tau^1$. In the Einstein case we are supposed to extend this so that S(N) is of maximum dimension 4. This would give

$$\tau = T^0 + T^1 + T^2 + T^3$$

where $T^0 = \tau^0 + \tau^1$ (the Newtonian special case) and where T^1, T^2, T^3 are various versions of the Relativity case. In this particular instance we already see that the representation of the now-concept is a graded pattern

$$\nu = \nu^0 + \nu^1 + \nu^2 + \nu^3$$

whilst the interval-concept is a graded pattern

$$\pi = \pi^1 + \pi^2 + \pi^3 + \pi^4$$

Our definition clearly gives us a genuine generalisation of the orthodox scientific view of time-coordinatisation and of time-interval. But now we can begin to analyse its significance in other areas of experience. If each individual can be regarded as traffic on some S(N) then his sense of time is linked to that particular structure. In addition it is likely that S(N) contains a classical physics-type structure which provides him with a Newtonian time $\tau^0 + \tau^1$. The culture of our civilization presently assumes that whenever we use the word "time" we are in fact referring to this Newtonian τ^1. But although this convention is a most useful one, by providing us with a common frame of reference, yet it nevertheless disguises the essential nature of our structurally-based experience of time.

To be precise, we consider the consequence of this cultural assumption. It consists in drawing the conclusions from a change in the time-pattern which "referring to Newtonian time" involves. Thus suppose an individual experiences a time traffic of dimension p, where $p > 1$. Then he finds it socially and culturally necessary to replace this by the Newtonian τ^1 , as follows.

time-pattern $\rightarrow$ $\tau^p + \tau^{p+1}$ (the "now" is on p-simplices)

changes to $\rightarrow$ $\tau^0 + \tau^1$ (Newtonian pattern)

But this means that the individual experiences a t-force of repulsion (where $t = p+1$), as far as the interval-pattern is concerned. This t-force is variously expressed by such phrases as "time flies" or "time drags" - an indication of the fact that τ^1 and τ^{p+1} are out of step.

This means that, because each of us is associated with some structure S(N), we feel that time (that is to say, Newtonian τ^1) is experienced as a set of t-forces in that structure. The concept of "event" is a peciliarly structural concept. Some events are 0-simplices, some will be 3-simplices; generally an event will be the "recognition of a p-simplex" in S(N). Now if we assume (and we stress that this can only be an assumption, without more research data) that, in S(N), there is a (1-1) correspondence between its 1-simplices and those of the Newtonian reference frame, then the time-intervals for the gap between one p-simplex σ_p and the next σ_p will bear a numerical relationship to the τ^1, viz.,

$$\text{time-ratio of} \quad "\binom{p+2}{2} \text{ to } 1"$$

because this $\binom{p+2}{2}$ of the number of edges in the least connection (a σ_{p+1}) between σ_p and σ_p'.

This would suggest that a p-event (recognition of a σ_p in S(N)) "takes longer to arrive" than does a 0-event by a <u>factor</u> of

$$(p+2)(p+1)/2$$

As an example, if the τ^1 unit is 1 day, then an event which needs 28 days to mature (to arrive) must be a 6-event, because of the equation

$$(p+2)(p+1) = 56$$

reducing to

$$(p-6)(p+9) = 0$$

Although we do not really know (without a detailed research programme to investigate it) at this stage whether or not the τ^1-on-S(N) is identical with the Newtonian-τ^1 (based as that is on some external point-particle-dynamics structure, such as the standard clock or pendulum) nevertheless it is of interest to list the time-intervals associated with various p-events, if that were in fact the case. In this list the interval-pattern is the corresponding τ^{p+1}.

p-event	ratio of $\tau^{p+1} : \tau^1$	(worst-ca
0-event	1	1 day
1-event	3	
4-event	15	
6-event	28	1 month
10-event	66	
18-event	190	
25-event	351	$\sim$ 1 year

If the unit of τ^1 is 1 day then a 25-event requires almost 1 year to "arrive" - that is to say, the interval between successive 25-events is 351 days.

The calculations assume that a p-event (recognition of a σ_p is S(N)) occurs by way of the "edges" which go to make up the σ_{p+1} which bridges the previous-σ_p and the present-σ_p. If, on the other hand, we assume that the situation is the worst possible then we might not be able to recognise a σ_p until all its faces have been separately recognised. But this would involve recognising all the intervals between the successive $\sigma_o, \sigma_1, \sigma_2, \ldots \sigma_p$. This number is the sum

$$\sum_{t=1}^{p+1} \binom{p+2}{t+1} \binom{t+1}{2}$$

since the number of t-faces of a σ_{p+1} is $\binom{p+2}{t+1}$, and the number of edges in each σ_t is $\binom{t+1}{2}$. This gives us a time ratio of structural-interval to Newtonian-interval, for any successive p-events in S(N), as

$$(p+2)\ (p+1)\ .\ 2^{p-1}$$

This worst-case ratio gives us the following list:

p-event	ratio of $\tau^{p+1} : \tau^1$	(worst-case)
0-event	1	1 day
1-event	6	
4-event	240	
5-event	672	$\sim$ 2 years

We notice too that the ratio contains the factor 2^{p-1} and this means that there is a natural log-scale relation between the two times (c.f. discussion of HOM and MOH in Research Report X.).

Of course, if we move up the hierarchy to S(N+1) then it will generally be the case that a σ_p at this new level will correspond to a much greater σ_q at the N-level ($q \gg p$). This would imply that, on any assumption about the ratio of time-intervals, a 1-event in S(N+1) requires a considerable time ratio when observed in S(N). If, for example, each vertex at (N+1) covers 6 vertices at N - and assuming a partition, then

a 0-event in S(N+1) $\equiv$ a 6-event in S(N)

and so the time-interval between successive 0-events in S(N+1) will be 28 units in S(N) - not using the worst-case. But an argument can be made out for associating this "worst-case" with the question of <u>moving up the hierarchy from</u> N <u>to</u> (N+1). This rests on the fact that at the (N+L)-level <u>all</u> the subsets of an N-level set can be seen as vertices (at (N+1)). The number of non-empty subsets of the N-level, (p+1) vertices, is (2^p-1) suggesting a log-relation once again. <u>If we therefore move from an N-level time-pattern to an (N+1)-level pattern we would be inclined to experience what we have called this worst-case correspondence</u>.

In either event we see that this idea of a graded time, associated with ideas of abstract geometrical structure, matches well our experiences of when events actually occur (in Newtonian time). Most events are not "points" and so cannot be fixed on a simple axis. At what precise point, for example, does one "grow up"; when "precisely" is a business successful ? These p-events are not 0-events and so Newtonian time is not applicable. Recent studies of the so-called "body-time" would seem to support this analysis. If bodily functions at certain physiological N-levels are characterised by the progression from one p-event to another p-event, then we would expect the time-intervals associated with these rhythms to be correspondingly proportional. If the pulse rate is used as a match with the Newtonian τ^1 of 1 second then the body's sense of an 8-hour day corresponds to the experience of a 240-event (approximately). Does this have physiological implications at that N-level defined by the "heart beat" . What is the set of 241 vertices (give or take a vertex) which constitute this 240-event ?

By using this kind of approach we can see that some new numerical relations can be calculated for time-intervals between different kinds of events (say, between one p-event and a second p-event). This has consequences for the standard time-series (dynamic modelling) analysis in predictive modelling .

But apart from the attempt to connect τ^{p+1} and τ^1 there is a general problem of time factors involved in the change of any pattern on a structure S(N).

If $\delta\pi = \delta\pi^0 \oplus \delta\pi^1 \oplus \ldots \oplus \delta\pi^p \oplus \ldots$

then the change $\delta\pi^p$ is idenfied with some $\Delta\mu^p$ and so is associated with the set of (p+1)-events (simplices) in S(N). If we observe these changes in Newtonian clock-time then we shall find that the ratio of clock-times needed to observe $\delta\pi^0 : \delta\pi^p$ is likely to be the ratio of $1 : \binom{p+2}{1}$ or $1 : (p+2)$. Does this give us an idea of predicting (in terms of clock-time) the consequences of planning decisions ?

[3.0] Decisions as traffic on a backcloth of p-events

In any situation the data at our disposal will constitute a hierarchy, H, (of various levels) which can be regarded as the backcloth structure S (actually it will be a set of structures, labelled S(N-1), S(N), S(N+1), etc.). And this backcloth will also carry relevant traffic - expressed dynamically as patterns which exist on the relatively static S . Such a collection can equally well be called a <u>system</u> , although conventional systems theory does not usually identify these structures in the way that has been done in this paper. [Notice that we are implicitly defining a "system" as data organised in this manner.]

But we shall now need to say that, at any particular N-level, the system will consist of a <u>graded set</u> of <u>events</u> (what we have called the p-events). Furthermore the backcloth, S(N), tells us how these p-events are topologically related to each other (via the geometry of the complexes which make up S(N)). More precisely, it tells us about the whole <u>range</u> of <u>possible events in the system</u> and how one p-event can be linked with another p-event, if at all. On top of this we must ask the question : "Who (or what) experiences these events ?" , and the answer to this must be : "The traffic which lives on this backcloth". Traffic experiences a p-event by being defined on it (by its pattern) : it experiences a second p-event by its change (via a $\delta\pi$) over S . In the simplest sort of case the traffic can be regarded as being defined by a pattern, π, which only takes the value "1" on some particular p-simplex σ_p and 0 on all other p-simplices. Then it is p-traffic (only) and located on one p-event in the backcloth. But its next eperience (consequent upon some $\delta\pi$) effectively moves this "1" onto another σ_p , leaving a 0 behind on the initial simplex. Of course this is only possible for this traffic if these two simplices are p-connected (by being faces of some (p+1)-simplex) in the structure of S(N).

In a more general case we would expect the traffic to be represented by a pattern which would have non-zero values on a number of p-simplices (and for different values of p). As far as the p-events are concerned, for one particular value of p, this traffic experiences a number of them simultaneously (or should we use the Jungian term of "synchronistically" ?) - but to differing degrees of intensity measured by the ratios of the pattern values (the "weights") on them. So now the traffic either sees a sort of smudge of p-events (which, we notice, constitute a <u>subcomplex</u> of S) or it picks out one as outstanding - by using some ranking of the pattern values to provide a <u>choice function</u> on this set of p-events which are suddenly available in S(N). <u>But is this not the essence of decision-making</u> ? Does not the traffic under discussion havd to "decide" how to identify its p-events in S(N) (how much of its subcomplex smudge it is going to see), and does this not greatly influence the possiblity of controlling the next set of p-events which can be brought into the domain of this particular traffic's experience ? For now the traffic must contemplate, in the framework of the geometry of S(N), how its present p-event subcomplex is connected to a future p-event subcomplex. If "it" makes the wrong "decision" then it will anchor its experience of p-events at the "now" in some part of S(N) which is so isolated and disconnected that it cannot move into any "future" p-events at all. On the other hand a judicious identification of its own p-event subcomplex (by suitable slicing of its pattern values, its π^p) will ensure the possibility of a wide horizon of p-events in the future. And of course, with an eye on some particular future p-event, it is essential to know enough about the geometry of S(N) to be able to say whether or not there is any possible decision which can ensure the traffic eventually arriving there. So the topology of the backcloth, over the whole of the hierarchy of data, plays a determining role in the decision-making process. And this process involves the interplay between changes in pattern, $\delta\pi$, and the obstruction vector $\hat{Q}$ for the backcloth, S .

But this also means that we must distinguish between the two cases, already outlined in previous sections, viz. :

Case I When the backcloth, S(N), for all relevant values of N, remains constant during the period of change $\delta\pi$. Now the decision-making process depends on a strategy of associating choice-functions (derived from the patterns of the relevant traffic) with a structural geometry (of each S(N)) which is constant ; so only s single analysis of the backcloth is necessary. This Q-Analysis will then provide sufficient information about the connections between the p-events for all possible (and envisaged) changes $\delta\pi$. This kind of situation seems relevant in many instances of urban planning (where the backcloth is defined by reasonably static Land-Use relations), industrial development (when the backcloth is constant - suggesting short-term economic forecasts ?), and of many games-type situations where the backcloth of possible events is unchanging (as in simple gambling situations - roulette, dice, etc..) - but significantly this is not the case with games like Chess (v. below).

Case II When the backcloth, S(N), changes and produces subsequent changes in all the patterns on it. This situation has been discussed in Section 1.7 above (and in greater detail in Research Report X). In this case we need to be able to move from changes $\dot{\delta}S$ to the induced changes δx on S - and this naturally involves more computing (but see Fig. 13 above). An important instance is provided by the game of Chess (see the attached paper : Fred CHAMP - positional-Chess analyst) This brings out the fact that, for a given background structure (for a given hierarchy H) it may well be quite feasible to have traffic which describes all the essential "positional" features of the system (in our case this traffic consists of well-defined patterns whose names are cval, pval, stval, sval) but that this does not ensure a decision (about the next move). Such a decision can only be taken at the meta-hierarchical level H+1, for it is at this level that strategies can be located. A typical strategy (in the game of Chess) will be a formula for deciding on positional play. Thus it will tell us how to look at the patterns under their various headings andthen how to proceed - by way of precise ordering of these values (at different levels of H). For example, in the crudest case, the strategy might say "order the patterns by placing the highest values associated with the Centre at the top of the list ; ignore all else.". Such a strategy is placed at the first level (say, M) of H+1 : it consists of a formula which allows us to study every possible tactical move (each one of which determines a possible change in H) and then to rank them. But if we want a number of possible strategies (so that the computer shall be able to change its mind about strategy during the course of the game) then they must be placed at level (M+1) of H+1 : for there we have sets of things at the M-level .

An important line of research in decision-making is now to make Fred CHAMP into a (decision-making) chess player -

as opposed to being merely an analyst. Using the interplay of these ideas of backcloth and traffic we can expect to free the computer chess player from the orthodox approach based on nothing but ad hoc tree-searching.

In the moregeneral case we would expect that a decision which attempts to control the succession of p-events by deliberately altering the backcloth S must itself be taken froma meta-hierarchical position, H+1 . And notice that the meta-hierarchy must itself be well-defined in terms of sets and relations. Indeed the sets in H+1 must consist of relations from the hierarchy H, and the relations in H+1 will then be relations between sets of relations (from H) . Pushing this on, we see that contemplating how one strategy can lead on to another strategy is equivalent to saying that one p-event (in the meta-hierarchy, H+1) is p-connected to another such p-event (in H+1). So as we contemplate changing our strategies we are moving about over the structure of the meta-hierarchy. Since each (fixed) strategy provides us with a decision in H it seems reasonable to talk about decisions about different strategies as being <u>meta-decisions</u> (about making decisions).

Perhaps the British House of Commons has long been an example of two alternative strategies at the H+1 (M) level, represented by the two main political party manifestoes. Then the General Election becomes a decision at the H+1 (M+1) level - where the electorate decide on the meta-question - a long way (structurally) from the daily routine of the citizen's decision-making (in the hierarchy H). This meta-decision literally happens by 0/1 pattern moving over two simplices (the two political parties) until the "1" settles on one of them (perhaps) ; this change of pattern occurring between successive General Elections (each of which, in the hierarchy H, is a p-event for some large value of p) .

Research into this analysis of decision-making would seem best served at this stage by a careful monitoring of such problems via specific cases and systems. This could be done for industrial organisations, political systems (at various social levels), military complexes, administrative systems, family groups, ... even down to studies of the individual psyche. Then, with an eye on computerising the process, the Chess game should be developed - and possibly extended to other board/field games ?

VOLUME 2A

ANNEX

Q-Analysis: Theory & Practice

Research Report X

(Study of East Anglia - 6)

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June 1977

Q-Analysis : Theory and Practice

Research Report X

(Study of East Anglia - 6)

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CONTENTS

[0] Acknowledgements

This Research Report is the tenth in a series which has described the development of the theory and application of Q-Analysis over the past six years. During that time the Social Science Research Council has supported the work by financial help and by its continuing interest. I would like to express my appreciation of that invaluable aid. My thanks are also due to my colleagues Jeff Johnson, Dave Rosen and Ges Westover for their theoretical and practical help in the preparation of this Report, and also to Jill Barber for her secretarial help in typing and producing it.

R.H. Atkin

[1] Introduction

In this Report I wish to examine some of the consequences, both practical and theoretical, of the structural analysis described in earlier Reports[1] and applied to this current Regional Study.

Data has been collected, under the broad heading of Land Use (meaning various forms of human activity), for the separate Parishes (about 1,000 of them) throughout the region of East Anglia - which comprises the present (1977) counties of Cambridgeshire, Norfolk and Suffolk. Prior to the reorganisation of local government (April 1974) Suffolk comprised East Suffolk and West Suffolk and Cambridgeshire did not include (as it largely does now) the county of Huntingdonshire. The number of vertices in the set of things called Land Use also amounted to about 1,000 and so the final data (for the year 1975) was a matrix array (approx. 1,000 x 1,000) of non-negative integers.

Qn. 1 Has it been a difficult or expensive task to collect this data ?

The answer to this is reasonably, No, on both counts. Of course the data is not being continuously monitored (and up-dated) but this is relatively easy to do. The basic set of data was collected by a team of one supervising research fellow and two research assistants working full time over a period of 3-4 months, and this was without the resources which would normally be available to the officers of (e.g.) a county planning department. The data was obtained by consulting large scale survey maps, Kelley's directories, yellow pages of district telephone directories, and by touring round selected parts of the region. Of course, an important point is to distinguish between the static backcloth of data and that which purports to describe the relatively dynamic traffic (of various kinds) on that backcloth.

In addition, the data must be collected with due regard to the <u>hierarchical structure</u> which is inherent therein - the structure which we constantly refer to as

level N, (N+1), (N+2), etc.

Qn. 2 <u>What is the significance of the hierarchical levels N, (N+1), (N+2), etc., and how are they defined in a practical way</u> ?

This raises an important and profound point of the methodology, and is the subject of the next section.

[2] The hierarchical set of levels, II.

We shall not expect to have a satisfactory mathematically based description of social data unless that data is identified in terms of sets and set membership. This view of data (this kind of data) is the basis of the so-called "hard" sciences (the physical sciences) and we can naturally refer to it as hard data. Lots of data is hard (for example), if we count the number of employed persons in the city of Norwich on any one day of the week we are identifying a set viz: {employed persons} . But lots of so-called data is not hard, it is "soft", because it fails the set-membership test. A good example of this occurs when, for example, good citizens are asked for their opinion about proposed urban and regional planning developments - opinions about mathematically ill-defined concepts such as "improvements in the environment", "more congenial amenities", "social opportunities", and other commonly occurring phrases. Of course these things have meaning, and the search for data based on them is surely an honest and well-intentioned one, but they do not qualify for the identification of set-membership data. Their "meanings" are ambiguous and commonly lead to inconclusive debate among the citizens. Because of the "soft" nature of much of the discussion (the strictly unscientific quality of the "data), everyone loses - the professional planners struggling to present a rational story to the public, the elected representatives of the citizens who carry the ultimate responsibility of decision-making, the conservationists and civic societies who see their roles as being guardians of (their own "meanings" of) these "soft" standards. Only a hard scientific and rational analysis of this whole field of social "data" can carry any hope of including all the interested parties. But such a view must not be developed by abandoning the soft concepts. It must be one which encompasses all the relevant data, which strives for a holistic analysis of the system (the town, the community, the county, the region).

But collecting hard data presupposes in the first instance that the set is well defined. By this we mean that, given a set X, it is possible to say of some proposed element, either that it is, or that it is not, a member of X. Such decisions are binary, being Yes/No or 1/0 type. In the physical laboratory it is common practice to delegate these binary decisions to the physical instruments themselves, thus removing the day to day problems from the scientist. Thus an ammeter says Yes/No to physical candidates for membership of the set of electrical charges. It not only identifies set-members (hard data) but also counts them. This kind of hard data collection is now so commonplace in our technological world that perhaps it induces us to regard the data problem as trivially easy to master. But in the medieval pre-Galilean study of motions of bodies (with its idea that "motion is the realisation of a body's potential") the distinction between soft and hard data was far from obvious and the resolution of the problem occupied many generations of scholars before the decisive break-through was achieved by Galileo. For example, to get some feeling of what the statement about "motion" and "realisation of potential" might have meant we need only move it into our own times and sideways into another (soft) science, whence it could easily become "education is the realisation of the individual's potential." To make this into a scientific statement, as hard for future generations as is classical kinetics to this one, we need a mathematical language which describes "education" in terms of relevant hard data. Such a description would have to provide a "kinematics" for that discipline, comparable to the kinematics of the motion of bodies.

Whereas a geographer can probably feel quite happy about the hardness of much of his data (for example, the set of streets in a town, or the set of contours over a region) there will be situations where he might be tempted to substitute soft data for hard, particularly in those areas where "human geography" creeps over into sociology, etc. In the field of urban studies how do we obtain hard data relevant to the planners asking the question "what urban structure is most conducive to making your town a pleasant place to live?" Here we need a set whose members are well defined things called "urban structures" and then we might imagine that we need a set whose members constitute well defined things called "pleasant places to live." The latter seems to lead inevitably to the question of "value judgements" and where are the hard data in that context? Many social scientists would probably argue that the presence of such judgements must mean that mathematical-set-data are fundamentally irrelevant and so their pursuit is an illusion. But this view ultimately negates the scientific approach and is not unlike the Aristotelian objections to Galileo's definition of velocity (as distance÷time) which, apparently, removed all the "content" (or poetry) from motion - in making it hard. But the poetry of the projectile was recaptured as the poetry of the mathematical parabolic curve.

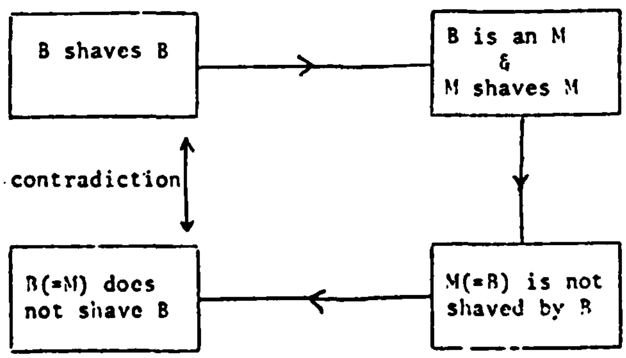
Where are the hard data in the replies to a questionnaire which asks, "do you think that Choice A is worse than/as good as/better than Choice B?" On the face of it this presents the citizen with a well defined set whose members are three in number, viz., worse-than, as-good-as and better-than. But is this set well defined among the observers (all of them) qua citizens ? Can any two peopoe agree as to worse-than or as-good-as ?

Also is it clear that any one choice is static, is it not often the case that one feels the need to answer "both worse-than and better-than "? In other words, one might really need to make a selection from the power set, P(X), of such a set X - selecting a subset, rather than a single element - or even from the power set of the power set, $P^2(X)$.

So, well defined sets certainly require (i) a common agreement about the sets (agreements among the scientific practitioners), and (ii) an appreciation of the logical (and therefore methodological) distinction between a set X and its power set P(X) and so of $P^2(X)$, $P^3(X)$, etc. This latter point is also fundamental to the avoidance of the many set-theoretical paradoxes which logicians have recently brought to light. A striking instance of this was provided by Bertrand Russell in his famous Barber Paradox, which runs as follows.

"In a certain town (Seville ?) every man either shaves himself or else he is shaved by the (male) barber. Does the barber shave himself ?"

If we assume that the barber shaves himself then we can deduce that he is a man who shaves himself and therefore he is not shaved by the barber. If we assume that the barber does not shave himself then we deduce that he is a man who does not shave himself and therefore that he is shaved by the barber.



We therefore obtain a logical paradox - which is illustrated by trying to list a relation λ between two sets Y and X. The set

$$Y = \{\text{barber}\} = \{B\}$$

while the set

$$X = \{\text{men}\} = \{M_1, M_2, \ldots\}$$

and the problem arises when we try to include B in the set X itself. Thus, let λ be a simple matrix of O's and l's as shown

λ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	B
B	1	0	1	1	0	1	0	?

In this scheme the men M_1, M_3, M_4, M_6 are shaved by B, but if $B \in X$ (if the barber is a man) then we cannot decide on the entry under B in the top row. This set $X = \{M,B\}$ is not well defined in terms of λ. The reason is that the barber, B, is not a member of X but is in fact a member of P(X). The barber, B, qua barber, is really the subset of X, viz., $\{M_1, M_3, M_4, M_6\} \in P(X)$, and because of that we cannot ask questions of B (of members of P(X)) as if B were a man, M_i (a member of X). This distinction, which is logically profound and of fundamental significance in any scientific methodology based on hard data, was classified by Russell in his "Theory of Types" - in which he insisted that we must not confuse

"elements" with "sets of elements" with

"sets of sets of elements" with ...

or

X with P(X) with $P^2(X)$ with ...

This suggests that the methodology we are seeking must depend on a hierarchical arrangement of data corresponding to these types, and the hierarchy, H, which I propose is one based on the quite general notion of mathematical cover sets. We can denote the hierarchical levels by N,(N+1),(N+2),etc.) with possible sets as follows:

H:	Level	Sets	Types
	N+2	...L...	$P^3(X)$
	N+1	A ... B...	$P^2(S)$
		μ θ	
	N	$X \overset{\lambda}{—} Y$...Z...	$P(S)$
	N-1	P ...	S

We shall expect the hierarchy of data sets, H, to be defined by mathematical relations $\lambda, \mu, \ldots$, in the following way.

If X is an N-level set and A a corresponding (N+1) level set then A must be a cover set for X, that is, the elements of A are subsets of X and if

$$A = \{A_1, A_2, \ldots, A_n\}, \quad X = \{X_1, X_2, \ldots, X_m\}$$

then

(i) $A_i \varepsilon P(X)$ for $i = 1, \ldots, n$,

(ii) $X = \bigcup_i A_i$

If, in addition, we know that $A_i \cap A_j = \emptyset$ when $i \neq j$ then of course A is a partition of X - the elements of A are quite distinct.

This idea of a <u>cover</u> (as something more general than a <u>partition</u>) is very important and gives a characteristic flavour to the way we have introduced the concept of hierarchy. But it is clearly the natural way of reading the data. A man can be a potential employee in more than one industrial enterprise if, for example, he is a trained lathe operator. In this case these enterprises have an overlap which is defined (partly) by sharing this vertex (set member) "lathe operator". The cover-set approach searches for these overlaps and needs to describe them, whereas the more orthodox approach based on partitions is concerned with separating (N+1) level words into quite distinct and disjoint classes.

In this Regional Study we have used the four levels in the following hierarchy:

<u>H</u>:	N+3	{East Anglia}	= typical set at this level
	N+2	{names of counties}	4 vertices
	N+1	{local authority names}	72 vertices
	N	{parish names}	1184 vertices

[3] The static backcloth for the region

In previous Research Reports[1] the relations λ between Land Use and local authority (geographical areas) areas at various levels have been analysed to give backcloth structures S(N), S(N+1), etc. The three levels of Land Use (using the sets X, P(X), $P^2(X)$) are:

(N+2)	-	Land Use set,	L(N+2):	8 vertices
(N+1)	-	Land Use,	L(N+1):	46 vertices
(N)	-	Land Use,	L(N):	966 vertices.

The vertices at (N+2) are:

Industry, Farming, Offices, Health & Education, Private Service, Public Service, Landscape, Transport.

The vertices at (N+1) are:

1. Chemical Manufacture
2. Vehicle Manufacture
3. Electrical Manufacture
4. Other Manufacture
5. Light Engineering
6. Heavy Engineering
7. Light Industry
8. Boat Building
9. Transport
10. Extraction
11. Food & Drink Industry
12. Construction
13. Construction Supplies
14. Construction Manufacture
15. Agricultural Service
16. Farming
17. Horticulture
18. Office Professional
19. Office Commercial
20. Office L.A. & Government
21. Office Other
22. Education - Primary
23. Education - Secondary
24. Education - Further
25. Health Hospital
26. Health Clinic
27. Residential Home
28. Residential Hostel & Hotel
29. Utilities
30. Church
31. Constabulary
32. Cultural Amenity
33. Popular Entertainment
34. Sports Facilities

35. Retail Hard
36. Retail Soft
37. Retail Food
38. Personal Service
39. Catering Licenced
40. Catering Unlicenced
41. Caravan Site
42. Woodland
43. Inland Water
44. Open Space
45. Beach - Sand & Shingle
46. Marshland

The vertices at N are to be found in Research Report VI on pages 21 to 28 (inclusive) some of which are reproduced here to illustrate:

1. Chemical Manufacture

7. [illegible] M & S
12. Agricultural & Horticultural Chemical Manufacturers
31. Anti-corrosive materials Man
65. Bottled Gas M & S
164. Chalk Man & Merchants
166. Chemical M & S
169. Chemist Analytic Consultants
176. Coatings Protective
202. Colours M & S
232. Cosmetic & Toilet Preparers
262. Detergent Man
265. Disinfectant M & S
292. Dust Collecting Chem & Materials
294. Dye M & S
319. Emulsifying Agents M & S
324. Engineers Chemical
361. Fireworks M & S
4[illegible]. Gases Industrial & Medical M & S
476. Insecticide Man
477. Insecticide Pest Control Materials & Equipment
523. Lime M & S
595. Nylon Manufacture
600. Oil Companies
602. Oil Lubricating M & S
628. Paint Varnish & Laquer M & S
631. Perfume M & S
636. Petroleum Products Man
637. Pharmaceutical Man
662. Polythene M & S
783. Soap Man
828. Synthetic Fibres M & S
829. Synthetic Paints Resins, etc.
835. Tar & Pitch Distillers & Merchants
859. Timber Preservatives M & S
900. Veterinary Products M & S
921. Whiting M & S

2. Vehicle Manufacture

16. Agricultural Implements & Machinery Manufacturers
23. Aircraft M & S
126. Car & Coach Body Builders
133. Car Man
137. Car Trimmers & Upholsterers
140. Caravan Manufacturers
192. Coach Painters
193. Coach Trimmers
204. Commercial Vehicle Manufacturers
206. Commercial Vehicles Body Building
240. Cycle Man
380. Fork Lift Truck Manufacturers
718. Road Making Machinery Manufacture
874. Trailer Equip Man
875. Trailer M & S

3. Electrical Manufacture

9. Aerial Manufacturers
47. Audio Visual Equip M & S
64. Battery Manufacturer
123. Capacitor Man
129. Car Electrical Equipment Suppliers & Repairers
203. Commercial Vehicle Electrical Equipment Man
208. Communications Equipment
210. Computers
211. Computing Machines
226. Control Panel Manufacturers
264. Dictating Machines
300. Electric Camp M & S
301. Electric Light Fitting & Fixtures
302. Electric Motors Rewinding
303. Electric Motors Dynamos & Generators
304. Electrical Component Manufacturers
305. Elec Control Gear Man
306. Electrical Domestic Appliances Man
308. Elec Element Man
310. Electrical Instrument Makers
311. Electrical Switchgear Man
314. Electronic Components
315. Electronic Equipment & Instruments
316. Electronic Systems M & S
317. Electro Platers & Metal Finishes
328. Engineers Electrical
329. Engineers Electronic
420. Glassware Man-Electric & Electronic
441. Hearing Aid M & S
519. Lighting Equipment M & S
699. Radar Equipment Man
700. Radio & Navigation Aids
701. Radio Communication Equipment M & S
702. Radio Frequency Heating Equip M & S
762. Semi-Conductor Man
787. Solenoid Man
789. Sound Equipment Systems Man
840. Telecommunication Equipment M & S
841. Telephone Equipment Man
843. T.V. & Radio Component Man
844. Television & Radio Component Wholesalers
845. T.V. & Radio Man
876. Transformer Manufacturers
934. X-Ray Electro Med App

4. Other Manufacture

2. Abrasive Mat Paper & Cloth Man
6. Adhesive Tape M & S
43. Asbestos Products
51. Baby Linen
53. Bag, Briefcase & Handbag Man
62. Basket Makers
70. Blind & Awning Man
84. Bottle M & S
88. Boxes Wooden
97. Brush & Broom Man Household
98. Brush Manufacturer
115. Cabinet Makers
121. Candle Makers
124. Car Accessory Man

4. [illegible] (cont)

[illegible]. Optical Goods Man & Wholesalers

[illegible]. Opticians Man & Wholesale

608. Organ Builders

610. Orthopaedic App Man

614. Packing Case Manufacturers Solid & Corrugated Fibre Board

615. Packing Case Manufacturers - Wooden

621. Pallete Manufacturers Wood

623. Paper & Board Man

624. Paper Bag Man

625. Paper Box Man

641. Piano Man & Dealers

642. Picture & Photograph Frame Makers

668. Pottery M & S

675. Poultry House Man

698. Racecourse & Sportsground Equip M & S

725. Rope Twine & String M & S

726. Rubber Manufacturers & Merchants

727. Rubber Product M & S

728. Rubber Products Synthetic

729. Sack & Bag M & S

730. Saddlers & Harness Makers

771. Shelving & Racking

790. Sports Goods Man & Wholesalers

793. Sportswear M & S

804. Stationery Man

822. Surgical Appliances M & S

836. Tarpaulin Man

847. Tent & Marquee Hire Service

848. Textile Man

850. Thermometer M&S

853. Tie Neckwear & Scarf Man

861. Toilet Articles M & S

862. Toilet Paper Man

870. Toys & Games Man

892. Upholstery Man

897. Veneers Man & Merchants

898. Venetian Blinds M & S

914. Waterproof Covers

916. Webbing Man

922. Windscreen Man

925. Wire Goods Man

926. Wireworkers & Weavers

927. Woodturners

929. Woodware

936. Yarn M & S

5. Light Engineering

5. Addressing Machine Manufacturers

9. Aerial Manufacturers

10. Aerosol M & S

19. Air Compresser Man

22. Aircraft Comp M & S

27. Amusement Caterers

29. Animal Sundries Man

30. Anodisers

48. Automatic Amusement Machines M & S

50. Baby Carriage Man

65. Bearings M & S

77. Bolt & Nut Man

89. Brake Linings M & S

92. Brassware Man

120. Can M & S

130. Car Engine Tuning & Conversion Specialists

153. Castors & Glides

179. Chrome Plating

182. Cinema Equipment

185. Clean & Polish Equipment Man

187. Clock & Watch Makers

223. Containers Metal

228. Conveyor Belting Man

229. Coppersmiths

230. Copying & Duplicating Machinery & Accessories

239. Cutting Machinery & Tools

241. Cycle Racks & Stands

243. Cylinder Boring - Regrinding

263. Diamond Tool Man

265. Die Sinkers & Stampers

269. Dispenser Man

277. Door Gear Sliding

287. Drilling Contractors

291. Duplicating Equipment & Accessories Man

293. Dust Extract & Vent Plant

299. Electric Cable & Wire M & S

335. Engineers Precision

338. Engineers Tools & Instruments

346. Fan & Blower Man

368. Flex Drive & Equipment Man

373. Food Cutting App Man

375. Food Processing Equipment

391. Fuel Injection Equipment

407. Garden Equipment & Tools Man & Wholesalers

431. Grinding Precision & Production

443. Heating & Ventilating Equipment Man

444. Heating Appliance Man

468. Hydraulic Flexible Pressure Pipe Man

478. Instrument Makers & Repairers

494. Jewellers Manufacturing

496. Tool & Fixtures

511. Lawn Mower Man & Servicing

535. Lubrication Equipment Manufacturer

537. Machine Tool Repair

538. Machine Tools Man & Merchants

541. Machine Repair & Recondition

552. Mechanical Handling Equipment Man

556. Metal Cutters & Shearers

557. Metal Perforators

558. Metal Spinning & Pressing

559. Metal Sprayers & Finishers

578. Mould & Tool Makers

579. Mould Makers - Bottle

598. Oil Burner M & S

617. Paint & Cellulose Spray Equip

630. Pattern Makers - Engineering

653. Plastics Mould & Tool Makers

654. Plastics Moulders

662. Polythene M & S

694. Pumps & Pumping Equipment Man

709. Refrigeration Equipment Commercial

710 Refrigerators Domestic M & S

738. Saw M & S

742. Scales & Weighing Equipment M & S

755. Scientific Apparatus M & S

756. Scientific Instrument Man

758. Screw Manufacturers

764. Sewing Machine M & S

773. Shim Man

779. Signal & Warning Device Man

794. Stampers & Piercers

799. Stapling Mach Tackers & Staples

810. Steel Office Furniture & Stores Equipment

818. Storage Equipment Man

834. Dies & Taps

852. Ticket Machine M & S

864. Tool M & S

865. Toolmakers

866. Torch Man

880. Tube Bender & Manipulators

883. Turned Parts Precision

885. Typeset Mach M & S

894. Valve Man & S

896. Vending Machine M & S

899. Ventilator Man

903. Vibrator Man Elec & Pneumatic

908. Washing Machine M & S

913. Water Treatment Equipment M & S

919. Welding Equip Man

6. Heavy Engineering

16. Agricultural Implements & Machinery Man

25. Aluminium & Alloy Rods & Sheet

58. Ball & Roller Bearings M & S

75. Roller Composition Man

76. Boiler Makers

Heavy Engineering (Cont).

91. Brass Founders & Finishers
[illegible]. Cable & Wire Fittings Man
152. Castings
158. Cement & Concrete Machinery
167. Chemical Plant & Equipment
168. Chemical Vat & Tank Makers
290. Collieries
266. Diecasters
287. Drilling Contractors
343. Expanded Metal Man
383. Founders Non-Ferrous
384. Founderies
395. Furnaces Industrial
403. Galvanisers
412. Gear Box Man
413 Gear Cutters & Makers
442. Heat Treatment Metal
489. Iron & Steel Man
491. Iron Founders
540. Machinery Man
560. Metal Stamp Man
562. Metal Workers
587. Non Ferrous Metal
600. Oil Companies
612. Oven Builders
613. Packaging Machinery
635. Petroleum Exploration Equipment Man
636. Petroleum Products Man
645. Pipes & Fittings
648. Plastic Pipes & Fittings
649. Plastic Sheeting
650. Plastics - Eng Materials
652. Plastics M & S
676. Power Transmission Equipment
677. Press Tools Man
678. Pressworkers
682. Profile Cutting
704. Railway Equipment
731. Safe & Vault M & S
739. Sawmills
760. Sheet Metal Work
769. Sheet Metal Working Machinery
780. Silo Man
795. Stainless Steel Man
796. Stainless Steel Metal Workers
809. Steel Founders
811. Steel Plate Workers
813. Steel Strip Man
814. Steel Tubes & Hollow Section Stockist
881. Tubes & Tube Fittings
887. Tyre Man & Wholesalers
888. Tyre Mould Makers
889. Tyre Retreading & Repairing

7. Light Industry

26. Aluminium Stockholders
32. Antique Furniture Repairing & Restoring
44. Asphalt & Coated Macadam Laying Man
[illegible]. Blacksmiths & Forgemasters
90. Brass & Copper Stockist
127. Car Breakers
134. Car Painters & Sprayers
135. Car Parking & Garaging
136. Rad Man & Repair
138. Car Wash & Polish
143. Caravan Repair & Servicing
160. Cemetaries & Crematoria
194. Coal & Coke Merchants
271. Display Models & Figure Man
320. Enamellers Stove
321. Engine Tuning & Reconditioning
322. Engineering Machine Shops
334. Eng Motor Cycle
385. Freight & Forwarding Warehousing
392. Funeral Directors
393. Funeral Directors Supplies
400. Furniture Repairers & Restorers
405. Garage Services
414. General Dealers
490. Iron & Steel Merchants
504. Label Printers
509. Laundries Industrial
526. Lithographic Printers
536. Lubrication Service
537. Machine Tool Repair
541. Machinary Repair & Recondition
561. Metal Treatment Specialist
572. Monumental Masons
586. Newspaper & Periodical Publishers
597. Oil Waste Disposal
601. Oil Fuel Distributors & Suppliers
616. Paint & Cellulose Spraying
622. Panel Beaters
633. Petrol & Oil Distributors
634. Petrol Filling Stations
655. Plastics Thermoforming
656. Plastics Vacuum Forming
657. Plastics Welding
679. Printers & Lithographers
691. Public Works Contractors
703. Rag Merchants
711. Removals & Storage Domestic
712. Removals Business & Industrial
716. Road Contractors
719. Road Surface Dressing Contractors
733. Salvage Dealers
757. Scrap Metal Merchants
759. Secondhand Dealers
763. Sewage Supply Authorities
767. Sheet Metal Merchants
777. Shotblasters
797. Stainless Steel Stockholders
805. Steam Cleaning Contractors
812. Steel Stockholders
816. Stone Masons
817. Storage Contractors & Services
857. Timber Importers
884. Typesetters
895. Vehicle Dismantlers
907. Warehousemen
909. Waste Clearance Contractors
910. Wast Merchants
911. Wastepaper Merchants
918. Welders
920. Wellborers & Sinkers
933. Wrought Ironwork
962. Local Authority Works

8. Boat Building

72. Boat Builders & Repairers
74. Boats & Smallcraft Sales & Accessories
333. Engineers Marine
467. Hovercraft Manufacturers
933. Yacht Builders & Repairers

9. Transport

18. Air Charter Rental
[illegible]. Air Services
24. Airfields
109. Bus & Coach Services
131. Car Ferry Operators
132. Car Hirers
149. Carriers Express
191. Coach Hire
205. Commercial Vehicle Towing
222. Container Trans operator
254. Delivery Services & Local Carriers
344. Express Freight Carriers
436. Haulage Contractors International
456. Horse Transport
529. Livestock Carriers
566. Minicabs
663. Port, Harbour & Dock Authorities
705. Railway Offices & Stations
708. Refrigerator Trans
717. Road Haulage Contractors
775. Shipping Companies & Agents
837. Taxis & Private Hire Cars
958. Rail Stations (**)

The (N+1) and (N+2) level Local Authority Areas, A(N+1) and A(N+2), can be seen in the accompanying map and also listed below. The A(N) set is listed in Research Report VI on pages 5 to 16.

The Q-Analysis of various relations between L(k) and A(h) have been listed as follows:

Research Report VI	:	relations between L(N+1) and A(N+1)
Research Report VII	:	relations between L(N) and A(N), for the counties of East Suffolk and West Suffolk
Research Report VIII	:	relations between L(N) and A(N), for the counties of Norfolk and Cambridgeshire
Research Report IX	:	relations between L(N+1) and A(N) for each (N+1) area in the Region.

[4] The idea of traffic on the backcloth S(k)

Since the backcloth S(N) with its characteristic topological properties is to act as the multidimensional space for the "action" (the dynamics) then it must provide the (only) routes for a general kind of traffic. This traffic must also be representable by hard data and must be less "static" than the backcloth itself. Since S(N), which is the union of complexes like KY(X) and KX(Y) is defined in terms of sets of vertices (e.g. the set X) so we would expect these vertices to play a role analogous to that of independent variables (in orthodox physical science - where the vertices become geometrical points). This means that what we would like to call traffic on S(N) must be entities which

(i) are determined in terms of the vertices of S(N)

and (ii) are representable by a pattern of set functions on a typical KY(X).

Such a pattern is what we have usually represented by a symbol π and noticed that it is defined on the simplices of KY(X) (on the set Y). Thus we usually write

$$\pi \; : \; Y \to J \qquad \text{(J is the set of integers)}$$

and its graded nature arises because of the members of Y are dimensionally graded, thus

$$\pi = \pi^0 \oplus \pi^1 \oplus \pi^2 \oplus \dots \oplus \pi^t \oplus \dots \oplus \pi^n \qquad (n = \dim K)$$

where $\pi^t = \pi|\{\text{the t-simplices of KY(X)}\}$.

Qn. 3 Can we have a practical example of traffic and pattern on a backcloth ?

Example 1. Let the backcloth be S(N), for the County of Norfolk (see Research Report VIII[2]), using 966 vertices (Human Land Use activities) and the 33 vertices (the Parishes) in the Rural District of Blofield & Flegg. In our analysis we regard the Parishes as the set X and so, in KY(X) the Land Use activities are the simplices whilst in KX(Y) the Parishes are the simplices. An example of traffic on this portion of S(N) is population distribution, on KX(Y). If the name of the associated pattern is pop then pop has a value on each of the 33 Parishes, viz., the number of people who are normally resident in each. The people living in this Rural District do so because of the activities (out of the 966 vertices) which are to be found there, so pop is graded as follows. All the people who live there because of (say) 10 activities (any 10 out of the 966) make a contribution to $\underline{pop}^9$, etc., so we can write

$$\underline{pop} = \underline{pop}^0 \oplus \underline{pop}^1 \oplus \underline{pop}^2 \oplus \ldots \oplus \underline{pop}^{51}$$

It stops at t = 51 because this is the highest dimension of any Parish in KX(Y), viz., that of Caister-on-Sea (the next highest is Acle at q = 44). It follows that (e.g.) all the $\underline{pop}^t$ - type people, where t has values 45 to 51, must live in Caister. Of course there might not be any such people, but then $\underline{pop}^{51}$ would simply be zero. Now Acle and Caister-on-Sea are q-connected at q = 25 (not before, i.e. not at q = 26), hence they share $\underline{pop}^{25}$-type people (and of course $\underline{pop}^{t}$-type, when $t < 25$).

Now we see the point of q-connectedness, which has been made many times in this published research, that the connectivity between Acle and Caister-on-Sea allows the free flow of people (traffic) between them - provided these people are in $\underline{pop}^{t}$ with $t \leq 25$. This is in spite of the fact that Acle and Caister are not geographical neighbours !

If the traffic we consider is (say) all 0-dimensional traffic (if we restrict ourselves only to people who live in Blofield & Flegg R.D. for one reason only (one vertex out of A(N)) then we naturally expect and find that there is the possibility of free flow over the whole District. Indeed this happens for all t-traffic when t = 7.

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But we realise also that this traffic of "people-residing" includes people who, whilst being in (say) pop^{4}, do not all share the same 5 vertices out of A(N). So how does this free flow (or possible free flow) affect this fact ? In other words, the Parishes of Acle and Caister share 26 vertices; if John Smith is a pop^{15}-type person but these 16 vertices are not to be found in the set of 26 then John Smith (as an individual) will not be free to move from Acle to Caister in this "flow". This situation, which points to specific vertices (suc h as Public Houses, General Stores, Junior School, Building Contractors) is naturally dealt with via the conjugate complex KY(X). For in KX(Y) the traffic is defined on specific Parishes, so in KY(X) it is defined on specific Activities. But there is therefore a natural question to ask.

Qn. 4 How is traffic on KX(Y) interpreted on KY(X), if at all ?

Example 2 Taking our traffic with pattern pop (on KX(Y)) and contemplate a particular person, John Smith, as part of (say) pop^{15}; then he can only move between Parishes which are connected (in KX(Y)) by a certain specific 16 (= 15+1) vertices out of L(N). This means he is restricted to all Parishes which share a certain 15-dimensional simplex (a face of, perhaps, Acle). Such Parishes constitute those simplices which possess this σ_{15} as a face (a subpolyhedron), where σ_{15} denotes the specific John Smith-type simplex (a face of Acle). This is, more precisely, given by

$$\{X^i : \sigma_{15} < X^i\} \qquad \text{for all appropriate values of } i.$$

In the mathematical literature this sequence of Parishes would also be called the Star of this σ_{15} , so that

$$\text{Star (Smith-type simplex, } \sigma_{15}) = \{X^i : \sigma_{15} < X^i\}$$

On the other hand this man Smith can be regarded as part of a traffic tpop on KY(X), where the simplices are the activities. Thus Smith's 16 vertices mean that he is part of the traffic on a subcomplex of KY(X) viz., all those 16 simplices which represent the individual activities, and each simplex is a set of Parishes. If (e.g.) two of these activities are Public Houses and Building Contractors we notice in Research Report VIII that they are, respectively, a σ_{21} and a σ_{20} . Thus "Public Houses" is the name of a simplex defined by 22 Parishes and because of this Smith, qua traffic on KY(X), is part of $tpop^{21}$ (and also $tpop^{20}$, because of the Building Contractors). Generally we deduce that each person (like Smith) in the original traffic on KX(Y) becomes "spread over" the grades of an associated traffic on KY(X). The precise grades are formed by examining, for each person, the vertices contained in the simplex σ_{15} (in KX(Y)) and relating them to the dimensions of the simplices (in KY(X)) representing each such vertex. Smith therefore becomes a graded pattern all by himself, on KY(X), like (e.g.)

$$\text{Smith} = (\text{Smith})^{0} \oplus (\text{Smith})^{q} \oplus (\text{Smith})^{r} \oplus \ldots$$

where the value of $(\text{Smith})^{t}$ on all h-simplices is zero if $h \neq t$ and equals 1 on those t-simplices which represent, in KY(X), the vertices of which Smith is defined as he makes a contribution to pop, on KX(Y).

Let us change the notation a little so as to relate more obviously to our Local Authority Areas $\{A_i\}$ and Land Use activities $\{L_i\}$. Now suppose that Smith makes a contribution to pop by residing in the Parish A_1; normally we would find that data is collected in such a way that Smith would contribute a count of 1 to pop, on A_1. That is to say, if

$$\text{pop}\ (A_1) = m \quad \text{(a positive interger)}$$

then this means there are m "Smiths", each residing in A_1, and each contributing a 1 to this value of pop. Now suppose that Smith is resident in A_1 because he values four "activities" $\{L_1, L_2, L_3, L_4\}$ and, in the backcloth S,

$$A_1 = \langle \ldots L_1 L_2 L_3 L_4 \ldots \rangle$$

i.e. $\langle L_1 L_2 L_3 L_4 \rangle$ is a face of A_1.

Then, in the conjugate complex (of the backcloth) $\overline{S}$, suppose we know that

$$L_1 = \langle A_{\alpha_1}\ \ A_{\alpha_2}\ \ A_{\alpha_3} \rangle\ , \qquad L_2 = \langle A_{\beta_1}\ \ A_{\beta_2}\ \ A_{\beta_3}\ , \ A_{\beta_4} \rangle$$
$$L_3 = \langle A_{\gamma_1}\ \ A_{\gamma_2} \rangle\ , \qquad L_4 = \langle A_{\delta_1}\ \ A_{\delta_2}\ \ A_{\delta_3} \rangle$$

Then in tpop, Smith makes a contribution on each of L_1, L_2, L_3, L_4. This contribution could be reasonably weighted as 1 on each of the L_i (but notice that this could be regarded as the result of Smith's ranking of these activities - therefore we can contemplate allowing for any possible ranking weight (say) k_i on each L_i) and that would give us

$$\underline{\text{tpop}}\ (L_i) = 1, \text{ due to Smith}$$

In this case we would regard Smith as a pattern, on $\tilde{S}$, graded as

$$\text{Smith} = (\text{Smith})^1 \oplus (\text{Smith})^2 \oplus (\text{Smith})^3.$$

It is then more reasonable to make Smith's contribution to pop the sum of these tpop values on the L_i, viz.,

$$\underline{\text{pop}}\ (P_1), \text{ due to Smith, } = \sum_{i=1}^{4} \underline{\text{tpop}}\ (L_i)$$

$$\text{which} = 4 , \text{ in this case.}$$

In this sense we are noticing that pop is a pattern which is a function of the pattern tpop, this being the expression of the (supposed) fact that Smith resides in parish A_1 because of its vertices $\{L_i\}$. Thus, for Smith, tpop comes first, pop is dependent on it.

This means that the simple "counting of heads" in collecting population data is insufficient to express the notion of graded traffic on a structured backcloth $S \cup S'$.

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[5] Patterns, generators and t-forces on the structure

The Land Use v. Areas (parishes) data defines the backcloth $S(N)$ as

$$S(N) - KL(A) \cup KA(L)$$

where L means the set L(N) and A means the set {Areas (parishes)}. If we have a pattern π_1 on the vertices of L(N), such as our previous example of tpop, then we can regard it as zero-graded and write

$$\pi_1 \equiv \pi_1^0 \ : \ KA(L) \rightarrow J \qquad \text{(J being the integers)}$$

meaning that π_1 has a value on every vertex L_i (L(N)), and not on any higher simplex of KA(L). But this π_1^0 naturally gives rise to an associated pattern π_1 on the whole of the complex KA(L) by way of the pattern generator Θ associated with the complex. This gives, formally,

$$\pi_1 \ \text{[on the whole of KP(A)]} = \theta\, \pi_1^0$$

[This generator θ is defined and discussed *in reference* [3]].
In brief, θ is the union of a set of exponential operators, e_i^{Δ}, one for each eccentric simplex in KA(L), and the operation of Θ on KA(L) provides an overall pattern π_1 which is compatible with the connectivity (the topology) of KA(L). For any one such simplex there is an associated coface operator Δ and the expression

$$e_i^{\Delta}(\pi_1^0) \equiv \{1 + \frac{\Delta}{1!} + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \ldots\}\, \pi_1^0$$

gives the values of (the new) π_1 on the t-faces of the simplex (as t takes values 0,1,2, ...).

Now with this π_1 , on KA(L), we have an associated pattern, say $\bar{\pi}_1$, on KL(A), the conjugate complex. This can be defined in the following way (c.f. the illustration in the previous section).

If $\pi_1^0\ (L_i) = k_i$, for each i,

and if, in KA(L),

$$A_j = \langle L_{\alpha_1} \quad L_{\alpha_2} \quad \ldots \quad L_{\alpha_j} \rangle$$

then

$$\pi_1\ (A_j) = \Delta^j\ (\pi^0) = \sum^{\alpha_j} k$$

In the example of the previous section the ranking of activities by an individual, Smith, provides the pattern π_1^o. This result S(1) gives the ranking as a graded pattern on KA(L).

But we can continue in this vein to notice that, in KL(A), if we know that

$$L_t = \langle A_{\beta_1} \quad A_{\beta_2} \quad \dots \quad A_{\beta_t} \rangle$$

we expect to define an associated pattern $\tilde{\pi}_1$ by

$$\tilde{\pi}_1 \; (L_t) = \sum_{j=\beta_1}^{\beta_t} \pi_1 (A_j) \qquad \text{S(2)}$$

If, for example, Smith is consistent then these values of $\tilde{\pi}_1$ (L_i) must be identical with his original π_1^o (L_i) - which is why we can regard the initial Smith rankings as being defined on KL(A).

It follows that, for this consistency, S(1) and S(2) must be equivalent to a map which, at first sight, might be regarded as the identify id and demonstrated by the following commutative (?) diagram.

$$\begin{tikzcd}
 & \pi_1 \; [KA(L)] \arrow[dr, "\beta"] & \\
\pi_1^o[L_t] \arrow[ur, "\theta"] \arrow[rr, "id(?)"] & & \tilde{\pi}_1 \; [KL(A)]
\end{tikzcd} \qquad (D_1)$$

where θ and β are defined by S(1) and S(2) respectively, and the question of commutativity depends on whether or not we can justify writing

$$\beta \, \Theta \, \pi_1^o \; \langle L_t \rangle \quad \equiv \quad \tilde{\pi}_1 (L_t)$$

abbreviated to

$$\beta \, \Theta = id(?)$$

If it were commutative the map β must be such as to allow us to identify an apparently multi-graded pattern on one complex as a multi-graded pattern on its conjugate.

The whole process exhibits how the structure of the backcloth creates a precise dependency between one pattern ($\tilde{\pi}_1$) and another (π_1).

The mathematical symmetry of the discussion can be illustrated by the "completion" of the diagram (D_1) to give the following diagram (D_1^2) - which is self-explanatory.

$$\begin{tikzcd}
\pi_1[KA(L)] \arrow[dr, shift left, "\beta"] & \tilde{\pi}_1^o\,[A] \arrow[l, "id(?)"'] \arrow[d, "\tilde{\theta}"] \\
\pi_1^\theta\,[L] \arrow[u, "\theta"] \arrow[r, "id(?)"] & \tilde{\pi}_1[KL(A)] \arrow[ul, shift left, "\tilde{\beta}"]
\end{tikzcd} \qquad \cdots \qquad (D_1^2)$$

with the possibility that: $\beta\theta = id(?)$ and $\tilde{\beta}\tilde{\theta} = id(?)$

In dealing with a pattern like pop we naturally regard it as the sum of all things like "Smith" and so we get a simple result, viz.,

$$\underline{pop} = \sum\{\text{patterns like } \pi_1\}$$

whilst

$$\underline{tpop} = \sum\{\text{patterns like } \tilde{\pi}_1\}.$$

We notice too, from (D_1^2) that from a simple "counting of heads" (to get data like π_1^o [L] or $\tilde{\pi}_1^o$ [A] we can deduce, via the backcloth structure S(N), appropriate graded patterns on the backcloth.

Qn. 5. Can the operators Θ and β be made operational in a simple way ?

The operator Θ is closely related to the incidence matrix Λ associated with the relation $\lambda \subset A \times L$[5]. For example, consider a simple situation in which there are two Areas $\{A_1, A_2\}$ and three Land Use vertices $\{L_1, L_2, L_3\}$ and that Λ is the following binary matrix

$$\Lambda = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

so that, A_1 is λ-related to L_1 and to L_3 (but not L_2) and A_2 is λ-related to L_2 and L_3 (but not L_1). Then Λ (and/or λ) defines two complexes

$$KA(L;\lambda) \quad \text{and} \quad KL(A;\lambda^{-1})$$

where Λ corresponds to λ and Λ' corresponds to λ^{-1} (Λ' is the transpose of Λ). Consider a pattern π^o on the vertices $\{L_i\}$ of $KA(L)$, e.g.

$$\pi^o\ (L_1) = 1, \quad \pi^o(L_2) = 2, \quad \pi^o(L_3) = 3$$

Then if we evaluate the matrix product, $\Lambda\pi^o$, by which we mean

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \pi^o\ (L_1) \\ \pi^o\ (L_2) \\ \pi^o\ (L_3) \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

we obtain the vector

$$\begin{pmatrix} 1 + 3 \\ 2 + 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} \pi\ (A_1) \\ \pi\ (A_2) \end{pmatrix}$$

since it corresponds to forming a pattern π on the rows (the A_i) via:

$$\pi(A_1) = \pi\langle L_1 L_3\rangle = \pi^0(L_1) + \pi^0(L_3) = 1+3 = 4$$
$$\pi(A_2) = \pi\langle L_2 L_3\rangle = \pi^0(L_2) + \pi^0(L_3) = 2+3 = 5.$$

It follows that, in general, if $\pi^0\{L_i\}$ is a set of values of a pattern on the vertices of KA(L) then

$$\Lambda\pi^0 = \pi$$

is the corresponding pattern π, on KA(L), <u>generated</u> by the structure of KA(L) (and represented by Λ).

But is this what Θ does, as an operator ? Well in fact Θ, as an operator defined by e^{Δ} (v. reference [3]), does rather more than this. It gives at each stage (obtained by applying successive terms of exponential operators e^{Δ}) the values on successive simplices (of dimensions 0,1,2,3, etc.) of patterns derived from π^0. Of course in the end it gives the pattern π, on KA(L); that is to say, on the simplices whose names are A_1, A_2, ... etc.. During this process it has given us a pattern on <u>all the faces</u> of each such A_i. *In comparison*, $\Lambda\pi^0$, gives the final values of π on the A_i, but not necessarily on the faces of A_i (the exception is when some A_j is a face of A_i).

But with this understanding about the difference we can see that the matrix Λ can be a useful short-cut tool for calculating, what is often the practically important end result of $\Theta\pi^0$. When that is justified, or adequate to our requirements, we can regard diagram D_1 as the following one:

$$\begin{tikzcd}
 & \pi_1[KA(L)] \arrow[dr, "\beta"] & \\
\pi_1^o[L] \arrow[ur, "\Lambda"] \arrow[rr, "id(?)"'] & & \tilde{\pi}_1[KL(A)]
\end{tikzcd} \qquad (D_1^1)$$

In this case it is clear that β çannot be viewed as a matrix since we shall not be able to satisfy the relation

$$\beta\Lambda \;=\; id(?) \;=\; \text{unit matrix } (?)$$

If Λ is an mxn-matrix, then β would have to be an nxm-matrix and the unit matrix would be I_m. But a simple computation shows that there does not exist a 3x2 matrix β such that

$$\beta \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What then is the map β and why does a matrix fail to represent it ? Since Λ (and also Θ) sums the values of $\pi^o(L_i)$ in suitable combinations to give (e.g.) $\pi(A_1)$ then β must act as some mapping (of maps) which factors through an underlying structure map and in such a way as to partition $\pi(A_1)$, whose value is the sum of $\pi^o(L_i)$ for appropriate values of i, and to re-allot the values $\pi^o(L_i)$. Hence we can try to introduce a mapping η as follows,

$\tilde{\beta} \;:\; \pi_1 \to \tilde{\pi}_2$ where $\pi_1 \;:\; KA(L) \to J$, $\tilde{\pi}_1 \;:\; KL(A) \to J$

via $\eta \;:\; KA(L) \to L$, where L is a cover of the set A when it acts as the domain of η.

But we can get to KL(A) from the set $A = \{A_i\}$ by the generator $\tilde{\Theta}$ (or $\tilde{\Lambda}$)

of the conjugate complex. Hence if we identify the values of π_1 (A_i) and use the fact that the A_i are vertices in KL(A) we can arrive at a new pattern $\tilde{\pi}_2$ on KL(A) via the diagram D_1"

$$\begin{tikzcd}
\pi_1[KA(L)] \arrow[r, "\tilde{\eta}"] \arrow[dr, dashed, "\tilde{\beta}"] & \tilde{\pi}_2^{o}[A] \arrow[d, "\tilde{\Theta}"', "\tilde{\Lambda}"] \\
\pi_1^{o}[L] \arrow[u, "\Theta", "\Lambda"'] & \tilde{\pi}_2[KL(A)] \arrow[l, "\eta"]
\end{tikzcd} \qquad (D_1'')$$

Is this "commutative" in the sense that, for example,

$$\tilde{\Lambda}\ \tilde{\eta}\ \Lambda\ \eta\ = \text{identity map} : \tilde{\pi}_2 \to \tilde{\pi}_2$$

Since $\Lambda, \tilde{\Lambda}$ are summation type operators <u>this condition cannot hold.</u> <u>Hence D_1" is not a commutative diagram</u>.

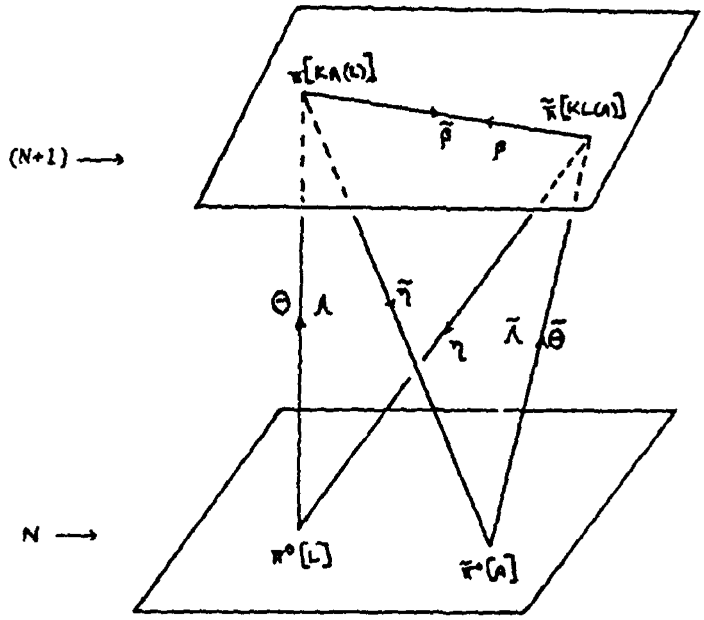
Cycling round D_1" is more like ascending a spiral of maps. The map $\tilde{\beta}$ must now be <u>defined</u> as a map : $\Lambda\pi^{o}[L] \to \tilde{\pi}\ [KL(A)]$ and such that

$$\tilde{\beta} = \tilde{\Lambda}\ \tilde{\eta}$$

We shall use the symbol β for the map which goes the other way

and write $\beta : \tilde{\pi}_1[KA(L)] \to \tilde{\pi}_2[KL(A)]$ and $\beta = \Theta\ \eta$

and these can be followed from the spiral nature of D_1" as represented in the diagram D_2, below.



(D$_2$)

The relations between the maps being

$$\bar{\beta} = \Theta\, \tilde{\eta} \quad ; \quad \beta = \Theta\, \eta \qquad 5(4)$$

or

$$\bar{\beta} = \bar{\Lambda}\, \tilde{\eta} \quad ; \quad \beta = \Lambda\, \eta$$

expressing (e.g) the fact that $\tilde{\eta}$ suppresses all those values of π on faces of the A_i (except when such a face is another distinct A_j).

Example 3 We can illustrate 5(4) by a simple numerical example, using our previous incidence matrix Λ, viz.,

$$\Lambda = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{with} \quad A = \{A_1, A_2\} \text{ and } L = \{L_1, L_2, L_3\}$$

and $\pi^{o}[L] = \{1,2,3\}$.

Then $\tilde{\Lambda} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $\Lambda\pi^{o}[L] = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} \pi^{o}(L_1) \\ \pi^{o}(L_2) \\ \pi^{o}(L_3) \end{pmatrix}$

giving $\pi[KA(L)] = \pi\{A_1, A_2\} = \begin{pmatrix} \pi(A_1) \\ \pi(A_2) \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Now $\tilde{\eta} : \tilde{\pi}[KA(L)] \rightarrow \tilde{\pi}^{o}[A]$

identifies $\pi^{o}\{A_1, A_2\}$ as $\{4,5\}$

Then $\tilde{\Lambda}\,\tilde{\pi}^{o}[A] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \tilde{\pi}^{o}(A_1) \\ \tilde{\pi}^{o}(A_1) \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 9 \end{pmatrix}$

Hence $\tilde{\beta} = \tilde{\Lambda}\,\tilde{\eta}$ is such that

$$\tilde{\beta}\,\pi\,[KA(L)] = \tilde{\pi}[KL(A)] = \tilde{\pi}\,\{L_1, L_2, L_3\} \quad (L_i \text{ is a simplex})$$

and so $\tilde{\beta}\,\pi[KA(L)] = \tilde{\pi}\,\{L_1, L_2, L_3\} = \{4,5,9\}$.

. . . .

Now we have seen elsewhere[3] that changes in patterns, things denoted by $\delta\pi$, correspond to the notion of t-forces. Precisely, if we begin with

$$\pi = \pi^{o} \oplus \pi' \oplus \ldots \oplus \pi^{t} \oplus \ldots \oplus \pi^{n}, \qquad n = \dim k$$

then a change $\delta\pi = \delta\pi^{o} \oplus \ldots \oplus \delta\pi^{t} \oplus \ldots \oplus \delta\pi^{n}$

is only possible if the structure S(N) is adequately connected. This be because "$\delta\pi^{t} \neq 0$" implies that the t-simplices are t-connected (and therefore must be faces of suitable (t+1)-simplices in the structure). Thus "$\delta\pi^{t} \neq 0$" means that we can regard this $\delta\pi^{t}$ as equivalent to $\Delta\mu^{t}$, where μ^t is a suitable pattern on K and where Δ denotes the coface operator. But this means that $\delta\pi^t$ is identified with a (t+1)-graded pattern, viz., $\Delta\mu^t$, and so (e.g) the structure cannot carry a change $\delta\pi^n$ - since it contains no (n+1)-simplices. Hence $\delta\pi^n = 0$ when $n = \dim K$, and with this proviso we can illustrate the t-force idea by the following diagram:

$$\begin{tikzcd}
\mu^t[KA(L)] \arrow[r, "\Delta"] & \Delta\mu^t[KA(L)] \\
\pi^t[KA(L)] \arrow[u, "\alpha"] \arrow[r, "\delta"] & \delta\pi^t[KA(L)] \arrow[u, "id"']
\end{tikzcd} \qquad (D_3)$$

where α is chosen so as to make (D_3) a <u>commutative diagram</u>, that is to say,

$$\Delta\, \alpha\, \pi^t(\sigma_t) = \delta\pi^t(\sigma_t) \qquad 5(5)$$

for each $\sigma_t \in KA(L)$.

When $\delta\pi^t > 0$ we speak of the traffic (represented by π^t) experiencing a <u>t-force of attraction</u>, and when $\delta\pi < 0$ we speak of a <u>t-force of repulsion</u>. This "attraction" is associated with those t-simplices in KP(A) where $\delta\pi^t > 0$, and repulsion likewise. <u>Hence the t-forces are located in the structure of S(N)</u> by this convention.

Qn. 6 Some illustrations of a t-force due to change $\delta\pi^t$?

Example 4. Suppose that Smith spends a total of $10 on leisure activities L_1, L_2, L_3 (in the Land-Use set) in the ratio of 2:3:5, so he is described by $\pi^o\{L_1, L_2, L_3\}$ viz., $\pi^o(L_1) = \$2$, $\pi^o(L_2) = \$3$, $\pi^o(L_3) = \$5$, and further suppose that these activities are all to be found in the parish A_o where Smith lives. This means that KA(L) contains the simplex

$$A_o = \langle \dots L_1 L_2 L_3 \dots \rangle$$

and that Smith makes a contribution to an overall pattern (say) ent which measures the total sum of money spent on a set of leisure vertices (containing, among others, L_1, L_2, L_3 of course) by the population which resides in the parishes of KA(L). Now suppose that Smith's tastes change so that he now spends his $10 in a new way, viz.,

$$\text{(new)} \quad \pi^o(L_1) = \$1, \quad \pi^o(L_2) = \$2, \quad \pi^o(L_3) = \$7.$$

This means that Smith exhibits a change of pattern defined by

$$\delta\pi^o(L_1) = -\$1, \quad \delta\pi^o(L_2) = -\$1, \quad \delta\pi^o(L_3) = +\$2.$$

In the first case we note that, on KA(L), the traffic (which is to be identified with "Smith's expenditure on leisure activities") experiences
a 0-force of repulsion on the vertex L_1,
a 0-force of repulsion on the vertex L_2, and
a 0-force of attraction on the vertex L_3.
[Incidentally we would say[3] that the intensities of these forces are -1/2,-1/3, and 2/5 respectively v. page 32].

By forming $\Theta\delta\pi^o$ (v. diagram (D_3) above and (D_4) below) we would find that these t-forces (t=o) would also be experienced (by Smith) on that part of the structure KA(L) denoted (above) by

$$\text{Star } \langle L_1 L_2 L_3 \rangle = \{ A_i : \langle L_1 L_2 L_3 \rangle \text{ is a face of } A_i \}$$

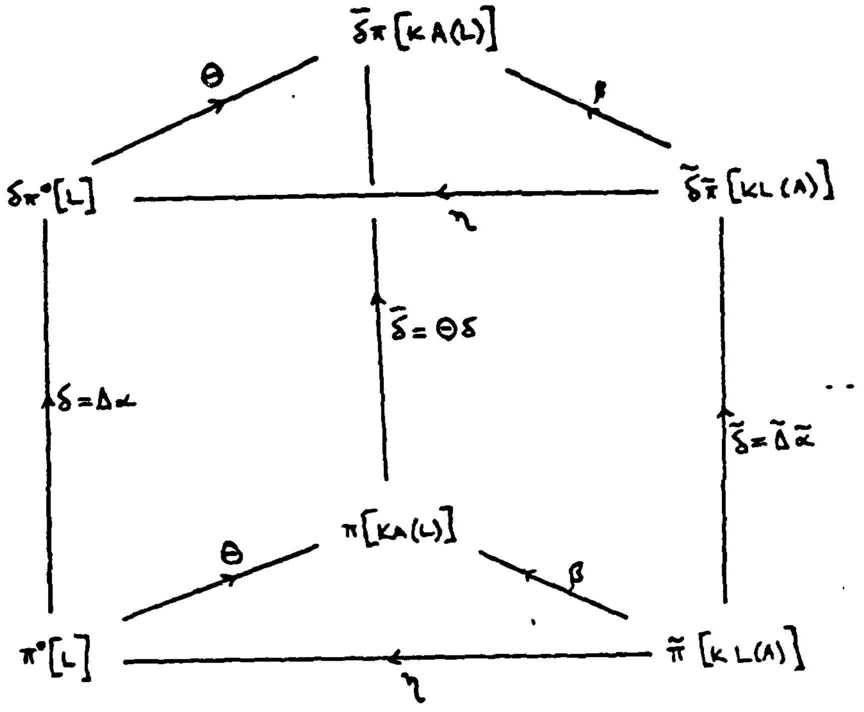
and that since, for example, there is a parish A_o containing this 2-simplex, $\langle L_1 L_2 L_3 \rangle$, so the traffic π_1 ($\equiv \theta\pi^o$) experiences a 2-force centred on A_o of intensity zero (it so happens) (because -1 -1 +2 = 0). At the same time this traffic experiences three instances of 1-forces (on the edges of $\langle L_1 L_2 L_3 \rangle$ in A_o, at least) of intensities -2/5. +1/7 and +1/8. [The intensity of a t-force $\delta\pi^t$ is taken as the ratio $\delta\pi^t/\pi^t$ when that exists. On the edge $\langle L_1 L_2 \rangle$ of $\langle L_1 L_2 L_3 \rangle$ this gives us the value (-1-1) ÷ (2+3) = - 2/5].

By the same sort of argument we can form $\beta\theta\delta\pi^o$ and deduce that Smith experiences a p-force of repulsion (of intensity 1/2), a q-force of repulsion (of intensity 1/3) and an r-force of attraction (of intensity 2/5) in the structure KL(A) - where L_1 is a p-simplex L_2 is a q-simplex, and L_3 is an r-simplex.

All these t-forces make a linear contribution to the pattern <u>ent</u> (and its conjugate <u>$\widetilde{\text{ent}}$</u>).

. . . .

The above example can be summarised by the following diagram of maps.



. . . . (D_4)

Regard this as a partly commutative diagram (v. diagram D_2 and relations 5(4)) and commence at the bottom left-hand corner, $\pi^{o}[L]$, then follow up the δ-operator and simultaneously the Θ-operator, etc., We therefore get $\beta = \Theta\eta$ and the increments $\tilde{\delta}, \tilde{\tilde{\delta}}$ are defined as in the diagram: δ is presumed given and its value as $\Delta\alpha$ is merely a reminder of diagram D_3.

But this example only illustrates a change $\delta\pi$ which is induced by the traffic itself (e.g. people like Smith who have the freedom to change their minds about their interests and use of resources). In addition, there will be all those changes $\delta\pi$ which are naturally induced by changes in the backcloth itself. This situation is commonplace in the context of planning and development, and so it is important for us to examine it in some detail. Perhaps the following example can bring out the concrete idea underlying it.

Example 5 Let us take the back·loth S(N+1) for the county of Cambridgeshire[4], being defined by the relation between (N+1)-Land Use, L(N+1), and (N+1)-Areas, A(N+1). There are 46 elements in the former and 12 elements in the latter.

L(N+1)

1. Chemical Manufacture
2. Venicle Manufacture
3. Electrical Manufacture
4. Other Manufacture
5. Light Engineering
6. Heavy Engineering
7. Light Industry
8. Boat Building
9. Transport
10. Extraction
11. Food & Drink Industry
12. Construction
13. Construction Supplies
14. Construction Manufacture
15. Agricultural Service
16. Farming
17. Horticulture
18. Office (Professional)
19. Office (Commercial)
20. Office(L.A. & Govn't)
21. Office (Other)
22. Primary Education
23. Secondary Education
24. Further Education
25. Hospital
26. Medical Clinic
27. Residential Home
28. Hostel/Hotel
29. Utilities
30. Church
31. Constabulary
32. Cultural Amenity
33. Popular Ent.
34. Sports Facilities
35. Retail-Hard
36. Retail-Soft
37. Retail-Food
38. Personal Service
39. Catering (Licensed)
40. Catering (Other)
41. Caravan Site
42. Woodland
43. Inland Water
44. Open Space
45. Beach
46. Marshland

A(N+1) Cambridgeshire

1. Cambridge MB
2. Chatteris UD
3. Ely UD
4. March UD
5. Whittlesley UD
6. Wisbech MB
7. Chesterton RD
8. Ely RD
9. Newmarket RD
10. N. Witchford RD
11. S. Cams. RD
12. Wisbech RD

Let us consider a pattern, π_1, which represents the traffic of people employed in those activities represented by the vertices {11,37,39,40} of L(N+1). We can therefore contemplate the initial pattern π_1^0 : {11,37,39,40} $\to$ J, where (e.g.) the value of π_1^0 on the vertex 37, $\pi_1^0(37)$, is the number of people employed (in Cambridgeshire) in that category. Using Q-Analysis to find the complexes, KL(A) and KA(L) as backcloth S(N+1), we are then able to induce patterns π_1 [KA(L)] and $\tilde{\pi}_1$ [KL(A)], as in the previous discussion (v. diagram D_4). The analyses shows[4] that, from the data, we deduce each of the elements 11,37,39 is an 11-simplex whilst 40 is a 10 simplex (these refer to the structure S(N+1) when the data is sliced at the minimum parameter value of 1). This shows that (e.g.) 40.Catering (non-licensed) is to be found in each Area except one (that one was N. Witchford RD). So, on this S(N+1), this gives a pattern $\tilde{\pi}_1$ (on KL(A)) graded as

$$\tilde{\pi}_1 = \overset{10}{\tilde{\pi}_1} \oplus \overset{11}{\tilde{\pi}_1}$$

Now suppose that this particular S(N+1) undergoes a change in its structure. For example, a transport strike results in the closure of all establishments providing 39.Catering (Licensed) in Cambridge MB, Wisbech MB, and Ely UD. Then {39} changes from being an 11-simplex and becomes an 8-simplex. We now get a natural change of $\tilde{\pi}_1$ to $\tilde{\pi}_1 + \delta\tilde{\pi}_1$ where

$\delta\pi_1^{11} < 0$ and $\delta\tilde{\pi}_1^{8} > 0$. The <u>new</u> $\tilde{\pi}_1$ now looks like $\tilde{\pi}^{8} \oplus \tilde{\pi}^{10} \oplus \tilde{\pi}^{11}$ and expresses the fact that the <u>domain</u> of the pattern has changed (the backcloth S has changed).

This induced change in $\tilde{\pi}_1$ is described as an 11-force of repulsion, in S(N+1), on the traffic of people employed on element 39 in Cambridgeshire. It introduces an 8-force of attraction, at the same time, throughout the county - on the whole (collective) employment traffic. If our individual Smith is part of this traffic then his "employment horizon" (the opportunities in Cambridgeshire) in this respect has shrunk from 11-dimensions to 8 and <u>this t-force (of repulsion) is experienced by him as a social and structural stress</u>. It is clear that in times of economic decline, when businesses are closing and factories are restricting their output, the employment pattern (like this one) changes in this sort of way. If we denote the structural change in S by $\mathring{\delta}S$ then in a well-defined way we can actually calculate any corresponding $\delta\pi$, when π was given on S. This could be written pictorially as

$$(S,\pi) \xrightarrow{\mathring{\delta}S} (S',\delta\pi)$$

where $S' = S \cup (S + \mathring{\delta}S)$, the term $S + \mathring{\delta}S$ denoting the new structure

To take another instance, suppose we consider the problem facing the County Council when it is concerned with that traffic which is described as "making provision for secondary and further education, cultural amenities and sports facilities" in the county of Cambridgeshire. This traffic is associated with the elements {23,24,32,34} out of the list L(N+1). It will probably have at least two relevant patterns associated with it - for example,

π_r = "annual recurrent expenditure on", and

π_c = "capital expenditure on".

Taking the same backcloth as before (data sliced at parameter = 1) we actually find that

$$\{23\} = \sigma_7, \quad \{24\} = \sigma_3, \quad \{32\} = \sigma_8, \quad \{34\} = \sigma_9$$

giving a grading for π_r as

$$\pi_r = \pi_r^{\,3} \oplus \pi_r^7 \oplus \pi_r^8 \oplus \pi_r^9$$

This expresses the fact that π_r is spread over the local authority areas according to these dimensions. "Sports Facilities" = $\{34\}$, presents a 9-dimensional space (horizon) to the Council's budgeting, etc. If $S(N+1)$ changes then π_r experiences t-forces whenever $\delta\pi_r^t \neq 0$, as a consequence, and of course it is highly likely that the Council itself will generate such changes - and this may well happen through a policy decision which involved π_c, in some such way as follows.

A general policy of "overall improvement in services" can better be viewed against a slightly different backcloth $S_1(N+1)$, viz., the one obtained by "slicing the data at the mean" (what is called the "characteristic backcloth" in [4]. This gives a structure S_1 which is such that (e.g.) Ely RD is only related to "Caravan Site" if the number of occurrences of this vertex in Ely RD (in the data file) is greater than or equal to the average number of caravan sites over the 12 areas of the County. A consequence of this is that (e.g.) a high dimension Land-Use (in S_1) is one which is more evenly distributed over the Areas, whilst a low dimension Land-Use reflects its concentration in a few Areas. In this particular case we find that, in $K_1\Lambda(L)$

$$\{23\} = \sigma_2, \quad \{24\} = \sigma_0, \quad \{32\} = \sigma_3, \quad \{34\} = \sigma_4$$

reflecting that 34.Further Education is concentrated primarily in only one Area (which is the city of Cambridge itself), etc.

Now this backcloth S_1 tells the County Council that its policy can be implemented by regarding π_c as a pattern on the <u>complement</u> $\bar{K}_1$ of this K_1, and so (effectively on the backcloth $S_2 = \bar{S}_1$) we would have

$$\{23\} = \bar{\sigma}_9, \quad \{24\} = \bar{\sigma}_{11}, \quad \{32\} = \bar{\sigma}_8, \quad \{34\} = \bar{\sigma}_7$$

and now it decides on a pattern π_c (on S_2) graded as

$$\pi_c = \pi_c^7 \oplus \pi_c^8 \oplus \pi_c^9 \oplus \pi_c^{11}$$

The consequence of this (in some suitable time period) is that the structure of S(N+1) becomes changed - via new *establishments etc.*, being created in the "needy" Areas of the County. Thus

$$\pi_c \text{ on } \{S_2\} \text{ induces } \dot{\delta}S$$

which then means that $\dot{\delta}S$ induces $\delta\pi_r$.

This example suggests strongly how policy matters can be *effectively* expressed in terms of backcloth structures (produced by slicing the data) and traffic patterns, and also how one kind of traffic (π_c) can, once more, seriously affect another (π_r).

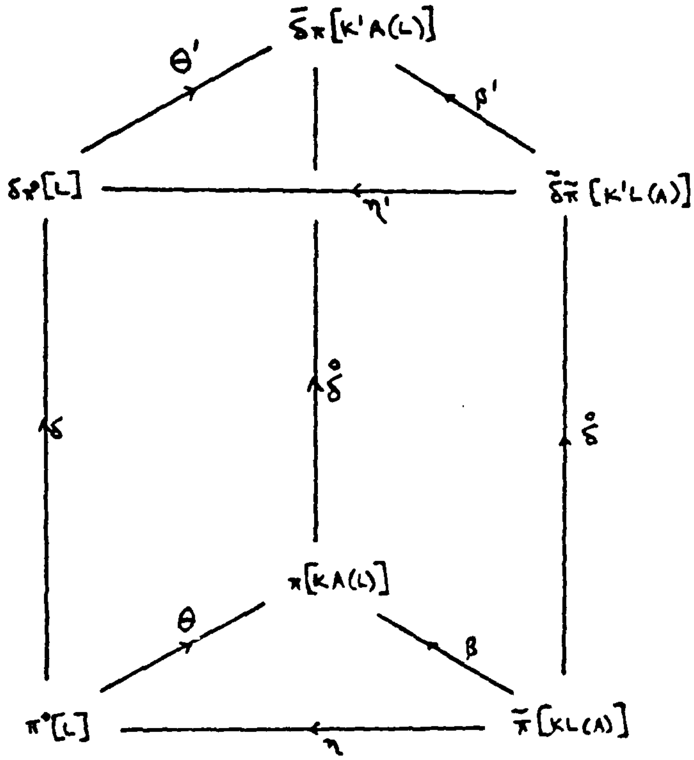
. . . .

We can now try to represent these ideas in a more general context and to compose them in a suitable scheme.

Denote the change in S, however it is brought about, by $\dot{\delta}S$ and let $S' = S \cup (S+\dot{\delta}S)$. This is necessary because the final pattern, following $\dot{\delta}$, is to be found in this union - sometimes $S \subset S+\dot{\delta}S$ and sometimes $S \supset S+\dot{\delta}S$. In the same way, for example, if $S = KL(A) \cup KA(L)$

let $S' = K'L(A) \cup K'A(L)$,   where $K' = K \cup \{K + \delta K\}$

and if Θ is the pattern generator on $KA(L)$ let Θ' be the one on $K'A(L)$; likewise let β' be such that $\beta'\Theta' = id = id'$. Then we can set up a partly commutative diagram D_5, comparable with D_4, as follows :



. . . . (D_5)

In this diagram we have $\beta = \Theta\eta$ and $\beta' = \Theta'\eta'$. In addition we regard $\mathring{\delta}$ as given (denoting change in the backcloth structure) and it then induces δ (change in pattern π^0) via the requirement that

$$\tilde{\delta} = \Theta'\delta$$

This $\mathring{\delta}$ produces the change from Θ to Θ', and $\tilde{\delta}$ is the effective change on a pattern π which is itself equal to $\Theta\pi^0$.

In reading this diagram we begin at the bottom corner where $\mathring{\delta}$ originates and we use the following definitions of $\bar{\delta}$, $\tilde{\delta}$, viz.,

$$\bar{\delta}\pi\,[K'] = \pi[\mathring{\delta}K], \qquad \tilde{\delta}\tilde{\pi}[K'] = \tilde{\pi}[\mathring{\delta}K] \tag{5(6)}$$

and δ is such that $\Theta'\delta = \mathring{\delta}\Theta$. Of course β, β' are defined as in 5(4).

. . . .

Now we must ask the following question.

Qn. 7 <u>How can we allow for the general dependence of one pattern on another - a sort of cause-and-effect</u> ?

Here we have in mind a generalisation of a simple dependence such as,

$\pi_1 : KA(L) \to J$, $\pi_2 : KL(A) \to J$ and (e.g)

$$\pi_1 + \pi_2 = \text{constant map} \quad c : KL(A) \to J.$$

By "constant" we mean that it does not vary with time. Thus, if π_1 and π_2 are time-dependent (relative to the static backcloth $S(N)$), then we would have

$$\delta(\pi_1+\pi_2) = \delta c = 0 \qquad \text{(zero map)}$$

or

$$\delta\pi_1 = - \delta\pi_2$$

The consequence of this is that t-forces of attraction (experienced by the π_1-traffic) are equal to t-forces of repulsion (experienced by the π_2-traffic). An example might arise by taking $\pi_1 \equiv$ the pattern of employed persons (in a particular region) whilst $\pi_2 \equiv$ the pattern of unemployed persons (in the same region). If we assume that, during some fixed time interval, the total number of employable people in the region is constant then

$$\pi_1 + \pi_2 = \text{constant.}$$

Every change which results in an extra n people being employed implies that an extra n people cease to be unemployed. Thus

$$\delta\pi_1 = - \delta\pi_2$$

The same situation naturally arises in a partiamentary/council chamber voting problem, where

π_1 = pattern of representatives voting <u>for</u> certain policy

and π_2 = pattern of representatives voting <u>against</u> that policy.

Now, if no one abstains, a change of policy-allegiance is expressed by the statement,

$$\delta\pi_1 + \delta\pi_2 = 0.$$

The same situation can of course arise with any individual constant pattern, e.g. the fixed total expenditure of our "traffic" Smith, on leisure pursuits. In this case we might have $\pi = \pi^8 \oplus \pi^9 \oplus \pi^{10}$ and if π = constant in the sense that

$$\pi^8 + \pi^9 + \pi^{10} = \text{constant}$$

then $\delta\pi^8 + \delta\pi^9 + \delta\pi^{10} = 0,$

and an increase in π^8 ($\delta\pi^8 > 0$) must be countered by $-(\delta\pi^9 + \delta\pi^{10})$. What Smith loses on the swings he gaines on the roundabouts !

More generally we consider a function (mapping)

$$F : \pi_1^0 \to \pi_2^0$$

where (e.g.) $\pi_1^0 : L \to J$ and $\pi_2^0 : L \to J$.

Then we suppose that F induces a mapping δF between the incremental changes in π_1, π_2 ;

$$\delta F : \delta\pi_1^0 \rightarrow \delta\pi_2^0$$

This now means that the 0-forces expressed by $\delta\pi_1^0$ (on L) manifest higher order t-forces, throughout KA(L), expressed by $\delta\pi_2$. The dependence, through the connectiv of KA(L), can be represented as follows:

$$\begin{tikzcd}
\delta\pi_2^0[L] \arrow[r, "\Theta"] & \delta\pi_2[KA(L)] \\
\delta\pi_1^0[L] \arrow[u, "\delta F"] \arrow[ur, "\rho"'] &
\end{tikzcd}$$

where we define $\rho = \Theta.\delta F$.

This represents a set of <u>induced t-forces</u> throughout KA(L) [v. [6], where Johnson has described this as q-transmission of forces].

Another possibility arises when we have

$$\pi_1^0 : L \rightarrow J \quad \text{but} \quad \pi_2^0 : A \rightarrow J$$

Now we get the arrangement

$$\begin{tikzcd}
\delta\pi_2^{\circ}[A] \arrow[r, "\tilde{\Theta}"] & \delta\pi_2[KL(A)] \\
\delta\pi_1^{\circ}[L] \arrow[u, "\delta F"] \arrow[ur, "\tau"] &
\end{tikzcd}$$

where $\tau = \tilde{\theta}.\delta F$, and

in which the induced t-forces are experienced by the π_2-traffic on the structure KL(A), the conjugate of KA(L).

Finally, if we know the graded $\delta\pi_1$ on KL(A) then we can express this as a $\delta\pi_1^0$ on L, via the map η, and hence obtain the induced t-forces on the π_2-traffic in the structure KA(L).

$$\delta\pi_1[KL(A)] \xrightarrow[\eta]{} \delta\pi_1^0[L] \xrightarrow[\delta F]{} \delta\pi_2^0[L] \xrightarrow[\Theta]{} \delta\pi_2[KA(L)]$$

or via

$$\begin{tikzcd}
\delta\pi_1^{\circ}[L] \arrow[r, "\delta F"'] & \delta\pi_2^{\circ}[L] \arrow[d, "\Theta"] \\
\delta\pi_1[KL(A)] \arrow[u, "\eta"] \arrow[r, "\nu"'] & \delta\pi_2[KA(L)]
\end{tikzcd}$$

[6] Hierarchy and Meta-hierarchy in Planning

We can illustrate the practical implications of the theoretical discussion by referring to the many problems experienced by professional planners in the Region. To do this we shall refer to the published Structure Plans for the two counties of Norfolk[7] and Suffolk[8] and illustrate by considering details of the first. These typical county Plans are required by the Secretary of State for the Environment and are expected to fulfil the following functions:

(i) "To state and justify, to the public and to the Secretary of State for the Environment, the County Council's policies and general proposals for the development and other use of land and thus to provide guidance for the development on issues of structural importance.

(ii) To interpret national and regional policies in terms of physical and environmental planning for the county.

(iii) To provide the framework and statutory basis for local plans which then in turn provide the necessary further guidance for development control at the more detailed, local level".

This means that a comprehensive analysis of "structure" seems to be essential before any discussion of new developments can be undertaken. But Planners have not been armed with any profound definition of "structure" - which they can use as a base for generating policies - other than, at best, a rather liberal use of simple (and often fuzzy) network ideas. There is no doubt that the responsibility for this lies primarily with research workers in this field. We cannot expect the practising professional Planners (public servants accountable to Local and Regional Authorities) to ignore their responsible priority of getting out the studies/reports and making regular decisions on a wide variety of day-to-day development and planning matters. But we who are engaged in the field of research have our own responsibilities to "extending the frontiers of knowledge" and to dig deeper into that area of concepts/techniques which is relevant to the general study of planning. If this particular line of research, the methodology of Q-Analysis, is to be helpful then it must be helpful at the daily practical level - as well as at a level of academic concepts - and one which can be of value to planning officers who do not necessarily possess a training in advanced mathematics. The "proof of the pudding is in the eating", of course, so that this particular thesis (a prescription for both theory and practice) can only be presented in that spirit.

But in the light of previous sections of this Report, and of the regional analyses previously published in Research Reports V, VI, VII, VIII, IX, it is now possible to offer a presentation for practical planning in the field and for the formulation of policies at the levels of the Parish, District, County and Region.

I. In the first place, using (e.g.) our three hierarchical levels, N, N+1, N+2, we require the structures of the backcloths S(N), S(N+1) and S(N+2). These are all obtained from a single collection of N-level data, and (e.g.) S(N) is the pair of conjugate complexes

KL(A) and KA(L)

where L = L(N) = Land Use set at N-level

and A = A(N) = Administrative Authorities at N-level (Parishes).

Since the data is not usually in the form of a binary matrix we can expect to obtain a selection of structures $S_i(N)$, where i = 1, 2, etc. each derived by slicing the data at the threshold value determined by i. The chief slicing parameters we have previously illustrated occur at $\theta = 1$ and $\theta = \bar{\theta}$(slicing at the mean). If the θ_i are an increasing set of such parameters they generate a "filtration" of structures

$$S \equiv S_1 \supset S_2 \supset S_3 \ldots\ldots$$

II. Armed with these backcloth structures S(N), S(N+1), S(N+2) we are then in a position to examine traffic on them. Any particular traffic will be represented by a pattern π on (say) S(N) - whose values will usually (but not necessarily) be integers. The traffic is dynamic (relative to S(N)) and so the various patterns we consider can all be monitored by regular data collection. Naturally too there will be observed or postulated dependencies between some of the dynamic patterns. Also the changes in any one π (with respect to time) naturally give rise to the interpretation in terms of t-forces on the backcloth S(N). These are all expressed by diagrams like D_4, D_5. (above).

III. We then expect that all actual development, proposed development, planning policies, or forecasts of a general nature, can be expressed in terms of:

(i) t-forces due to $\delta\pi$ on a constant backcloth S(N)

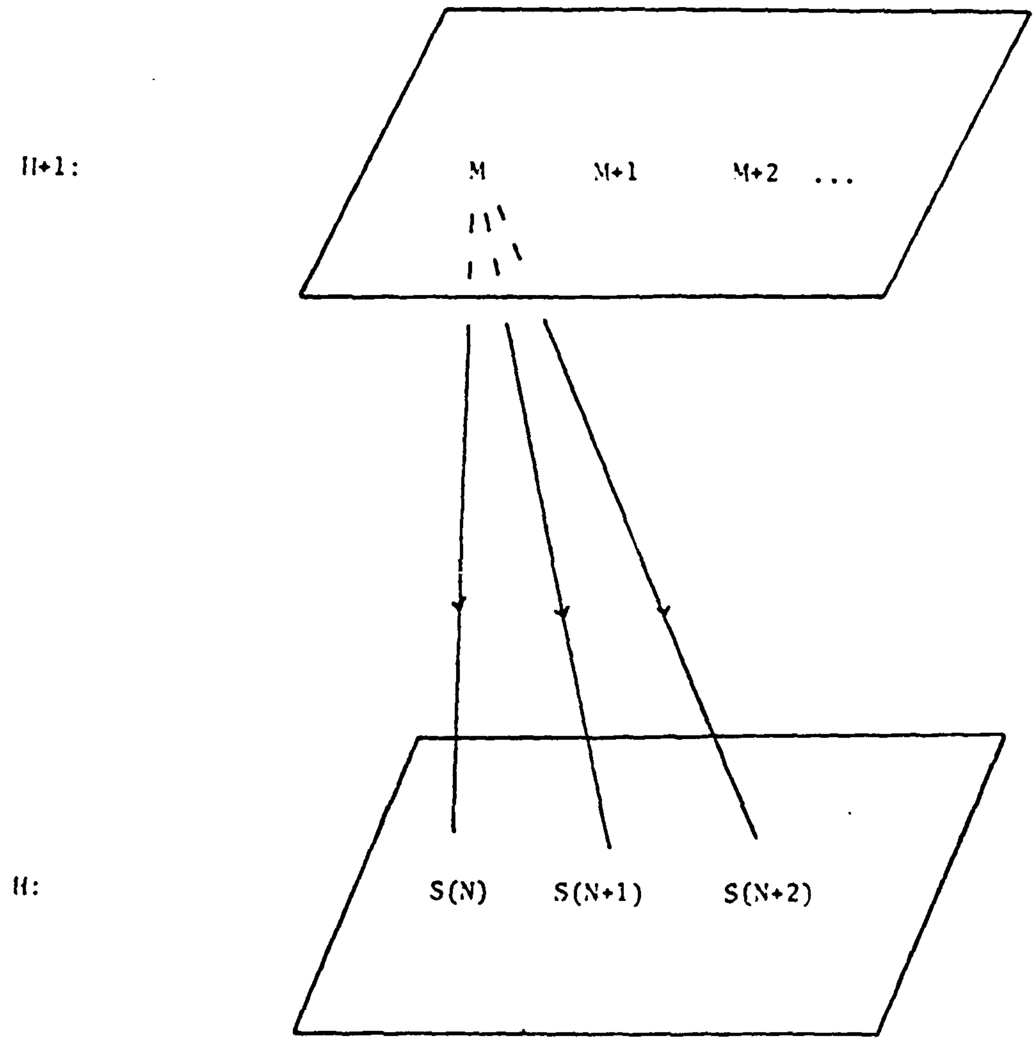
or (ii) t-forces induced by a change δS in the backcloth

or (iii) a combination of the above.

Included in these will be the mutual interaction of various patterns, resulting (e.g.) in the transmission of many t-forces throughout the structure.

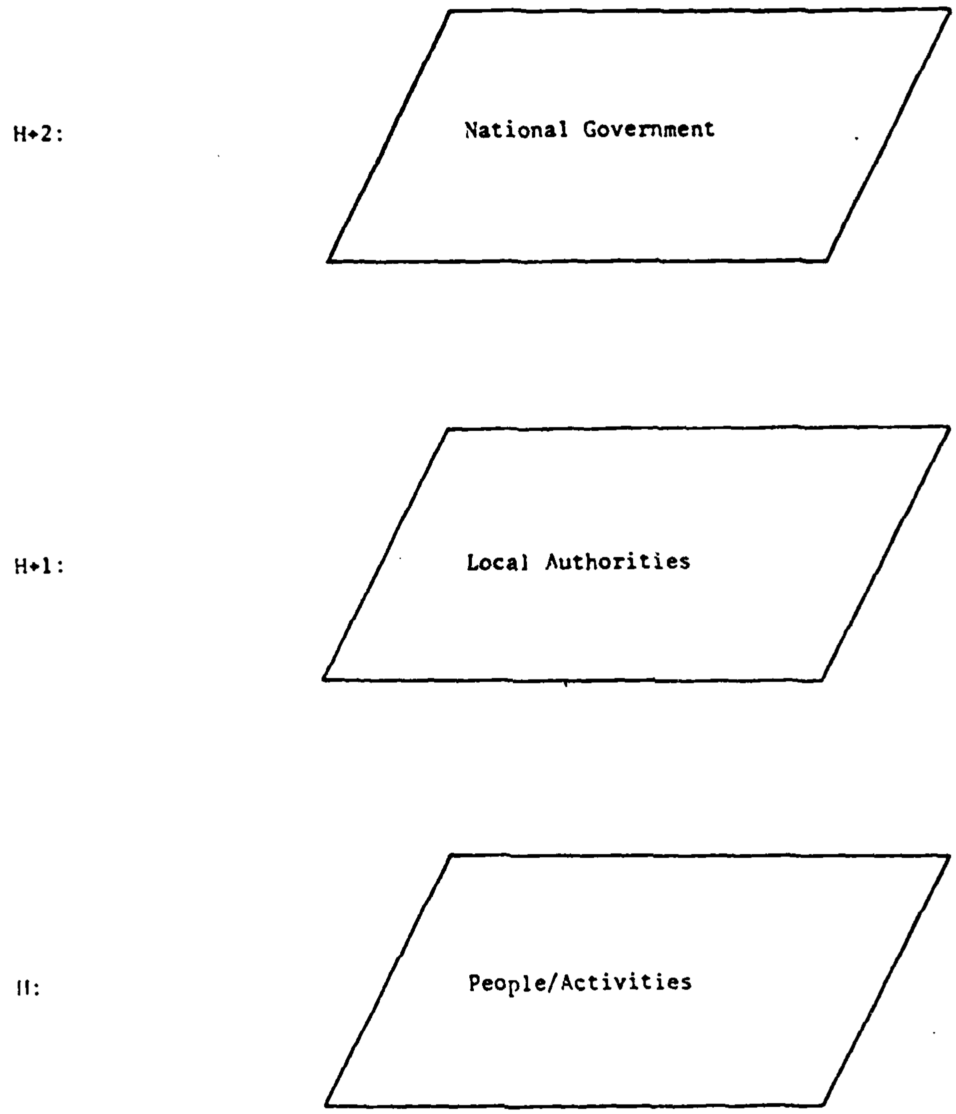
But in this schema where does the political action occur (the peculiar role of the various decision-making bodies) ?

If we denote the hierarchy of structure by H, then the action which can alter (e.g.) S(N) is such that it is placed so as to alter H (to alter the relations between L(N) and A(N)). This means that the political decisions occur at a meta-hierarchical level, say, H+1, giving something like the following schema.



For example, District Council policy (producing δs) formulated at the M-level (in H+1), or at the (H+1,M)-level can affect all the structures in H (by altering the relations between L and A). Then County Council policy must be effectively formulated at the (H+1,M+1) level, because there it can not only effect all the structures in H but it also constitutes a natural cover of the District Council bodies - assuming that the political demoncracy built-in to the system ensures that. The (H+1,M+2)-level must then be available for Regional Policy bodies.

Who has the power to alter the political organisation of these various councils at (H+1)? Why, the national government of course (just illustrated by the reorganisation of local government in 1974). When it is doing just that the national government must be sited in a meta-meta-hierarch (H+2)! Thus we get something like:



Naturally, (H+2)-authorities can act at (H+1) and at H, but (H+1)-authorities cannot act at (H+2) - only at H. The resulting "weight of authority" felt at H can be considerable.

East Anglia showing local authority areas

[7] The Norfolk County Structure Plan (1976)

This Plan[7] is a response to studies made jointly by the Department of the Environment, the East Anglia Economic Planning Committee and the East Anglia Consultative Committee[11]. The resulting Government "Response" provided national level guidelines for the regional Planners and the first things described in the Structure Plan are these broad guidelines under the headings of Population, Agriculture, Employment and Industry, Housing, Communications, Education, Public Utility Services, Conservation/Tourism and Recreation, Resources and Monitoring. In this the context Planners are acknowledging (in effect) what I have already called the meta-meta-hierarchy, H + 2. The statements of policy (either established, proposed or guessed at) emanating from (H+2) are a strange mixture of hierarchies and of levels to be found therein. The county Planners then proceed to describe their own policies and forecasts and in doing so they are inevitably led to adopt the same sort of hierarchical mix in the language and concepts they use.

The whole range of levels at our disposal are based on previous analysis published in Research Reports (this Project) and are probably the least in number which are required. They are, precisely,

(H+2, G) : The national government at the level G in the $(\text{meta})^2$- hierarchy H+2.

(H+1,M+2): The Regional Planning Authority at (M+2)-level in the meta-hierarchy H+1,(exemplified by the East Anglia Economy Committee and the East Anglia Consultative Committee, and (of course) the national government through its Civil Service officers responsible for the Region.

(H+1),M+1): The County Planning authority

(H+1, M) : The District Planning authority

(H, N+2) : The (N+2)-level of structure S(N+2) together with appropriate traffic (of people, goods, service, investment, communications).

(H, N+1) : The (N+1)-level of structure S(B+1), etc.

(H, N) : The N-level of structure S(N), etc.

Now the Structure Plan is produced at (H+1, M+1). In it we must look upwards to (H+1, M+2) and to (H+2, G) for the "guidelines". Then the Plan must look downwards to (H+1, M) and thence to (H, N+2), (H, N+1), (H, N).

(i) The message from (H+2, G) in the matter of population is contained in the following quotations.

(a) "Rate of Growth

The Government accept that the population of the Region will continue to grow substantially and consider that it is the duty of the planning authorities to guide and accommodate this growth. The Government do not think that there is any need for new policies, or changes in existing policies, designed to accelerate the growth of population in East Anglia. They recognise that population growth puts resources under pressure and can endanger the environment, but in their view difficulties in this connection can be overcome by careful planning."

(b) "The "Response" notes that the latest official population projections suggest a growth in East Anglia towards the upper end of the range forecast in "Strategic Choice for East Anglia." Population growth will be dominated by migration, as follows:

(a) planned overspill

(b) retirement migration

(c) miscellaneous movements

(d) natural increase

It is the Government's view that planned overspill from London involved in the current Town Development schemes should continue as programmed, but that any further schemes or phases will have to be considered on their merits in the light of the current economic, industrial and demographic situation. Of retirement and "miscellaneous" migration (which in Norfolk in recent years has included a substantial amount of employment-based inward migration unrelated to Town Development schemes), the "Response" says that the scale of movement will be subject to the prevailing economic situation, and may lessen in the longer term."

(c) "Location of Growth

The "Response" recognises that, in planning for continued population growth, the planning authorities in the Region might wish to develop small growth points, or to begin a new phase of a Town Development scheme. The choice of location for such small developments is considered to be a matter for Structure Plan, Local Plans, and Development Control. The "Response" does not inhibit well-judged proposals for growth in individual places, but it lays emphasis on the severe limitations of resources which are likely to persist for some time yet."

(d) "The "Response" refers to the special problems of the more remote rural areas, where the economic and social fabric is being weakened, and states that the primary need is "to sustain and diversify the population base". The Government draw attention to the powers and activities of the Development Commission and the Committee on Small Industries in Rural Areas to assist small manufacturing industries with advice and finance."

If we consider the traffic of population on the backcloth, these "guidelines" are statements about the associated pattern (say) pop. But then we are driven to make (on behalf of the Planners at (H+1, M+1) the following points:

Ref. (a):

On what structure is pop defined ?

Is it a zero-order pattern pop^0 (defined on KL(A) - the most plausible) ?

If so, how is the growth interpreted ?

Is it δpop^0 [A] at level N, (N+1) or (N+2) ?

How is δpop^0 interpreted as a new pattern $\oplus\ \delta\text{pop}^0$ on the structure KL(A) ?

If the government does not see the need for δpop to accelerate does it specifically assert that $\delta^2\text{pop} \equiv 0$ (zero pattern) ?

If δpop endangers the environment how does it do so .

Presumably via the induced $\dot{\delta}S$ on the backcloth ?

If it also "puts resources under pressure" then how is this to be measured - should it not be done by considering the interdependence of different patterns, in which δpop^0 induces $\delta\pi_2$, $\delta\pi_3$ ... via a scheme such as D_4, D_5 or D_6 ?

What are "difficulties in this connection" except unacceptable (or at the least irritating) t-forces experienced through H by the citizens ?

If these can be "overcome by careful planning" does it not mean that the t-forces can be restricted to acceptable limits ?

In this case, what are the acceptable limits and who is to determine them ?

Ref. (b) :

What is the measure of the growth δpop^0 [A] ?

Is it forecast by the standard time-series analysis (v. later sections of this Report) and if so how does it take into account the effect of the topology of the backcloth ?

How is the population growth related to other traffic (economic investment, rising costs of developments, employment prospects, housing, education, amenities) and to changes in the backcloth $\mathring{\delta}S$?

Also, are these statements to be read at level N, (N+1), or (N+2), or at all three ?

Ref. (c) :

Locating the growth of populations is particularly dependent on the backcloth S of the structure. Presumably it is done by providing housing, services and employment in the Region. But this means producing change $\mathring{\delta}S$ at (say) the N-level. How does this affect existing patterns, the induced t-forces in the Region ?

This can be answered by appealing to diagram D_5. Is there any criterion (in the shape of policy) which should be applied when choosing between one location and another, any policy expressed in terms of limitations on the permissible t-forces experienced by specific traffic ?

Clearly there is not, but there could be !

Ref. (d) :

What are the "special problems of the rural areas" and in what sense is the economic and social fabric being weakened ?

These concepts are glaringly "soft", but they could be made "hard" by being expressed in terms of pronouncements on the existing backcloth S(N) (about the dimensions of areas A_i and the connectivity of KA(L) or of KL(A) and on the allowable t-force stresses (allowed changes $\delta\pi^t_i$, for various t and i) experienced by traffic on S(N). If there are powers available to assist small manufacturing industries with finance then the implication is that the traffic of employment can be altered (some $\delta\pi_i$) and that possibly $\mathring{\delta}S$ can be induced - thus causing other changes in patterns on $S' = S \cup (S+\mathring{\delta}S)$.

. . . .

When the County Planning Authorities, at the level of (H+1) (M+1), attempt to come to grips with their own tasks - and incidentally trying to follow the above "guidelines", however inadequate they might be - then they need to move the discussion down to the hierarchy H and to the backcloth structures S(N), S(N+1), S(N+2). This attempt is summarised in the "objectives" quoted below.

"The strategy described can be expressed in terms of the following major objectives. These objectives help to give an understanding of the strategy and they will provide the basis for monitoring and the assessment of performance in the various sectors of Structure Plan policy:

(i) to ensure that population and employment growth is of a rate and distribution commensurate with the most effective use of financial resources from both national and local sources;

(ii) to ensure an adequate choice of employment over the whole country by the widening of opportunities in areas of deficiency;

(iii) to provide for adequate housing to cater for existing and future need, with a range of choice consistent with the provision of services;

(iv) to improve the level of access to jobs, services and facilities and to maintain, and where possible, enhance, the current level of mobility of residents of the rural areas;

(v) to achieve a gradual shift in public investment to support the rural areas, having regard to the need for continuing substantial investment in the major urban areas;

(vi) to protect and enhance the physical character of the county in terms of the landscape and buildings;

(vii) to avoid the loss of good quality agricultural land to development wherever possible;

(viii) to avoid the wasteful use of energy resources."

<u>Ref. (i)</u>: This is a statement about the patterns <u>pop</u>, <u>emp</u>, describing the traffic of population and employment. The implication is that δ<u>pop</u> > 0 and that it can be located on the backcloth S by planning decisions. Since <u>pop</u> and <u>emp</u> are highly dependent, with respect to S, the "effective use of resources" must refer to investment traffic which induces δ<u>emp</u> and δ<u>pop</u>. What is this other traffic and how are the dependencies evaluated and related to S or to δS ? (v. diagram D_5 above). In any event how are these patterns obtained at the different H-levels ? Is the statement meant to refer to N, (N+1) or (N+2) ?

<u>Ref. (ii)</u>: The idea of a "choice of employment" refers directly to the <u>dimension</u> of the backcloth KA(L) - with specific employment vertices specified in the set L. An "adequate choice" requires a policy and definition of "adequate". Perhaps it means an "improved" choice - for men/women throughout the County. This particular aspect of employment (discussed in terms of the <u>q-connectivities</u> of the Local Authority Areas) has been illustrated in a previous Research Paper[12]. By using data from the Census of 1966 and 1971 that particular study gave a Q-Analysis at the (N+1)-level of the <u>emp</u> pattern (and separated the areas of males/females). In this respect we found that, slicing the pattern $\underline{emp}^{0}$[L] at the <u>average value</u> (over the areas A_j, at (N+1)) we obtained a detailed picture of the "employment opportunities" throughout the county of Norfolk, for the years 1966 and 1971, and for the separate cases of males and females. In (e.g.) the year 1966 we obtained a feeling of the t-forces over the geographical areas A_i (there are 29 of these). These results are reproduced here.

<u>Qn. 8</u>: Where are the comparable results for 1974 etc., as a basis for the policies outlined in the Norfolk Structure Plan (1976) ?

Norfolk

1. Gt. Yarmouth CB
2. Norwich CB
3. Cromer UD
4. Diss UD
5. Downham Market UD
6. E. Dereham UD
7. Hunstanton UD
8. Kings Lynn MB
9. N. Walsham UD
10. Sherringham UD
11. Swaffham UD
12. Thetford MB
13. Wells-N-T-Sea UD
14. Wymondham UD
15. Blofield & Flegg RD
16. Depwade RD
17. Docking RD
18. Downham RD
19. Erpingham RD
20. Forehoe & Hemstead RD
21. Freebridge Lynn RD
22. Lodden RD
23. Marshland RD
24. Mitford & Launditch RD
25. St. Faith & Aylsham RD
26. Smallburgh RD
27. Swaffham RD
28. Walsingham RD
29. Wayland RD

Cambridgeshire

1. Cambridge MB
2. Chatteris UD
3. Ely UD
4. March UD
5. Whittlesley UD
6. Wisbech MB
7. Chesterton RD
8. Ely RD
9. Newmarket RD
10. N. Witchford RD
11. S. Cams RD
12. Wisbech RD

Standard Industrial Classification at (N+1)-level

1. Agriculture, Forestry, Fishing
2. Mining and Quarrying
3. Food, Drink and Tobacco
4. Coal and Petroleum Products
5. Chemicals and Allied Industries
6. Metal Manufacture
7. Mechanical Engineering
8. Instrument Engineering
9. Electrical Engineering
10. Shipbuilding and Marine Engineering
11. Vehicles
12. Metal Goods not elsewhere specified
13. Textiles
14. Leather, Leather Goods and Fur
15. Clothing and Footwear
16. Bricks, Pottery, Glass, Cement, etc.
17. Timber, Furniture, etc.
18. Paper, Printing and Publishing
19. Other Manufacturing Industries
20. Construction
21. Gas, Electricity and Water
22. Transport and Communication
23. Distributive Trades
24. Insurance, Banking, Finance and Business Services
25. Professional Scientific Services
26. Miscellaneous Services
27. Public Administration and Defence

The 1966 data gives the following illustration of t-forces across the county, with respect to the "employment opportunities".

County of Norfolk

From A	To B	Structural t-forces: Attraction to B	Structural t-forces: Attraction to A
Norwich CB	Kings Lynn MB	nil	8-force
Norwich CB	Cromer UD	nil	7-force, 8-force
Norwich CB	Wayland RD	nil	7-force, 8-force
Kings Lynn MB	Swaffham UD	8-force	nil
Kings Lynn MB	Thetford MB	5-force, 6-force	5-force, 6-force, 7-force
Kings Lynn MB	Depwade RD	nil	6-force, 7-force
Thetford MB	Wymondham UD	5-force	5-force, 6-force
Thetford MB	Downham RD	nil	4-force, 5-force, 6-force
E. Dereham UD	Diss UD	5-force	5-force
Swaffham UD	Norwich CB	8-force	8-force
Diss UD	Kings Lynn MB	6-force, 7-force	nil
Gt.Yarmouth CB	Norwich CB	7-force, 8-force	nil
Gt.Yarmouth CB	Lodden RD	nil	5-force, 6-force

We read the above table as follows:

Thetford MB and Downham RD attract (via t-forces) employment (males and females) in the relative proportions of 4-, 5-, 6-forces compared to nil. Thus we see that (e.g.) being an employable person in Downham RD means that (as an illustration) you are attracted to Thetford MB by a 6-force of attraction. Being employable in Thetford means that you experience zero attraction to Downham RD. The other entries are similarly interpreted.

The 1971 data gives the following illustration of such t-forces.

County of Norfolk		Structural t-forces	
From A	To B	Attraction to B	Attraction to A
Norwich CB	Kings Lynn MB	7-force, 8-force	7-force
Norwich CB	Cromer UD	nil	6-force, 7-force
Norwich CB	Wayland RD	nil	6-force, 7-force
Kings Lynn MB	Swaffham UD	5-force, 6-force	5-force, 6-force, 7-force, 8-force
Kings Lynn MB	Thetford MB	5-force, 6-force 7-force	5-force, 6-force 7-force, 8-force
Kings Lynn MB	Depwade RD	nil	5-force, 6-force 7-force, 8-force
Thetford MB	Wymondham UD	5-force, 6-force	5-force, 6-force, 7-force
Thetford MB	Downham RD	nil	5-force, 6-force 7-force
E. Dereham UD	Diss UD	7-force	7-force, 8-force
Swaffham UD	Norwich CB	5-force, 6-force 7-force	5-force, 6-force
Diss UD	Kings Lynn MB	7-force	7-force, 8-force
Gt.Yarmouth CB	Norwich CB	6-force, 7-force	nil
Gt.Yarmouth CB	Lodden RD	nil	5-force

Pursuing this analysis of the emp traffic we can see the t-forces induced via changes $\delta\dot{S}(N+1)$ between the years 1966 and 1971 for Norfolk.

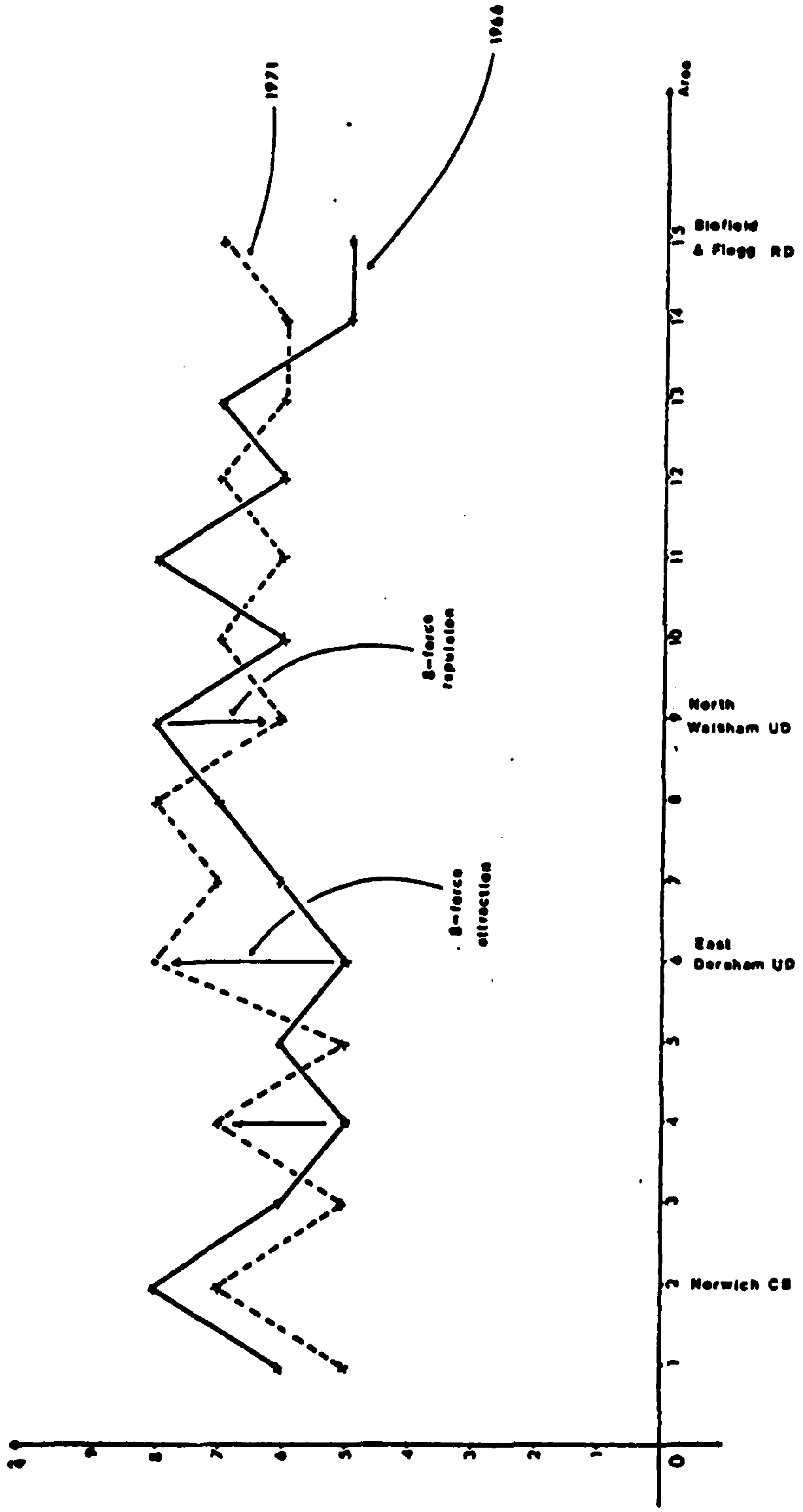


Fig.8 Norfolk — t-forces due to $\Delta S(N+1)$

A comparison between males and females can equally well be obtained for Norfolk. Research Report V only shows this comparison for Cambridgeshire (for 1966 and for 1971). They are reproduced below:

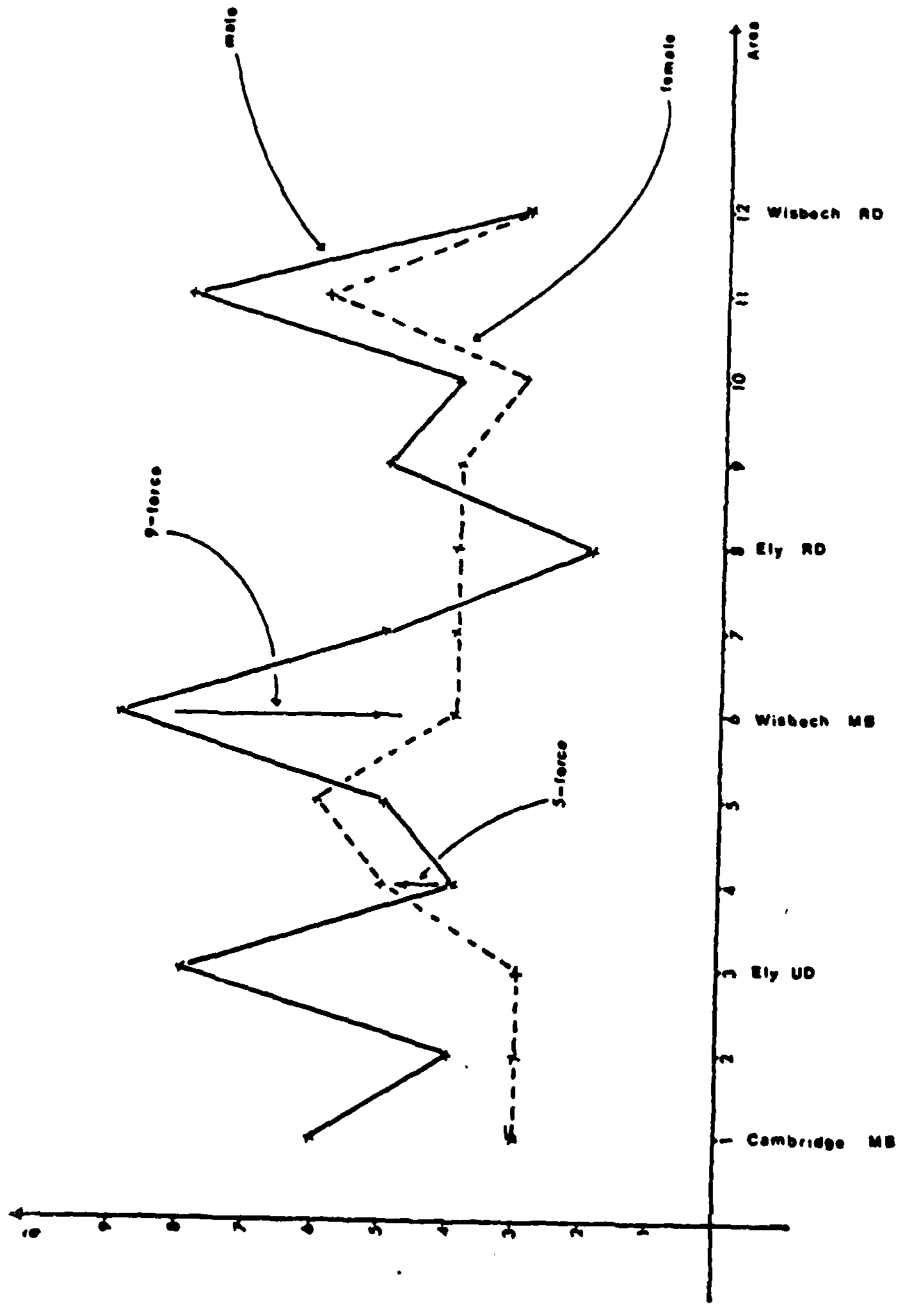


Fig.13 Female — Male I-forces 1966 Data

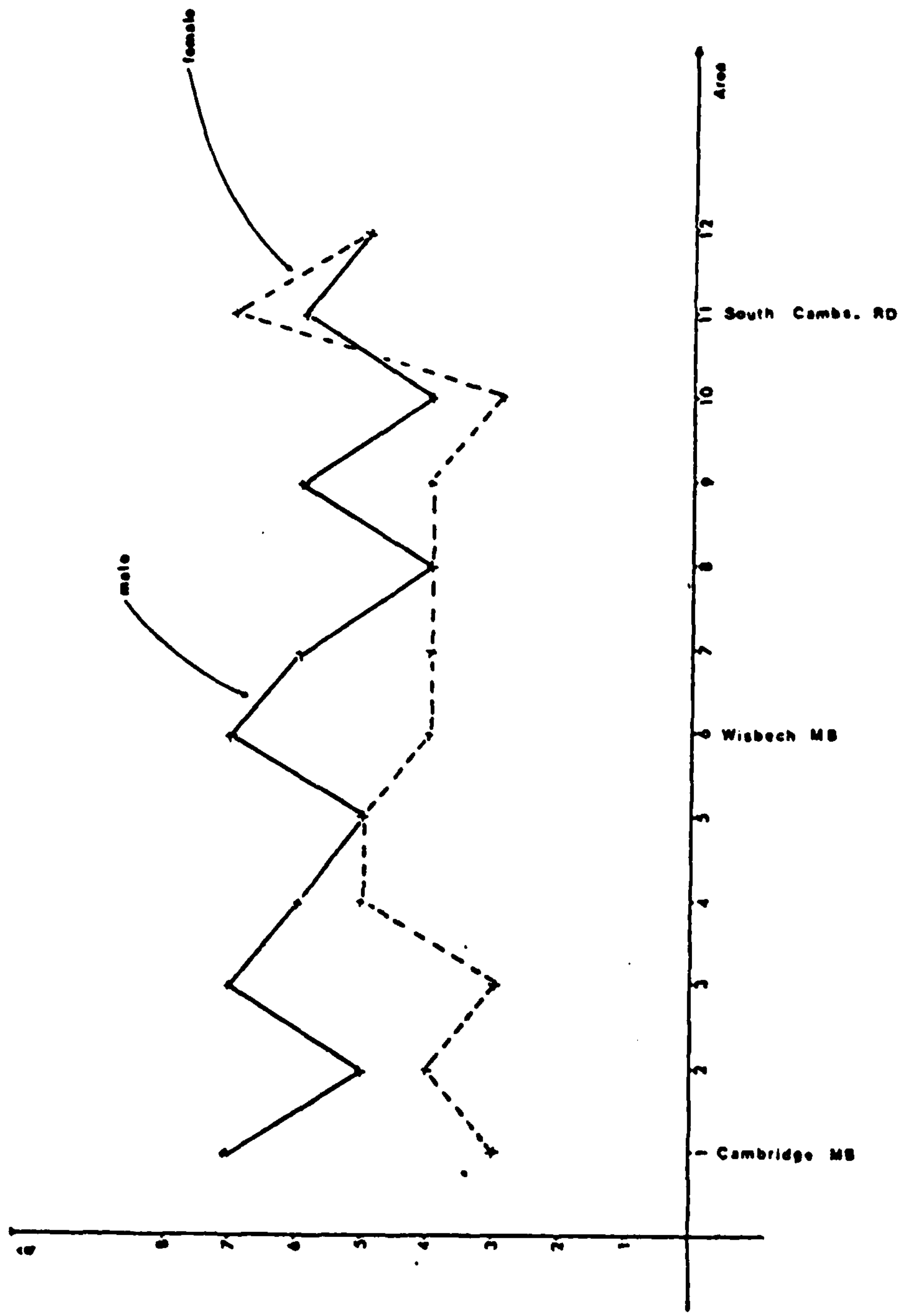


Fig.14 Female — Male t-forces 1971 Data

We notice here that the change from emp (male) to emp (female) is quite striking. This is a typical δemp and shows definite forces of repulsion (from the female).throughout 75% of the county in 1966, and throughout 90% in 1971. If this was true for Cambridgeshire, what is the picture for Norfolk and why is it not displayed in the Structure Plan ?

Ref. (iii),(iv): The problem of "adequate housing", whether it be for the present or future need, can only be discussed at the N-level of data. Where is the data on housing at this level ? It requires a set of vertices which describe housing in some detail. Some of this is available from the Census Data (such as provision of separate toilets, baths, space) but, by and large, it is not data which is currently collected. This makes it particularly difficult for the County Planning Authorities to make a serious policy out of it.

To "improve the level of access to jobs, ..." brings us back to statement (ii) and should be discussed in the same terms - of dimensions of Areas A_i in the structure KA(L). A policy based on this analysis would naturally be expressed in terms of allowed dimensions [e.g. should any area A_i be of dimension less than q_o (for some agreed q_o) in KA(L) ?]

Ref. (v),(vi): Any investment to "support the rural areas" needs to be specified in greater detail - maybe, even, at (N-1)-level. It implies a planned change in the backcloth, $\dot{\delta}S(N)$ or $\dot{\delta}S(N-1)$, and over some agreed time-span (what precisely does "gradual" mean ?). What effects will this have on the basic patterns of pop, emp ? Should we not employ diagram D_5 to monitor any proposal or any actual implementation of such a policy ? Certainly that diagram will provide us with a practical algorithm to so monitor the policy.

As far as (vi) is concerned we need data (at N or at (N-1) ?) which properly describes the "physical character" of the county in terms of "landscape and building". Where is this data and at what level has it been collected ? Typical examples of what such data involves have been provided in earlier Research Reports in this series[13,14]. This is a non-trivial exercise for Planners and Policy-makers, but not one which is outside the scope of practicality.

Ref. (vii,(viii): The first of these statements ties up with statement (vi) and also with any policies (resulting in δS) which are designed to induce δemp and δpop. The economy of the County is clearly dependent on the agricultural wealth therein and so policy must reflect this. The dominent role of agriculture is manifest throughout all the Q-Analysis to be found in Research Reports VI, VII, VIII and IX of this series. "Farming" appears as a high dimensional simplex in all the KL(A) of these analyses. Conversley, they show that all areas (rural areas) are connected through this vertex, in KA(L), showing that "frrming traffic" is very mobile throughout the County. This dominates the economic well-being of Norfolk and in this sense a statement like (vii) is either unnecessary or it verges on the banal.

Statement (viii) requires data of patterns to make it meaningful and relevant to the Structure Plan. The use of "wasteful" is not clear - except in the obvious sense of "being against sin". What are the energy resources of the County and how are they used ? Where is the traffic which means "using energy resources" and what is its pattern energy ? How is energy defined over the backcloth S(N), etc., and what are the causes of changes δ(energy) ? All of these questions need to be answered in detail at the appropriate levels.

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When the Structure Plan descends to the details of the N-level data, and the backcloth S(N), we see it struggling to find a coherent policy through the maze of "soft data" and ill-defined concepts. But the Planners have the unenviable job of having to produce Policy in spite of this lack of scientific tools. Here we can only indicate how new tools can be used - whereas we cannot guarantee providing them with a policy yet we certainly claim to be providing them with a disciplined language with which to search for such a policy and therein to express it.

When it comes to actual Proposals for the rural areas we are definitely in the realm of the N-level backcloth S(N) and, to illustrate the problems, we can consider two of the Structure Plan's main proposals:

(i) the identification of six towns as "growth centres".

(ii) the identification of twenty-six village service centres.

The first picks out the following six towns:

Fakenham, North Walsham, Swaffham, Diss, Downham Market and East Dereham.

They are to be especially favoured as "growth centres" and consequently have high priority in the matter of housing, education, industry, amenities. The criteria for selecting these towns are twofold:

(a) the need (relative) for expansion in employment (numbers and types of jobs) and the expected occurrence of an increase in employable young persons.

(b) possibilities for expansion in terms of communications and freedom from constraints (to development ?)

An important idea is that each of these six towns is a focal point for a kind of "problem rural area" which is in especial need for growth[10].

A Q-Analysis of the backcloth S(N), for these six towns only, gives us the following structure (based on our data of Land Use versus Authorities).

<u>KA(L)</u>:

q = 152 (East Dereham)

q = 139 (East Dereham), (Downham Market)

q = 137 (East Dereham), (Downham Market), (Diss)

q = 135 (East Dereham), (Downham Market), (Diss),(Fakenham)

q = 131 (East Dereham), (Downham Market), (Diss),(Fakenham), (N.Walsham)

q = 108 (East Dereham), (Downham Market), (Diss),(Fakenham), (N.Walsham), (Swaffham)

etc.

q = 73 (East Dereham, Downham Market, Diss, Fakenham, North Walsham, Swaffham)

Thus we notice that these six towns are all connected at the dimension of q = 73 (not before). By any standard this is a high connectivity. The six towns therefore form a sort of county (using the 1975 data) which gives them a character of their own. But in all of the towns we notice that East Dereham has the greatest top-q, and the eccentricity of each is listed below:

Ecc (East Dereham) = 0.67 → (152,91)

Ecc (Downham Mkt) = 0.69 → (139,82)

Ecc (Diss) = 0.62 → (137,84)

Ecc (Fakenham) = 0.58 → (135,85)

Ecc (N.Waltham) = 0.43 → (131,91)

Ecc (Swaffham) = 0.47 → (108,73)

Thus Diss has a top-q of 137, a bottom-q of 84 and its eccentricity is Ecc (Diss) = (137,84) ÷ 85 = 0.62. It follows that, although the six towns have a unity due to their connectivity at q = 73, yet each is eccentric, each possesses its own individuality in the group. Furthermore, the values of the top-q ($\hat{q}$) gives an indication of the t-forces acting across the geography (between the separate towns). East Dereham acts as the major attraction (to 152-traffic, or to all t-traffic for t > 139). Then Downham

Market, Diss, Fakenham and North Walsham are much on a par, with Swaffham somewhat below at $\hat{q} = 108$.

We can illustrate the "problem areas" by considering two of these towns, viz., Fakenham and East Dereham.

A Q-Analysis of "Fakenham + Environs" shows that Fakenham is most closely connected (in the Land Use vertices) to Burnham Market, Great Walsingham and Little Walsingham, at $q = 24$ (compared with $\hat{q} = 135$ for Fakenham). It is least connected to Pudding Norton ($q = 2$), to Great Snoring, Washam, Houghton, Burnham Norton and Barsham (at $q = 3$). At intermediate values of q (between 3 and 24) it is variously connected to other rural parishes.

This means (e.g.) that if the idea is to make Fakenham a centre of attraction for "its area" then *the Q-Analysis suggests a set of priorities* for that area. Proposals for increasing the q-connectivities between Fakenham and its environs can then be expressed as $\dot{\delta}S$ with its consequent influence on patterns thereon (v. diagram D_5). By pursuing the operations of D_5 we can immediately calculate the structural t-forces throughout the County. This effect is exactly that which pertains to vehicular traffic in a town[6]; the operations of D_5 tell us how (e.g.) a new set of traffic lights affects the flows on the other side of town.

But policy proposals about (e.g.) employment opportunities need to be specified in detail at the N-level. At the moment this particular structure KL(A), relevant to the Fakenham "growth centre", is dominated by Farmers, Building Contractors, Junior/Infant Schools, Inland waterways, and Public Houses (all of which occur between q=35 and q=17). At low $\hat{q}$-values we find (e.g.) Agriculture Machien Dealers and Repairers, Builders Merchants, Bus & Coach services, Concrete Deady-Mix Suppliers, Dairies, Painters & Decorators, Ladies Hairdressers (each of which has a top-q value of 2 - which means they are to be found in only 3 of the rural areas in the Fakenham sub-complex of S(N)). If employment is to be improved how is that spectrum to be enlarged ? Does the Policy suggest that these low q-value simplices be increased (and if so by how much) or are new vertices to be introduced ? The first of these ways would certianly preserve the character of the structure. How would the second affect that character ? Is this where the source of conflict between Planners and People really lies ?

Now let us look at the structure of the East Dereham[10] sub-complex of S(N).

In KA(L), East Dereham has a top-q of 152 and the next Parish to enter the list is Shipdham, at q = 33. Of course we expect East Dereham to dominate this sub-complex but these q-values give a quantitative measure to that situation. Shipdham and East Dereham are connected at q = 26, showing that there is already a strong link between the two - enabling traffic (such as employable persons) to move from one to the other. Other Parishes enter much lower down and (e.g.) the components at q = 8 are the following:

(1) East Dereham, Shipdham, North Elmham, Foulsham, Mattirhall, Swanton Lorley, Beeston with Bittering, Yaxham, Gressenhall, Beetleg, Lyng, Scarning, North Tuddenham, Wendling, Hockering, Mileham

(2) Garvestone

(3) Cranworth

(4) Fransham

(5) Elsing

(6) East Tuddenham

(7) Brisley

(8) Bawdeswell

All 34 Parishes in this sub-complex fall into one connected component at $q = 2$. So how is the "growth" to be planned ? Is it to be the increase in the bottom-q-values for Parishes other than East Dereham ? Is it to be an increase in the top-q value for East Dereham ? Is it to be a bit of both ? What levels of q (bottom-q values) are to be aimed at if the Policy is the former ? What new vertices are to be introduced to East Dereham if the latter ? How would this affect $\delta S(N)$ and the rest of the County ?

Once more we can see that these choices in Policy (which seem to me to be inadequately discussed in the Norfolk Structure Plan) seriously affect the character of the Area and the prognosis of the "disease" (because increasing q-value for East Dereham might well affect its connectivity with different county areas and so act to the detriment of this particular sub-complex - such an effect can of course be detected by applying D_5).

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Now we can take a look at the other example of the Planning Proposals, viz., the 26 Village Service Centres selected for residential and small workshop development. All 26 will involve urban employment growth and various services investment policies (such as public transport, schools, medical services). Some of these 26 are near to the urban areas of Norwich, King's Lynn and Great Yarmouth. The rest (15) are properly in rural areas and comprise:

Docking, Feltwell, Harling, Heacham, Hingham, Marham, Mattishall, Melton Constable/Briston, Methwold, Mundesley, Necton, Reepham, Shipdham, Stalham/Sutton, Upwell/Outwell.

The following selections from the Q-Analyses of KL(A) and KA(L) illustrate how these 15 Parishes are structured in S(N).

KL(A) - in which the Land Use names denote simplices/Parishes are the vertices

q-value	components
16	(Farmers, Building Contractors, Butchers, Garage Services)
12	(Farmers, Building Contractors, Butchers, Garage Services, Police, General Stores, Schools Infants & Juniors, Woodland, Grocers Retail, Physicians and Surgeons)
4	(As above, together with Banks, Public Houses, Ladies Hairdressers, Inland Waterways, Coal & Coke Merchants, Newsagents, Hotels, Road Haulage Contractors, Painters & Decorators, Bakers & Confectioners, Car Hirers, Inns, Poultry Farmers, Car Distributors & Dealers, Carpenters & Joiners, Dairies, Florists, Fruiterers & Greengrocers, Garden Centres, Market Gardeners, Nurserymen & Seedsmen, Old Peoples Homes, Television & Radio Shops)

KA(L) - in which the Parishes are simplices/Land Uses are vertices

q-value	components
69	(Heacham)
66	(Heacham), (Stalham)
53	(Heacham), (Stalham), (Mundesley)
44	(Heacham), (Stalham), (Mundesley), (Reepham)
38	(Heacham, Stalham), (Mundesley), (Reepham)
32	(Heacham, Stalham, Mundesley), (Reepham), (Harling), (Hingham), (Methwold), (Shipdham), (Feltwell)
16	(Heacham, Stalham, Mundesley, Hingham, Harling, Feltwell, Docking, Reepham, Methwold, Shipdham, Upwell, Marham) (Outwell), (Necton), (Briston), (Coltishall), (Melton Constable)

In KL(A) we see (e.g.) that General Stores is a 12-simplex (is to be found in 13 of the Parishes). It follows that a Policy which controls this simplex will affect 13 Parishes simultaneously. For example, already the structure experiences a 12-force of repulsion when the General Stores close down for a Bank Holiday (or rather, the traffic of shoppers experiences that force). Thus detailed Policy (e.g.) to encourage expansion of job opportunities needs to take into account not only the dimension of such new jobs but also their connectivities throughout KL(A). By applying D_5 we can then find the effect of any such proposals on the pattern <u>pop</u>, on KA(L). <u>Any house building programme should be linked closely with that calculated</u> δ<u>pop</u>, illustrating our previously noted procedure.

$$\underline{\mathrm{emp}}^{o}[L] \xrightarrow[\theta]{} \underline{\mathrm{emp}}[KA(L)] \xrightarrow[\delta F]{} \underline{\mathrm{pop}}[KA(L)]$$

In KA(L) we see that Heacham and Stalham dominate the structure, followed by Mundesley and then Reepham. The character of these Parishes is well brought out by their non-zero eccentricies. Notice that Heacham ($q = 69$) and Stalham ($q = 66$) are not connected until we reach $q = 38$. Thus they share 39 Land Use vertices (giving scope for cross traffic) but separately possess 70 and 67 vertices.

Once more we can see how Policy needs to be expressed very carefully in a way compatible with these structures. Building houses (e.g.) in various villages because there might be some convenient pieces of land available is not necessarily a responsible thing to do. <u>It should be part of a controlled plan which includes the structural effect of the implied dependencies</u>.

$$\delta(\mathrm{emp}) \xrightarrow{\delta F_1} \delta(\underline{\mathrm{pop}}) \xrightarrow{\delta F_2} \delta(\underline{\mathrm{services}})$$

each of which induces changes in the whole backcloth S(N) - over the whole

County, in effect - via

$$\delta F_1 : S \rightarrow S \cup \dot{\delta}_1 S$$

and

$$\delta F_2 : S \rightarrow S \cup \dot{\delta}_2 S$$

the induced calculations being based on diagram D_5 above. The Norfolk Structure Plan recommends that "workshop scale industries" may be allowed in these Village Service Centres. This implies an extension of the vertices in L(N) which are relevant to this sub-complex KA(L). The success of such a scheme would seem to depend on which vertices are chosen since a few mistakes could easily disconnect one or more of the Village Centres from the group - by connecting them more with nearby urban areas. Choosing the vertices so as to increase the connectivity between the villages could have a marked *effect on the sense of community therein*, without destroying the essential character of the areas.

. . . .

Can we fairly draw any conclusions from this sort of analysis of a published Structure Plan (and we must not regard the Norfolk Structure Plan as in any way different from other such productions by other counties) ?

The Q-Analysis is of course very demanding and "hard" (as opposed to "soft"), but it is claimed here that it can be seen to exhibit those <u>practical</u> features which make it a useful tool for Planners (and People) in both the formulation of Policies and in the Monitoring of change throughout the whole field of social activities. For the most part it is clear that Planners (at any level) have only a crude kind of information at their disposal - and this is because the traditional method of producing plans contains no theoretical (or practical) need for anything better. A simple illustration is provided by typical statistics of (say) population in a county. In Norfolk Rural Areas (1961) the Census data tells us that[7] the population was 215,900, whilst by 1974 it had become 237,000. These figures are a simple sum of all the things we have here denoted by $\underline{pop}^t$ (as t goes from 0 to (say) n), because vis-a-vis the structure KA(L), in S(N), we know that

$$\underline{pop} = \underline{pop}^o \oplus \ldots \oplus \underline{pop}^n$$

The "data" tells us that in 1961

$$\sum_o^n \underline{pop}^t = 215,900$$

and in 1974

$$\sum_o^n \underline{pop}^t = 237,000$$

This is a sort of "horizontal arithmetic" which destroys the <u>structure</u> of the pattern and loses the significance of the backcloth. What, for example, can we say about the increase in <u>pop</u> (that is to say about $\delta(\underline{pop})$) when all we know is that

$$\delta(\text{population}) = 21,100$$

It is little wonder that the Planners' Policies have to be stated in high level "soft" language ((N+3)-level concepts ?) and why it is difficult for them to provide a disciplined (a scientific ?) story which links (H,N) to (H+2,G) ? But should we be satisfied with this situation, which surely produces a surfeit of frustration and social conflict ?

[8] Theoretical features of the structural approach

The mathematical foundations of this methodology have been published in earlier Reports and elsewhere[5]. For convenience some of the basic definitions are repeated here.

Resumé

Given a relation λ between finite sets Y and X (Y contains m elements Y_i and X contains n elements X_j) so that $\lambda \subset Y \times X$ and $KY(X) \equiv KY(X,\lambda)$ and $KX(Y) \equiv KX(Y,\lambda^{-1})$ are the conjugate simplicial complexes. The incidence matrix Λ(representing λ) is an (mxn)-matrix with entries 0/1 and $\bar{\Lambda}$ is the transpose of Λ (and represents λ^{-1}).

There are two distinct representations of a particular KY(X), viz.,

(R1): The geometric representation in E^{2n+1}, where n = dim K, and where each $\sigma_p \in K$ is represented by a unique convex polyhedron in a p-dimensional subspace of E^{2n+1}. A pattern (representing some specified traffic on K) is a mapping

$$\pi : \{\sigma_p \in K\} \rightarrow J \qquad \text{(J is usually the integers)}$$

which is naturally graded via the values of p.

If $\sigma_p = \langle X_1 X_2 \ldots X_{p+1} \rangle$ is a p-simplex in K, then the X's correspond to the vertices of the representative convex polyhedron, and the <u>face operator</u> f acting on a σ_p is defined as a set of (p-1) simplices by

$$f\sigma_p = \bigcup_i \{\langle X_1 \ldots \hat{X}_i \ldots X_{p+1} \rangle\}$$

here $\langle X_1 \ldots \hat{X}_i \ldots X_{p+1} \rangle$ is that polyhedron whose vertices are all these X's except for X_i (a sub-polyhedron of dimension (p-1)).

The <u>coface operator</u> Δ is associated with patterns defined on k and defined by

$$\sum_i (\langle X_1 \ldots \hat{X}_i \ldots X_{p+1} \rangle \pi^{p-1}) = (f\sigma_p, \pi^{p-1}) \stackrel{\text{Def}}{=} (\sigma_p, \Delta\pi^{p-1}) \qquad \ldots 8(1)$$

Thus $\Delta\pi^{p-1}$ is a p-dimensional pattern (a "π^p") whose value on any σ_p is the sum of the values of π^{p-1} on all the faces (that is to say, all the σ_{p-1} which are faces) of σ_p.

The supra-face operator f_{+1} is defined by

$$f_{+1}\,\sigma_p = \bigcup_i \{\sigma^i_{p+1} \text{ such that } \sigma_p \in f\,\sigma^i_{p+1}\}$$

[N.B. This f_{+1} is what, in earlier papers [3,5], was written as f_{-1} (under the illusion that it could be regarded as some sort of "inverse".) But this former notation is too bad to continue with; f_{+1} seems too sensible to ignore.]

(R2): The algebraic representation in $\bigwedge(X)$, the exterior algebra formed by the variables X_i, naturally represents the oriented complex K (in which the vertices have the natural ordering of the integers). Each simplex $\sigma_p \in K$ is now represented by (e.g.) the algebraic element $X_1X_2 \ldots X_{p+1}$ and a pattern π is a polynomial in the algebra with coefficients out of J. The definitions of f, f_{+1}, Δ are the same as in R1 except that we can write sums instead of unions, viz.,

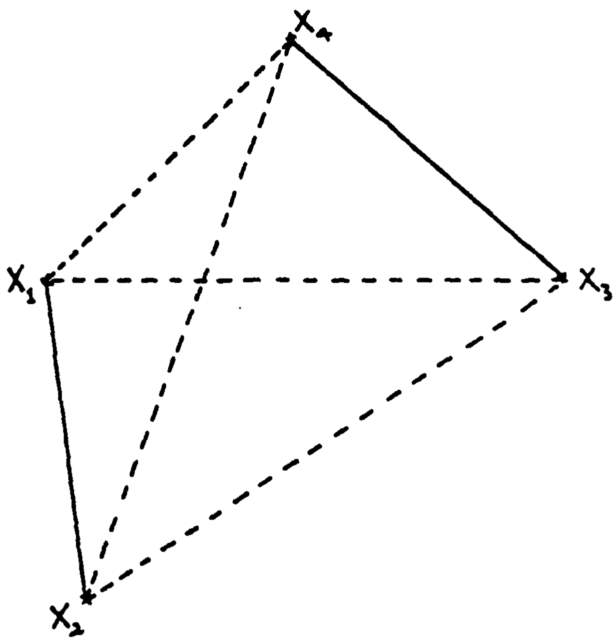
$$f\sigma_p = \sum_i (X_1 \ldots \hat{X}_i \ldots X_{p+1}) \text{ etc.}$$

(end of Resumé)

Now in R2 we also have the possibility of a product of two simplices (or of two patterns). For example, if $\sigma_1{}^1 = X_1$ and $\sigma_1{}^2 = X_2$ then the product $\sigma_1{}^1 \wedge \sigma_1{}^2 = X_1X_2$ represents the 2-simplex $\langle X_1X_2 \rangle$. Similarly, if we have two simple patterns (say) $\alpha = a\,X_1X_2$ and $\beta = bX_3X_4$ their product in R2 gives $\alpha \wedge \beta = ab\,X_1X_2X_3X_4$, which means that $\alpha \wedge \beta$ takes the value ab on $\langle X_1X_2X_3X_4 \rangle$. This suggests that we can fine some correspondence between patterns which are closely associated with the additive structure (in R1 or in R2) and those associated with the multiplicative structure in R2.

In R1 this product corresponds to forming what is known as the Lefschetz prism (between, say, σ_p and σ_q). Thus the tetrahedron

$\langle X_1X_2X_3X_4 \rangle$ is the prism formed by the edge $\langle X_1X_2 \rangle$ and the edge $\langle X_3X_4 \rangle$,



Tetrahedron $\langle X_1X_2X_3X_4 \rangle$ as a Leftschetz prism on $\langle X_1X_2 \rangle$ and $\langle X_3X_4 \rangle$.

Now suppose we begin with a zero-order pattern π^0 (with values on the vertices X_i, only). Then, for example, on $\langle X_1X_2 \rangle$ we can find the value of a particular π^1 (viz., $\Delta\pi^0$) by the equation,

$$(\langle X_1X_2 \rangle \ , \ \Delta\pi^0) = (f \langle X_1X_2 \rangle \ , \pi^0) = (\langle X_1 \rangle \ , \pi^0) + (\langle X_2 \rangle \ , \pi^0)$$

<u>thinking of it in R1</u>. But <u>in R2</u> we can utilize the product to say that this $\Delta\pi^0$ corresponds to some other 1-pattern π^1 given by

$$(\langle X_1X_2 \rangle \ , \ \pi^1) = (\langle X_1 \rangle \ , \pi^0) \ . \ (\langle X_2 \rangle \ , \pi^0)$$

and so we expect to find a natural homomorphism (actually an isomorphism, if we insist on using oriented complexes) between patterns $\{\pi^0\}$ on R1 and those on R2 via what is essentially an exponential map, exp: R1 → R2. This is to be interpreted as follows:

$$\left.\begin{array}{ll} \text{if } (\langle X_1\rangle, \pi^0) = a_1, \ (\langle X_2\rangle, \pi^0) = a_2 & \text{in R1} \\ \text{and since } e^{a_1+a_2} = e^{a_1}.e^{a_2}, \text{ the } \Delta\pi^0 \text{ in R1} & \\ \text{corresponds to a } \pi^1 \text{ in R2 which is defined} & \\ \text{by} \quad e^{a_1}.e^{a_2} \overset{\text{Def}}{=} (\langle X_1X_2\rangle, \pi^1) & \text{in R2} \\ \text{so that} & \\ \qquad \exp: (a_1 + a_2) \to e^{a_1}.\ e^{a_2} & \end{array}\right\| \quad \text{(HOM)}$$

Using this HOM we can see that any π^0 on KY(X), in R1 corresponds to e^{π^0} on the 0-simplices in R2 and products of these e^{π^0} can be interpreted as the image of $\Delta\pi^0$ in R2, whilst the π^0 and $\Delta\pi^0$ are found in R1. In the same way we know[15] that if π^0 has values on (p+1) vertices $\{X_1, X_2 \ldots X_{p+1}\}$ then

$$(\langle X_1 \ldots X_{p+1}\rangle, \frac{\Delta^p\pi^0}{p!}) = \sum_{i=1}^{p+1} (\langle X_i\rangle, \pi^0)$$

and so the relations in HOM can be naturally extended to higher order simplices, giving the following general result,

$e^{\Delta}(\pi^0)$ <u>is a sequence of patterns,</u> viz.,

$$\pi^0, \Delta\pi^0, \frac{\Delta^2}{2!}\pi^0, \ldots, \frac{\Delta^p}{p!}\pi^0, \ldots \frac{\Delta^n}{n!}\pi^0 \ \ldots \ \underline{\text{in}} \text{ R1}$$

<u>which are such that (e.g.)</u> $\frac{\Delta^p}{p!}\pi^0$ <u>is that</u> π^p <u>whose value on any</u> σ_p <u>is the sum of the values of</u> π^0 <u>on the vertices of that p-simplex.</u> <u>Furthermore these have images under HOM which form the sequence,</u>

e^{π^o}, $e^{\bar{2}\pi^o}$, $e^{\bar{3}\pi^o}$, ... $e^{\bar{p}\pi^o}$, ... $e^{\bar{n}\pi^o}$ ... <u>in</u> R2

<u>where (e.g.) by</u> $e^{\bar{p}\pi^o}$ <u>we mean the set of terms like</u>

$e^{a_1+a_2 \cdots a_p}$, <u>the p-selection being over distinct vertices</u>

<u>and</u> $a_i = (< X_i > , \pi^o)$.

We notice that, with this interpretation, there are ${}^nC_p$ terms in the set $e^{\bar{p}\pi^o}$, corresponding to the number of p-selections out of n, and so the total number of such terms in the whole sequence is 2^n-1. Furthermore, if we find a canonical representation $\hat{Y}$ (in which every element Y_i has non-zero eccentricity) then we know[15] that the <u>pattern generator</u> Θ is a collection of suitable operators e_i^Δ (one for each member of $\hat{Y}$); precisely

$$\Theta = \sum_i e_i^\Delta - \sum_{i<j} e_{ij}^\Delta + \sum_{i<j<k} e_{ijk}^\Delta - \text{etc.}$$

where (e.g.) e_{ij} is the operator applicable to that simplex which is $Y_i \cap Y_j$. This generator contains the structure of KY(X) in its definition and has already been referred to in previous sections of this Report.

But Θ also has an image under HOM, say Φ , where we can agree to write

$$\Phi = \text{Exp}\ \Theta$$

This generator Φ acts in R2 and relates the multiplicative structure there to the additive one in R1.

The analogue, in functions of a real variable, of π^o, defined on the finite set X, is the ordinary idea of a function f(x), defined on a (continuous) domain D. Then HOM corresponds to the ordinary exponential mapping

$$\exp: \; f(x) \to e^{f(x)} \quad (= F(x))$$

or equivalently, expressing the inverse of HOM by the homomorphism MOH which gives

$$\ln: \; f(x) \to \ln F(x).$$

The operator Δ (which sums values of π^o over subsets of X) corresponds to the operation of <u>integration</u> over D. The idea of an integral can in fact be used precisely in our discrete theory if we use the <u>Riemann-Stieltjes Integral</u>, for then (e.g.)

$$\sum_{r=1}^{n} f(r) = \int_{0}^{n} f(x) \, d\,[x]$$

where [x] = greatest integer not exceeding x.

It follows that we can write

$$(<X_i X_j>, \; \Delta\pi^o) = \int_0^2 f(x) \, d[x] \qquad \text{(C1)}$$

where $f(r) = (<X_r>, \pi^o)$ and correspondingly

$$(<X_1 \ldots X_{p+1}>, \; \frac{\Delta^p}{p!}\pi^o) = \int_0^{p+1} f(x) \, d[x]$$

This suggests that there should be an operation "inverse" to Δ, say Δ^{-1}, which can be the analogue of differentiation - the "inverse" of integration. We do this by the following definition,

$$(f\sigma_p, \Delta^{-1}\pi^p) \underline{\underline{\text{Def}}} (\sigma_p, \pi^p)$$

which means that

> $\Delta^{-1}\pi^p$ <u>is a (p-1)-pattern and the sum of its values on the faces (the</u> σ^i_{p-1}) <u>of any</u> σ_p <u>equals the value of</u> π^p <u>on that</u> σ_p.

We can therefore regard Δ^{-1} as an operator which <u>partitions</u> the value of π^p (on any σ_p) into a set of values on the faces of that σ_p. How does it partition this value ? Let us agree, in this discussion, to regard it as partitioning this value (σ_p, π^p) into <u>equal parts</u>, viz., $(\sigma_p, \pi^t)/(p+1$
It follows then that $\Delta^{-1}\Delta$ = identity <u>only</u> when π^p has equal values on p-simplices. In general $\Delta^{-1}\Delta \neq$ identity. But, interestingly, it is true that

$$\Delta\, \Delta^{-1}\, \pi^p = (\text{identity})\ \pi^p = \pi^p$$

when both sides are applied to any one p-simplex.

So under these conditions our Δ^{-1} is a <u>right-inverse</u> of Δ, but not a left-inverse. This should be compared with the analogous results for the differentiation operator D and the integration (indefinite) operator D^{-1} viz.,

$$D\, D^{-1} = \text{identity}, \quad D^{-1}D \neq \text{identity}.$$

Having defined Δ^{-1} we can now consider $\Delta^{-p} = \Delta^{-1}\Delta^{-1}\Delta^{-1} \ldots \Delta^{-1}$ (p terms) and obtain its meaning by the following result,

$$(f^p\sigma_p,\ \Delta^{-p}\pi^p) = (f^{p-1}\sigma_p,\ \Delta\Delta^{-p}\pi^p) = (f^{p-1}\sigma_p,\ \Delta^{-(p-1)}\pi^p) = \ldots = (\sigma_p, \pi^p)$$

Thus $\Delta^{-p}\pi^p$ is a 0-pattern (on vertices only) and its values on the $f^p\sigma_p$ add up to the value of π^p on σ_p. But $f^p\sigma_p$ counts each vertex p! times and so (c.f. the introduction of $\Delta^p/p!$ above) the values of $\frac{\Delta^{-p}}{p!}\pi^p$ on the vertices of σ_p add up to the value of π^p on σ_p.

This means that

> $\frac{\Delta^{-p}}{p!}$ is an operator which divides the value of π^p (on any σ_p) into equal parts on the vertices of σ_p.

Example: Take $\sigma_3 = \langle X_1X_2X_3X_4\rangle$ and π^0 such that $(\langle X_r\rangle, \pi^0) = r$ for $r = 1,2,3,4$. Then

(i) $(\langle X_1X_2\rangle,\ \Delta\pi^0) = 1 + 2 = 3$ (and similar)

(ii) $(\langle X_1X_2X_3\rangle,\ \frac{\Delta^2}{2}\pi^0) = 1 + 2 + 3 = 6$ (and similar)

(iii) $(\langle X_1X_2X_3X_4\rangle,\ \frac{\Delta^3}{6}\pi^0) = 1 + 2 + 3 + 4 = 10$

(iv) Now take π^3 such that $(\langle X_1X_2X_3X_4\rangle, \pi^3) = 10$ whence
$\Delta^{-1}\pi^3$ takes the value 10/4 on each face like $\langle X_1X_2X_3\rangle$,
$\frac{\Delta^{-2}}{2!}\pi^3$ takes the value 10/6 on each edge like $\langle X_1X_2\rangle$,
$\frac{\Delta^{-3}}{3!}\pi^3$ takes the value 10/4 on each vertex $\langle X_i\rangle$.

(v) In this case $\Delta\Delta^{-1}\pi^3$ takes the value 10 on $\langle X_1X_2X_3X_4\rangle$, that is to say, $\Delta\Delta^{-1}$ = identity. But
$\Delta^{-1}\Delta\pi^0$ takes the value 3/2 on each of the vertices X_1 or X_2, that is to say,
$\Delta^{-1}\Delta \neq$ identity

(end of Example)

Qn. 9: What happens to Δ^{-1} under the transformation HOM ?

If dim K = n and if e^x_n is the truncated exponential series (last term is $x^n/n!$) we have the following result.

$e_n^{\Delta^{-1}}(\pi^n)$ is a sequence of patterns, viz.,

$\pi^n, \Delta^{-1}\pi^n, \frac{\Delta^{-2}\pi^n}{2!}, \dots, \frac{\Delta^{-n}\pi^n}{n!}$ in R1

which are such that (e.g.) $\frac{\Delta^{-p}\pi^n}{p!}$ is that π^{n-p} whose value on any (n-p)-face of $a\sigma_n$ is $\left[{}^{n+1}C_p\right]^{-1}$ times the value of π^n on that σ_n. These have images under HOM which form the sequence

$$e^{\pi^n}, e^{\underline{2}\pi^n}, e^{\underline{3}\pi^n}, \dots, e^{\underline{p}\pi^n}, \dots e^{\underline{n}\pi^n} \text{ in R2}$$

where (e.g.) by $e^{\underline{p}\pi^n}$ we mean the set of equal terms $e^{ka}n$, where $k^{-1} = {}^{n+1}C_p$ and $a_n = (< X_1 \dots X_{n+1} > , \pi^n)$ for all the p-selections out of (n+1).

A similar statement, with HOM replaced by MOH, can be constructed by simply replacing the exponential map by the logarithmic one and interchanging R1 and R2.

[9] The idea of a potential pattern

Since Δ^{-1} is the analogue of the "derivative" ($\frac{d}{dx}$, $\frac{\partial}{\partial x}$, or grad) we can contemplate the question:

Qn. 10: Given a pattern π^t on KY(X) is there a pattern μ^{t+1} such that

(a) $\pi^t = \Delta^{-1}\mu^{t+1}$ and (b) $\Delta\pi^t = \mu^{t+1}$?

If such a pattern μ^{t+1} exists for a given π^t we shall naturally call the former (the μ^{t+1}) a potential pattern for the latter (the π^t). We notice that (a) and (b) are only equivalent in the special case that $\Delta^{-1}\Delta = \Delta\Delta^{-1}$ = identity, and that this cannot be guaranteed in an arbitrary structure KY(X).

Notice too that altough we have already introduced the idea of a force as some $\delta\pi$ (which we identify with a $\Delta\mu$ - v. D_3 on page 31) we have not previously introduced the idea of a potential (which would have required identifying $\Delta\delta\pi$ with some μ). Our proposed potential μ^{t+1} is analogous to the potential function V(x) of a field of force F(x) in classical mechanics. Actually, in mechanics it is usual to define V(x) by requiring $F(x) = -V'(x)$ because of the statement of conservation of energy. This means that our μ^{t+1} is more like the "work function" W(x), which equals - V(x). The two conditions (a) and (b) do in fact hold in mechanics in the case of a conservative field of force, because then the Work Done by the field in moving a particle from P to Q equals the Work Function (with W(P) = 0)

(a) $F(x) = \frac{d}{dx}W(x)$ implies (b) $\int_P^Q F(x)dx = W(Q) - W(P) = W(Q)$

c.f. $\pi^t = \Delta^{-1}\mu^{t+1}$ implies $\Delta\pi^t = \Delta\Delta^{-1}\mu^{t+1} = \mu^{t+1}$ provided $\Delta\Delta^{-1}$ = identity.

But does μ^{t+1} always exist, given π^t ?

Clearly μ^{t+1} does not exist if t = n = dim K because KY(X) does not contain any (n+1)-simplices. Apart from this case, we can make the following points

(i) If KY(X) is only a simplex σ_n (and all its faces) then, if we are given some π^{n-1} we can always find a μ^n such that

$$\pi^{n-1} = \Delta^{-1}\mu^n \quad \text{and} \quad \Delta\pi^{n-1} = \mu^n.$$

There is no obstruction to the operation of Δ^{-1}, which simply partitions the value of (σ_n, μ^n) into equal parts and distributes them over the (n-1)-faces of σ_n. Then both conditions (a) and (b) clearly apply. But if we are given some π^t, with $0 < t < n - 1$, then as far as this pattern is concerned KY(X) effectively consists of a number (${}^{n+1}C_{t+1}$ precisely) of t-simplices which are shared by a number of (t+1)-faces. This is therefore the case in which KY(X) has a non-zero obstruction vector (at this (t+1)-level) and, although the operation of partitioning (via Δ^{-1}) is quite possible we notice that there is difficulty in arranging for $\Delta\Delta^{-1}$ = identity (which therefore refers us to the next case).

(ii) If KY(X) is such that the obstruction vector $\hat{\underline{Q}} \neq \underline{0}$ we are faced with a particular difficulty in finding a potential μ^{t+1} for an arbitrarily given π^t. It is not generally possible to find such a μ^{t+1} when $\hat{Q}_{t+1} > 0$. We can illustrate the difficulty with a numerical example.

<u>Example</u>: Take KY(X) = {faces of a hollow tetrahedron} , so that at t+1 = 2 we have $\hat{Q}_{t+1} = \hat{Q}_2 = 3 > 0$. In fact $\hat{Q} = \{3\ 0\ 0\}$. Suppose we are given a pattern π^1 such that

$$(< X_1 \ldots \hat{X}_i \ldots \hat{X}_j \ldots X_4 >, \pi^1) = i \times j$$

for $1 \leq i \leq j \leq 4$. Then (e.g.) $(< X_1X_2 > , \pi^1) = 3 \times 4 = 12$ and

$$(< X_2X_4 > , \pi^1) = 1 \times 3 = 3, \text{ etc...}$$

Then we look for some 2-pattern μ^2 such that (a) $\pi^1 = \Delta^{-1}\mu^2$ and (b) $\Delta\pi^1 = \mu^2$. Let us begin to find μ^2 by relation (b). Then we easily find that by 8(1)

μ^2 takes the value 24 on $< X_1X_2X_3 >$, 21 on $< X_1X_2X_4 >$,
16 on $< X_1X_3X_4 >$, and 9 on $< X_2X_3X_4 >$.

But then we find that $\Delta^{-1}\mu^2$ is a pattern which is the set of 4 patterns (because there are 4 components at the q=2 level) viz., $\Delta^{-1} = (\Delta_1^{-1}, \Delta_2^{-1}, \Delta_3^{-1}, \Delta_4^{-1}$

This means that we have the following list:

$\Delta_1^{-1}\mu^2$ associated with the 1-faces of $< X_2X_3X_4 >$

$\Delta_2^{-1}\mu^2$ associated with the 1-faces of $< X_1X_3X_4 >$

$\Delta_3^{-1}\mu^2$ associated with the 1-faces of $< X_1X_2X_4 >$, and

$\Delta_4^{-1}\mu^2$ associated with the 1-faces of $< X_1X_2X_3 >$.

Since each Δ_i^{-1} $(i = 1,2,3,4)$ partitions the values of (σ_2,μ^2) into equal parts on the 1-faces of σ_2, we easily find that

$\Delta_1^{-1}\mu^2$ takes the value 9/3 on the edges $< X_2X_3 >$, $< X_3X_4 >$, $< X_2X_4 >$.

$\Delta_2^{-1}\mu^2$ takes the value 16/3 on the edges $< X_1X_3 >$, $< X_3X_4 >$, $< X_1X_4 >$.

$\Delta_3^{-1}\mu^2$ takes the value 21/3 on the edges $< X_1X_2 >$, $< X_2X_4 >$, $< X_1X_4 >$.

$\Delta_4^{-1}\mu^2$ takes the value 24/3 on the edges $< X_1X_2 >$, $< X_2X_3 >$, $< X_1X_3 >$.

If we regard $\Delta^{-1} = \Delta_1^{-1} + \Delta_2^{-1} + \Delta_3^{-1} + \Delta_4^{-1}$ (what else ?) we find that

$\Delta^{-1}\mu^2$ takes the value

45/3 on $< X_1X_2 >$, 33/3 on $< X_2X_3 >$, 25/3 on $< X_3X_4 >$,

40/3 on $< X_1X_3 >$, 37/3 on $< X_1X_4 >$, 30/3 on $< X_2X_4 >$.

But then it is clear that $\Delta\Delta^{-1} \neq$ identity, since (e.g.)

$$3\,(< X_1X_2X_3 > ,\ \Delta\Delta^{-1}\mu^2) = 45 + 33 + 40 = 118 \neq 3\,(< X_1X_2X_3 > ,\ \Delta\pi^1) = 72.$$

We conclude that in the case that $\hat{\underline{Q}} \neq \underline{0}$ and, in particular, $\hat{Q}_{t+1} > 0$ then an arbitrary π^t does not generally possess an associated potential pattern μ^{t+1}.

(iii) But if we consider a pattern π which has been generated by the pattern generator Θ (associated with the structure KY(X)) from a zero-pattern π^o, then we find that a potential may exist. This follows because we begin with

$$\pi = \Theta\pi^o$$

and Θ consists of a set of exponential maps e_i^{Δ}, one for each non-eccentric simplex γ_i in the representation γ. Then each component π^t in π is the direct sum of terms like $\frac{\Delta^t}{t!}$, one for each t-component. But if (e.g.) we have $\pi^t = \frac{\Delta^t}{t!}\pi^o$, *can we find* a μ^{t+1} such that (a) $\pi^t = \Delta^{-1}\mu^{t+1}$ <u>and</u>

(b) $\Delta\pi^t = \mu^{t+1}$? The second condition is met by identifying

$$\mu^{t+1} \quad \text{with} \quad \frac{\Delta^{t+1}}{t!}\pi^o$$

whilst the first is also satisfied by writing $\mu^{t+1} = \frac{\Delta^{t+1}}{t!}\pi^o$, <u>provided</u>

$\Delta^{-1}\Delta$ = identity. When is this true ? It is true when π^t has <u>equal values</u> on the t-faces of any σ_{t+1}, for then

$$\sum_i (\sigma_t{}^i, \Delta^{-1}\pi^t) = (f\sigma_t, \Delta^{-1}\Delta\pi^t) = (\sigma_t, \Lambda\Delta^{-1}\Delta\pi^t)$$

$$= (\sigma_t, \Delta\pi^t) = (\sigma_t, \mu^{t+1})$$

This must hold for every value of t and so for t=0.

When this condition holds, viz., <u>when</u> π^o <u>takes a constant value on every vertex (0-simplex) of KY(X), then it possesses an associated potential</u> μ^1 and

$\pi = \Theta\pi^o$ <u>possesses an associated potential μ given by</u>

$\mu = \Delta\pi = \Delta\Theta\pi^o$.

Whilst π is graded from 0 to n, μ is graded from 1 to n, and π^n does not possess a potential. At any t-value between 0 and (n-1) we have

$$\mu^{t+1} = \underset{t!}{\Delta}^{t+1}\pi^o$$

(iv) The discussion above is clearly a special case of a more general result, which depends on a pattern π taking constant values k_t on every σ_t in KY(X), for t = 0, 1, ... (n-1), where n = dim K. It is not necessary for k_t to equal k_s, when $t \neq s$. Under these circumstances it follows that we can take

$$\mu^{t+1} = \Delta\pi^t \qquad 0 \leq t < n$$

and then

$$\Delta^{-1}\mu^{t+1} = \pi^t$$

because $\Delta^{-1}\Delta$ = identity. (Notice that we are not concerned with the product $\Delta\Delta^{-1}$ in this discussion). This relation is true whether or not $\hat{\underline{Q}}$ is or is not zero, because generally $\Delta = \{\Delta_i\}$ and $\Delta^{-1} = \{\Delta_i^1\}$, the values of i indexing the distinct elements of the canonical set $\mathring{Y}$. If $\sigma_p = \mathring{Y}_i \cap \mathring{Y}_j$ we have

$$(\sigma_p, \Delta\pi^{p-1}) = \{(\sigma_p, \Delta_i\pi^{p-1}), \ (\sigma_p, \Delta_j\pi^{p-1})\}$$

and

$$(\sigma_p, \Delta^{-1}\pi^{p+1}) = \{(\sigma_p, \Delta_i^{-1}\pi^{p+1}), \ (\sigma_p, \Delta_j^{-1}\pi^{p+1})\}$$

and since $\Delta_i^{-1}\Delta_i$ = identity = $\Delta_j^{-1}\Delta_j$.

But neither $\Delta_i^{-1}\Delta_j$ nor $\Delta_j^{-1}\Delta_i$ are defined on (things like) σ_p, and so

$$\Delta^{-1}\Delta = \{\Delta_i^{-1} , \Delta_j^{-1}\}\{\Delta_i , \Delta_j\}$$

$$= \{(\text{identity})_i , (\text{identity})_j , \Delta_i^{-1}\Delta_j , \Delta_j^{-1}\Delta_i\}$$

giving

$$\Delta^{-1}\Delta = \{(\text{identity})_i , (\text{identity})_j \}$$

We conclude that, independently of $\hat{Q}$, whenever π^t takes constant values on the σ_t in $KY(X)$ for any t, then μ^{t+1} exists as an associated potential pattern and is given by $\Delta\pi^t$.

This also means that the property of possessing a potential is independent of changes in $\hat{Q}$ (provided π remains defined on K). So this idea of a potential pattern transcends the idea of particular structural backcloth. Traffic which is so represented by a potential pattern is always so represented.

Example: We have seen in the previous section that any pattern π on $KY(X)$ can be regarded as being defined in R1 or in R2 and that these are related by HOM or MOH. It follows that (e.g.) any pattern of traffic might naturally arise as a "function"

$\pi_1^o = \ln x$ (x taking values on the vertices of K) in R1 or as

$\pi_2^o = e^x$ (x taking values on the vertices) in R2. There is no substantial difference between these (only the shadow of a mathematical homomorphism !). If the values of x are such that the pattern π_1^o takes a constant value on each, say k, then $\pi_1 = \Theta\pi_1^o$ possesses an associated potential μ_1 where $\mu_1 = \Delta\Theta\pi_1^o$. Then $\Theta\pi_1^o = \Delta^{-1}\mu_1$ and similarly for π_2 and μ_2. Equally well, if we now regard μ_1 as (say) a logarithmic function (in R1) it is the potential of another pattern π_1 where

$$\Delta^{-1}\mu_1 = \pi_1$$

But this is the analogue of $\frac{d}{dx}(\ell n(x)) = \frac{1}{x}$. The potential is the analogue of the Newtonian gravitational potential. It follows that, under those conditions which ensure the existence of a potential, the (analogue of the) gravitational potential (the so-called "gravity models" for patterns) is <u>the potential</u> for <u>any pattern</u> (satisfying those conditions). This is a mathematical property, not a scientific law which needs to be discovered.

[10] Relevance of entropy-maximising and gravity models

The development of an analysis, based on an analogue of classical statistical mechanics, known now as the entropy-maximising method is due primarily to Wilson[16,17] - and can be described by reference to the transportation problem of journey-to-work trips over a region which has been zoned into (say) N areas. Each zone is regarded as an element O_i in the set of origins O and as an element D_j in the set of destinations D. Furthermore, it is assumed that there is a notional cost c_{ij} associated with each possible trip from O_i to D_j, and that T_{ij} is the number of trips (in any one period) between these zones. The theory proposes the following constraints:

$$\sum_j T_{ij} = \bar{O}_i = \text{total trips leaving area } i \quad \dots (1)$$

$$\sum_i T_{ij} = \bar{D}_j = \text{total trips entering area } j \quad \dots (2)$$

$$\sum_{i,j} T_{ij}c_{ij} = C = \text{total notional cost (all trips)} \quad \dots (3)$$

where C is a given constant. If the total of all trips is T it is required to find a most probable distribution (of that T) over the N^2 "cells" (i,j) of the problem, subject to the constraints (1), (2), (3).

Write $m = N^2$, regard T as fixed, and let y_r denote the rth cell (in some sensible ordering of all the cells (i,j)). Then since any one trip can be allocated to only one cell this allocation can be represented by a cell-occupying function, viz.,

$$y_1 + y_2 + \dots + y_m$$

which represents the (assumed) mutually exclusive events.

(y_1 occupied) (others unoccupied) + (y_2 occupied) (others unoccupied) + etc...

Since the allocations are independent (it is so assumed) the total of all distributions of the T trips is found via the coefficients of the terms of the expansion of the generating function

$$(y_1 + y_2 + \ldots + y_m)^T$$

The coefficient of (e.g.) $y_1^{T_{11}} y_2^{T_{12}} \ldots y_m^{T_{NN}}$ in this multinomial expansion gives the number of ways in which T_{11} trips are allotted to cell y_1, T_{12} to cell y_2 ... and T_{NN} to cell y_m. This number is

$$V = \frac{T!}{T_{11}!T_{12}! \ldots T_{NN}!} \quad \text{where } T_{11} + T_{12} + \ldots + T_{NN} = T.$$

The theory then proceeds to maximise the logarithm of this V (and $\ln V$ looks like a typical "entropy" function) relative to all partitions $\{T_{ij}\}$ of T. This is done by the standard use of Lagrangian multipliers λ_i, μ_j, β in the expression

$$L = \ln V - \sum_i \lambda_i \left(\sum_j T_{ij} - \bar{O}_i\right) - \sum_j \mu_j \left(\sum_i T_{ij} - \bar{D}_j\right) - \beta\left(\sum_{ij} T_{ij}c_{ij} - C\right)$$

The details arise by considering $\frac{\partial L}{\partial T_{ij}} = 0$, all (i,j),

and produce the result

$$T_{ij} = A_i B_j \bar{O}_i \bar{D}_j \exp(-\beta c_{ij}) \quad \ldots (4)$$

where

$$A_i = \{\sum_j B_j \bar{D}_j \exp(-\beta c_{ij})\}^{-1}$$

and

$$B_j = \{\sum_i A_i \bar{O}_i \exp(-\beta c_{ij})\}^{-1}.$$

The result (4) can be seen as typical of what are called "gravity models" in which $\exp(-\beta c_{ij})$ is replaced by a problem-dependent function $f(d_{ij})$, d_{ij} being the distance between O_i and D_j in the set of cells (i,j). In its turn this $f(d_{ij})$ is a generalisation of the simple (Newtonian) case when $f(d_{ij}) = d_{ij}^{-2}$.

[Note: There are two critical assumptions in this Wilson theory and these could well be altered to obtain new mathematical results. They have already been referred to above and are the following.

(i) The trips are mutually exclusive, as expressed in the cell occupancy function $y_1 + y_2 + \dots y_m$. This means that if Smith is in a trip from O_i to D_j (in cell (i,j)) then he is not in any other cell - that is to say, he cannot be a trip from O_i to D_t where $t \neq j$. This seems a particularly rash assumption in the context of a transportation study, if only because Smith might well need to travel through zones $(t,h,k\dots)$ before arriving at his destination of zone j.

(ii) The allocations are independent, as expressed in the resultant generating function (using the Rule of Product), viz., $(y_1 + y_2 + \dots + y_m)^T$. Again, in the context of a transportation study, this is probably not justified - chiefly because heavy traffic in some cells (i,j), or in some sequence of cells like $(i,j) \to (j,h) \to (h,k) \to \dots \to (p,q)$, must affect the possibility of trips in other cells (via intersection of routes etc.). This sort of consideration will certainly affect the quantities c_{ij}, and probably limit the extent to which C can be regarded as a constant.

The criticism under (i) can be met by replacing the cell occupancy function by something like (e.g.)

$$y_1 + y_1y_2 + y_2 + y_1y_2y_3 + \dots + y_m$$

where (e.g.) the term $y_1y_2y_3$ means that a trip is possible which occupies (during the period of study) cells y_1, y_2 and y_3. Various accepted routes could thus be built in to this function and then the distribution (ignoring criticism (ii)) will be given by the generating function

$$(\text{cell occupancy function})^T.$$

(end of Note)]

A straightforward interpretation of this particular "model" in terms of our structure is as follows.

(1) Let Z be the set of zones and $Z = \{Z_i,\ i = 1,\ldots N\}$ and let λ be the relation (between Z and Z) which says "Z_i and Z_j are regarded as possible origin O_i and destination D_j, for some trip". In fact the Wilson theory assumes that every pair $(Z_i, Z_j) \equiv (O_i, D_j)$, so that $\lambda = Z \times Z$, and Λ is a matrix of 1's.

(2) This relation λ defines two simplical complexes KO(D) and KD(O). In the former, each O_i is an (N-1)-simplex (it is related to all N of the D_j) and any two of these share all N vertices (the D_i). This means that the structure vector for KO(D) (and incidentally for KD(O) as well) is

$$\underline{Q} = \{\overset{N-1}{1}\ 1\ 1\ \ldots\ \overset{2}{1}\ 1\ \overset{0}{1}\}$$

and KO(D) $\equiv$ KD(O) $\equiv$ a single (N-1)-simplex, with all its faces. [A small example would be 1 tetrahedron, with 4 different names].

(3) The Wilson theory ignores these structures (their dimensionality) and proceeds as if KO(D) were embedded <u>as a network</u> in the cartesian plane $Z \times Z$. This means that T_{ij} is effectively viewed as a number attached to an <u>edge</u> (Z_i, Z_j) - in which the first member is an O_i and the second

is identified as D_j. Thus we have

$$T : (O_i, D_j) \rightarrow T_{ij}, \varepsilon J \quad \text{(J non-negative integers)}$$

In this network (call it $K_1Z(Z)$) the trips form a 1-pattern as do the others

$$\{T_{ij}\} \text{ is a 1-pattern, } \pi_T^1 \text{ on } K_1Z(Z)$$

$$\{\bar{O}_i\} \text{ is a 1-pattern, } \pi_O^1 \text{ on } K_1Z(Z)$$

$$\{\bar{D}_j\} \text{ is a 1-pattern, } \pi_D^1 \text{ on } K_1Z(Z)$$

and $$\{T_{ij}c_{ij}\} \text{ is a 1-pattern, } \pi_C^1 \text{ on } K_1Z(Z)$$

There are N^2 edges (1-simplices) in $K_1Z(Z)$ together with the formal identification $O_i = D_i = Z_i$, for $i = 1,2, \ldots N$.

Since there are no 2-simplices in $K_1Z(Z)$ it is trivially true that $\Delta\pi^1$ is not defined. The theory regards this as allowing arbitrary (given) values for $\Delta\pi^1$ and this is the sense in which we can view <u>the equations of constraint</u> (1), (2), (3), <u>together with</u>

$$\sum_{ij} T_{ij} = T$$

Since there are no trips on the vertices ($\pi_T = \pi_T^1$ only, it is <u>not</u> $\pi_T^o \oplus \pi_T^1$) we can identify the Lagrangian multiplier method with the following statement,

$$\Delta^{-1} \{\pi_T^1 - \lambda\pi_O^1 - \mu\pi_D^1 - \beta\pi_C^1\} = 0$$

This is the analogue of $\frac{\partial L}{\partial T_{ij}} = 0$ (the relation $\lambda\pi_0^1$ is short for

$\sum_i \lambda i \left(\sum_j T_{ij} - \bar{O}_i \right)$).

(4) We see that the theory is peculiarly attached to the consequences of specifying a backcloth S ($\equiv K_1 Z(Z)$) for which dim S = 1, carrying traffic which is necessarily represented by a set of patterns π^1 which must then be subject to the statements

$$\Delta\pi^1 = \text{constant}, \quad \text{and } \Delta^{-1}\pi^1 = 0 \qquad \ldots (5)$$

The emphasis on using the logarithmic function ℓn V in the theory is, as we have seen, only an expression of the homomorphism MOH and ensures that the patterns are defined in R1.

It is also clear that there is no potential pattern for π_T^1 since there cannot be any μ^2 defined on this structure. In no way therefore can we regard this trip-distribution as the analogue of a classical conservative field and the distribution is highly dependent on the actual structure of KZ(Z).

(5) What happens if we return to the structures KO(D) and KD(O) and consider the trips-distribution in that context ? Since these structures are isomorphic ($O_i = D_i = Z_i$) we need only consider one of them, and call it KZ(Z). Now a particular trip (from Z_i through Z_r, Z_s to Z_j) makes a contribution to a pattern of trips π_T and that particular contribution will be to π_T^3 in the grading

$$\pi_T = \pi_T^0 \oplus \pi_T^1 \oplus \pi_T^2 \oplus \ldots \oplus \pi_T^{N-2} \qquad \ldots (6)$$

This is because the trip from Z_i is identified with the destinations D_r, D_s, D_j, and these are vertices of a simplex in K. Precisely, the trip is associated with the face $< Z_i Z_r Z_s Z_j >$ of σ_{N-1}, it is therefore a π^3_T. In the Wilson theory (see the Note above) this possibility of grading is excluded and, in fact, in KZ(Z) each trip contributes only to π^1_T, so that in that theory we would be writing

$$\pi_T = \pi^1_T \qquad \ldots (7)$$

We also notice there that (6) is compatible with (and suggestive of) the introduction of the modified cell-occupancy function of the Note above, viz.,

$$y_1 + y_1 y_2 + \ldots + y_m$$

is replaced by something like $x_h x_k + x_h x_k x_p + \ldots$,
which corresponds directly with a polynomial in ΛX in the representation in R2. Thus the grading of π_T, via (6), already meets a basic criticism of the entropy-maximising theory. If we stay with that theory and use (7), together with (1), (2), (3) (which are now π^1_O, π^1_D, π^1_C) we obtain equations like (5), viz.,

$$\Delta\pi^1 = \text{given constant, and } \Delta^{-1}\pi^1 = 0 \qquad \ldots (8)$$

but this time $\Delta\pi^1$ takes a constant value in a legitimate way because 2-patterns are allowed (by Q-Analysis) on KZ(Z), and $\Delta^{-1}\pi^1 = 0$ because there are no 0-patterns allowed in the structure KZ(Z). Equations (8) give the standard result, as before.

Using (6) obviously requires a different attitude to the data needs, but that is the price to be paid for getting closer to reality. One consequence of better data would probably be that the structure vector $\underset{\sim}{Q}$ for KO(D) would be much different - expressing, for example, the fact that

O_1 is not related to D_{10} etc., and resulting in $KO(D) \neq KD(O)$.

(6) If we consider only the structure as derived from the λ of (2) above (other structures would be problem-oriented) and follow the procedure of the Wilson theory we can deduce the following.

The cell-occupancy function should be

$$g(\underline{x}) = (x_1x_2 + x_1x_3 + \ldots + x_{N-1}x_N) \oplus (x_1x_2x_3 + \ldots + x_{N-2}x_{N-1}x_N) \oplus \ldots$$
$$\ldots \oplus (x_1x_2 \ldots x_N)$$

$\underline{x} = \{x_1, \ldots, x_N\}$ and

where we interpret the presence of x_ix_j as meaning a trip from O_i to D_j, the presence of $x_ix_jx_k$ as meaning a trip which starts from O_i and includes D_j and D_k as destinations and so forth. This is a simple polynomial in ΛX (in R2) and effectively describes the structure of $KO(D)$. In other words

$$g(\underline{x}) = S_2 \oplus S_3 \oplus \ldots \oplus S_N$$

where S_r = the elementary symmetric function of degree r in N variables. Now, for a given total of trips T, we get the distributions as coefficients in the generating function

$$G(\underline{x},T) = [g(\underline{x})]^T$$

which is interpreted as

$$G(\underline{x},T) = S_2^{T_2} \oplus S_3^{T_3} \oplus \ldots \oplus S_N^{T_N} \qquad \ldots (9)$$

where $\qquad T_2 + T_3 + \ldots T_N = T$ (constant)

Notice that (e.g.) $S_2^{T_2}$ gives the distribution of all partitions of T_2 over the trips contained in S_2.

Of course it would not be unreasonable to expect some practical modification of this - such as limitations on S_r as r increases in value or the modification of any one S_r. Such modifications would in a practical way express the underlying structure of KO(D). For example if S_2 did not contain any terms of the form $x_i x_j$ where $j > i + 2$ it would represent a structure in which no trips were feasible between the zones i and (i+3). Such conditions could clearly involve constraints on the numbers c_{ij} in (3). But now we would require additional equations of constraint, viz.,

$$\sum_k \sum_j (T_k)_i^j = (\bar{O}_k)_i \qquad i = 1,2, \ldots N \quad k = 2,3, \ldots N \qquad \ldots (1)$$

$$\sum_k \sum_i (T_k)_i^j = \bar{D}_k^j \qquad j = 1,2, \ldots N \quad k = 2,3, \ldots N \qquad \ldots (2)$$

$$\sum_t \sum_{i,j} (T_t)_i^j (C_t)_j^i = C_t \qquad t = 2, \ldots N \qquad \ldots (3)$$

together with

$$\sum_t \sum_{i,j} (T_t)_i^j = T \qquad i,j = 1,2, \ldots N \quad t = 2,3, \ldots N \qquad \ldots (4)$$

These equations, together with (9) may then be used to find the most probable distribution on KO(D). The symbol $(T_k)_i^j$ means the number of trips from Z_i to Z_j to be found in the part T_k of T (associated with the occupancy function S_k). The number $(\bar{O}_k)_i$ is the total of all trips leaving area O_i and which are also trips identified via the function S_k; similarly for $\bar{D}_k{}^j$.

The prospect of maximising the possible distribution in a general case is clearly formidable, and also pointless (?) since the "general case" is pointless. In this context the probabilistic basis for the Wilson theory should be considered by reference F Section 13 below.

[11] How to deal with data which is non-binary

Although we find the backcloth structures S(N) from binary matrices it is common for us to obtain data in the form of a weighted relation μ, whose matrix M contains non-binary entries (usually non-negative integers). This research has produced three ways of dealing with this kind of data, but so far only one of them has been actually used.

Fundamental to all three is the idea of "slicing at parameter value 1" to provide us with a basic backcloth $S(N) = KY(X) \cup KX(Y)$. This means that, assuming the entries in M are to be found in J_o^+, we create a binary (incidence) matrix $\Lambda = (\lambda_{ij})$ where

$$\lambda_{ij} = 1 \text{ if } \mu_{ij} \geq 1$$

and $$\lambda_{ij} = 0 \text{ if } \mu_{ij} = 0.$$

Then Λ defines S(N).

Now there are three ways of proceeding.

(1) We can "slice" at different parameter values, say,

$$\lambda_{ij} = 1 \text{ if } \mu_{ij} > \mu_1, \mu_2, \ldots \qquad (\text{where } 1 = \mu_1)$$

and these give a sequence of backcloths $S_i(N)$ such that, in R1.

$$S(N) \equiv S_1 \supset S_2 \supset S_3 \supset \ldots$$

These backcloths correspond to various "norms" which are relevant in the particular context (or not). Thus in Research Reports V, VI[4,12] we have illustrated the case when μ_i is chosen as $\bar{\mu}$, the mean of the column values per row (of matrix M). A sequence of slicings has also been found significant in another discipline, viz., medical diagnosis[18].

(2) In this case we regard the entries in (say) row i of the data matrix M (corresponding to y_i-values on the columns X_j) as defining a π^o on the basic backcloth S ($\equiv S_1$). Then each such Y_i produces an effective pattern π_i on the whole of S via the pattern generator Θ (characterising the structure of S), viz.,

$$\pi_i = \Theta\pi^o \equiv \Theta Y_i$$

Now we can compare the row values (opposite Y_i and Y_j) by noticing the difference between π_i and π_j. This can be interpreted as a set of t-forces (over the structure S) given by

$$\delta\pi = \pi_i - \pi_j = \Theta(Y_i - Y_j) = \text{graded pattern.}$$

This gives a value for t-forces (t = 0,1,...) on S as we contemplate changing from the Y_i data to the Y_j data (over the columns X). This has been illustrated in this Report (v.pp. 56,57 above).

(3) In this case we can use the matrix M as a means of defining an operator Δ , on S. So far we have concentrated exclusively on the simple case where (e.g.)

$$(< X_1X_2 > , \Delta\pi^o) = (< X_1 > , \pi^o) + (< X_2 > , \pi^o)$$

we simply add the vertex values to obtain the edge values.
But why not add <u>weighted</u> vertex values to get the edge values ? If the data is referred to a canonical set $\mathring{Y}$ (with respect to the basic backcloth S(N)) then $\Delta = \{\Delta_i\}$, one for each element of $\mathring{Y}$. If the entries, in M, opposite

Y_i are the numbers $a_{i1}, a_{i2} \dots a_{in}$ then we can use these a_{ij} as weights in defining a new coface operator Δ_i (which is significant for that data matrix). This would give (e.g.)

$$(\langle X_1 X_2 \rangle \ , \Delta_i \pi^o) = a_{i1} (\langle X_1 \rangle \ , \pi^o) + a_{i2} (\langle X_2 \rangle \ , \pi^o)$$

for all $i = 1, 2, \dots m$.

<u>Example</u>: $\dot{Y} = \{Y_1 Y_2 \mid \text{names of streets}\}$ and

$X = \{X_1 \text{ (butcher)}, X_2 \text{ (grocer)}, X_3 \text{ (inn)}\}$

and the data is given by the weighted relation μ with matrix M, where

$$M = \begin{pmatrix} 0 & 2 & 4 \\ 1 & 0 & 3 \end{pmatrix}$$

showing that there are 2 grocers and 4 inns in street Y_1, and 1 butcher and 3 inns in street Y_2. The basic backcloth S is defined by the matrix

$$\Lambda = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

and Δ is now the pair (Δ_1, Δ_2).

If π^o is the pattern: "numbers of shoppers per outlet" on this structure (in any one day) then if $(\langle X_1 \rangle \ , \pi^o) = 10$, $(\langle X_2 \rangle \ , \pi^o) = 15$, $(\langle X_3 \rangle \ , \pi^o) = 5$ we get a generated pattern π^1 via

$$\pi^1 = \Lambda\pi^o = \Lambda \begin{pmatrix} 10 \\ 15 \\ 5 \end{pmatrix} = \begin{pmatrix} (2 \times 15 + 4 \times 5) \\ (1 \times 10 + 3 \times 5) \end{pmatrix} = \begin{pmatrix} 50 \\ 25 \end{pmatrix}$$

which gives the total numbers of shoppers per kind of shop per street for the two streets Y_1 and Y_2, where

$$\Delta_1 \ (15,\ 5) \rightarrow 2 \times 15 + 4 \times 5$$

and $$\Delta_2 \ (10,\ 5) \rightarrow 1 \times 10 + 3 \times 5.$$

With this understanding the generators e_i^{Δ} ($\equiv c^{\Lambda}i$) can also be interpreted. What remains to be investigated is the possibility of defining an associated Δ_i^{-1} .

[12] The question of prediction vis-a-vis structure

We cannot form any sensible idea of theory-predictive matters without, in the last analysis, "taking on" the notion of time. In this research work, as it has developed over a number of years, the idea of "space" has been developed in a way which is Aristotelian/Leibnitzian as opposed to the orthodox Newtonian/Cartesian one[15]. This view says that our awareness of space ("ordinary" one) is that derived from an awareness of the relations (things like λ) which exist between objects. The orthodox (physics-type) view says that space is a priori and objects just sit in it, occupying pieces of it. The other view is rapidly gaining acceptance by social scientists - under the pressure of necessity. "Social spaces", "functional spaces", "mental maps", are all examples of this fact.

But now, if we propose that space is not just an old wastepaper basket for holding objects - but is the manifestation of relations between a priori (observable) objects, then how can we avoid saying the same sort of thing about our awareness of time ? The orthodox (Newtonian) notion of time is of a sort of ever-rolling stream of vacuous "moments" waiting to be occupied by "events" - a sort of temporal wastepaper basket.

Now we must look at the consequences of supposing that time is the manifestation of relations between events.

This must mean that time will correspond to some sort of traffic on a structure of events, a backcloth S. In this case time will be graded into multidimensional features, dependent on the multidimensional nature of S. We shall see that in this context the idea can be seen as a natural mathematical extension of that view of time which even the physical scientist has found to be mutable, c.f. the radical change from Newton to Einstein, in this respect.

Time in Physics

The idea of time which is current in scientific circles is almost unchanged since the days of Isaac Newton. This is probably due to the fact that it has hardly been questioned by the practising scientist. All he has really required of the notion is that it shall fit into his theoretical discussions as easily as other quantities do. By "fitting in" he means only that there shall be, in some standard manner, a mathematical way of representing the thing called time and that this mathematical way shall be no more complicated than the way which represents his orthodox view of space. This has meant that, for the theoretician, it has been quite sufficient for him to be able to represent his time by a time axis. This means that when he is drawing his typical cartesian picture by using standard geometrical representations then he is quite content to have, as a representation of his time, a line in space - one which starts as a convenient origin and which extends outwards vaguely forever, marked with an arrow at the far end and with a small letter t printed beside the arrow head. Thus the scientist begins to use an algebraic symbol, such as the letter t, and as with his discussion of space and other quantities, this letter t is to stand for a real number. His vague idea that time is an ever-rolling stream is adequately represented in his mind by some imaginary variable progressing through the continuum of the real numbers. The idea of the progression through these values becomes modelled as an imaginary point which is allowed to move along this particular time-axis in his geometrical representation. In this respect his use of the variable t, and the assumption about time on which it is based, are still anchored in the description of time which was given by Isaac Newton in his Principia.

> " Absolute, true and mathematical time, of itself, and by its own nature, flows uniformly on, without regard to anything external. It is also called duration."

Since these early days of Newtonian mechanics, which require a variable for the time t which could be comparable with similar variables for distance, velocity, etc., the practical scientist has regarded his prime function in this context as one of refining mechanical measurements of time. This has resulted in a quite fascinating technological development throughout the centuries of devising new chronometers and other horological devices. But we must not let the scientist get away with it completely, chiefly because we can hardly be expected to accept in such a glib way the idea that, for example,a grandfather-clock is measuring"the time". This belief that it actually is measuring the time is a clear expression of the above philosophical position expressed by Isaac Newton. This position is one which asserts the existence of some "absolute time". It is as absolute and as rigid and as given as similar ideas about "absolute space". Indeed the whole of the Newtonian mechanics, and sunsequent theoretical physics, is peculiarly attached to this sense of absolutism. It has all the overtones of the platonic philosophy and bears witness to the belief that somewhere there is "a great pendulum in the sky" which is beating out a thing called time.

But we cannot have an absolute time in this sense, if we are to be practical scientists. This is because we can only accept as universal facts things which we can observe. We cannot measure the absolute time unless we have some means of knowing that the thing which we call absolute time exists independently of the way we can observe it. This is really an impossible feat and so we are faced with the need to keep on measuring something which we intuitively believe, or hope, to be time. Thus a grandfather-clock certainly seems to have a kind of regularity about it, as it cycles through its repetitive conditions, and this intuitive sense of its regularity is what we must rely on as an expression of our inbuilt sense of time. It is quite idle at this stage to ask "how regular is the grandfather-clock". This cannot be answered without referring to another kind of clock. In the end, we find ourselves in a vicious circle chasing the idea of an absolute time. Hence we are destined, as practical scientists to press on with various devices for measuring this <u>sense of sequential ordering</u> and to rely on our intuitive senses in the last analysis. We have of course a great deal of personal experience of sequential ordering and this ranges over the experience of day-followed-by-night to the seasons of the year and the simple repetitive motion of a pendulum. It is in fact commonly believed among scientific historians that the great Galileo first observed the isochronous behaviour of the pendulum by watching a chandelier swinging to and fro in the cathedral at Pisa. How on earth he knew that it was <u>isochronous</u> is speculative, although it has been suggested that he timed it by his pulse rate. But presumably he was willing to believe that it was worth while using the words "of equal duration" to describe what he was observing and that if he followed this plan he would not come up against any serious conflict with his intuitive sense of what equal time intervals consisted. No one of us would presumably wish to question this kind of simple scientific faith, and had we been present at the time and been as inquisitive as Galileo each of us would presumably have thought about the chandelier the same as he did.

This Newtonian idea of time can be quite adequately represented by the following diagram, in which the numbers denote successive moments of time measuring.

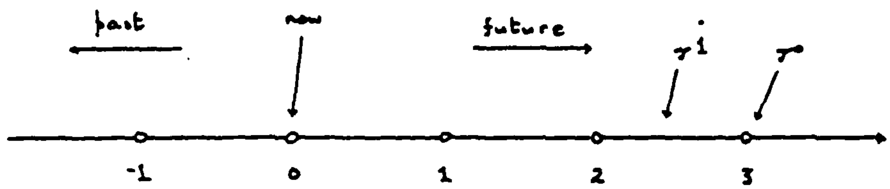


Figure 1 : Newtonian time structure

These time measurements might be, for example, the completion of the swing of a simple pendulum or the successive jumps of a modern electric clock. They are denoted in the Figure by points, or vertices, whilst the lines which join them up are indicative of our sense of time duration. What we have of course is a particularly simple complex. This complex has an unending set of vertices and the numbers which are attributed to the vertices form a pattern τ on the vertices, that is to say on the zero-simplices. If we are traditional physicists who use this Newtonian representation of time then we usually take the next step of subtracting the successive pattern values and thus obtaining a new set of numbers which we attribute to the edges which join them. These numbers we refer to as the time intervals between the successive moments of measurement. Thus we get along scientifically with 1) a simplical complex K consisting of a continuous zero-connectivity as shown in Figure 1 and

2) a pattern on this complex which represents both "moments of time" and "time intervals".

This pattern of time is therefore a graded pattern as shown.

$$\tau = \tau^0 + \tau^1$$

It is because of this kind of representation that we can refer to the Newtonian view of time as a peculiarly linear concept. The linearity of the complex K is precisely the one-dimensional property which characterises it. Even here we see that the pattern which the scientist has had to use to represent his idea of time is one which has certain dimensional characteristics. Thus the Newtonian use of the time variable is a peculiarly one-dimensional idea. When the Newtonian time-axis is used to represent a set of observed events the idea behind it is to somehow produce a kind of "clock" whose time-moments (the vertices in Figure 1) can be matched up into a one-to-one correspondence with the set of events. The idea of a time interval between two events is automatically provided by appealing to the corresponding term in the time-pattern, τ^1. An important feature of the Newtonian notion of time, and how it could be used mathematically in a discussion of physics, was the idea of complete independence between this parameter t and all other parameters - such as those which were to represent the geometry of space or other properties of matter. In terms of our structural analysis this is equivalent to saying that the complex K represented in Figure 1 above is not connected to any other complex of operations - such as the complex of points obtained by observing 3-dimensional space.

Relativity Theories

This technical and philosophical position remained adequate for over

200 years until it had to be modified at the beginning of this century. This modification came about through measurements of the velocity of light and via the subsequent theoretical contributions of Albert Einstein. In his Theory of Relativity in 1905 and 1915, he found it necessary to break this idea of independence between the time structure and the space structure. This was expressed by way of the so-called space-time continuum, a concept which was essentially mathematical in its nature and which combined the traditional 3-dimensions of geometrical space with an additional dimension which was naturally associated with the observer's sense of time[19]. This combination of geometrical and time axes, particularly in the General Theory of Relativity, had to be arranged in such a way that although for one possible observer in the universe the time and the space might be quite separate and independent, yet for any other observer this would be quite impossible. Hence the time-pattern which any particular observer (a pseudonym for an experimental physicist) will find is itself dependent upon, that is to say, being a function of, space-like properties of the system wherein the observer is situated. Thus the time intervals will be a function of velocity and of position. This strange mix-up cannot be unravelled for all observers at all positions and all moments of time. Indeed it becomes impossible to speak of a "moment of time" as if it were something that could be simultaneously observed and understood by different observers. Thus the idea of Newtonian absolutism as well as the idea of independence had to be eliminated from the theoretical structure. But it is perhaps ironic to notice that this was only achieved by replacing one kind of absolutism by another. In place of the idea of an absolute time and an absolute space the theory of relativity introduced the idea of an absolute signal velocity. This refers to the velocity of light usually denoted by the letter c, which plays a fundamental physical and mathematical role in the formulation of the Special Theory of Relativity. This is because, in a practical situation, it had to be assumed that all the information about the geometry of a system is carried to the observer by the light signal. If this signal has a finite

velocity, which it has, then it builds into all the observations its own sense of delay. The absolutism of this philosophy is to be found in the observation that however it is measured it turns out to have a constant value. Thus it does not matter whether the observer is moving relative to the source of light or not. The value which he observes as the velocity of the light he receives always remains constant. It is clear that this situation is bound to affect the time-pattern τ which we have introduced above.

In order to illustrate how this happens we need to describe, if only briefly, the simplest results in Einstein's Special Theory of Relativity. To do this we consider the basic situation which is illustrated in Figure 2.

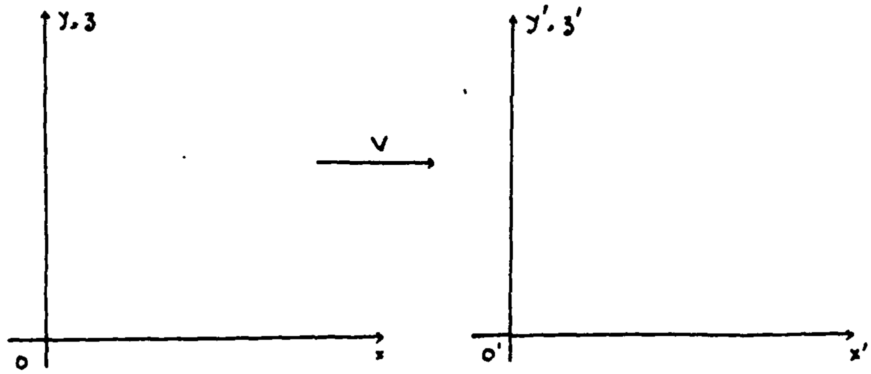


Figure 2 : Relativity frames of reference

This is a standard diagram which represents the possibility of two observers who are busy trying to observe the geometrical position and time coordinate of some moving particle. The one observer O' is supposed to be moving with a uniform steady velocity of V relative to the other observer O. The direction of this relative V is taken as the common direction of the X-axis of both observers. Then, in this simplified case, the other two geometrical axes - the Y-axis and the Z-axis - do not enter into the description in any significant way. But each observer needs to use his own frame of reference. Observer O uses coordinates $\{X, Y, Z, t\}$ whilst observer O' uses coordinates $\{X', Y', Z', t'\}$. The basic theory of this relative notion, taking into account the fact that only the velocity of the light signal itself can be regarded as constant for the two observers, gives us the standard relation between these two sets of coordinates[15].

$$x' = \beta(x - Vt) \qquad y' = y, \; z' = z \qquad t' = \beta\left(t - \frac{Vx}{c^2}\right)$$

where $\beta^2 = \left(1 - \frac{V^2}{c^2}\right)^{-1}$

The number denoted by β is a constant of proportionality and since it involves taking the square root of a certain expression it is clear that if V is greater than c then this expression is negative and so β does not exist as a real number. Herein lies a simple mathematical explanation of why in this theory it is not possible to observe anything moving with a velocity greater than that of light.

Now we can see that it is possible to represent a time-pattern which is compatible with the structure inherent in a relativistic system, by the following considerations.

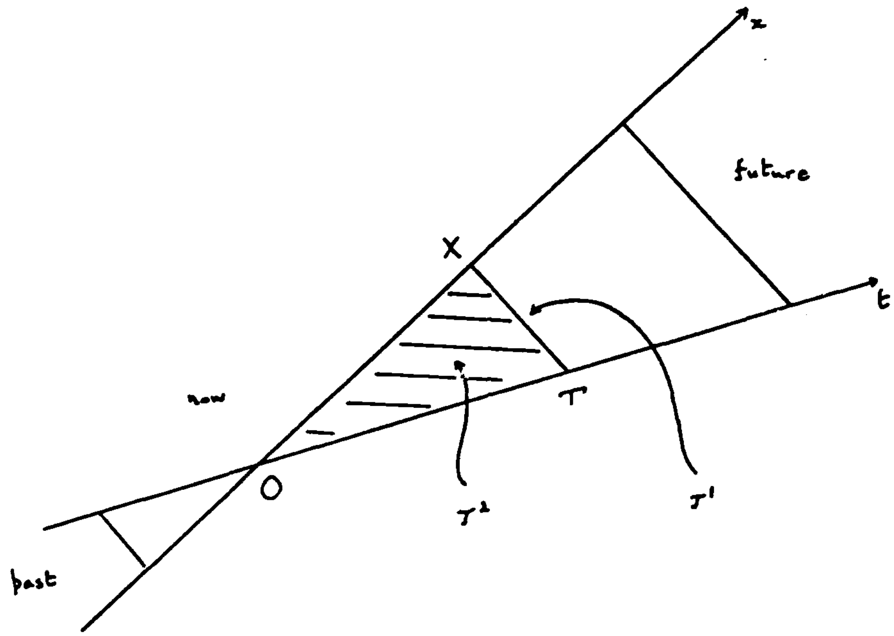


Figure 3 : Simplified Relativity $\tau = \tau^1 + \tau^2$

In the above diagram, which is based on the simplified version of Special Relativity, the Newton time-pattern $\tau = \tau^0 + \tau^1$ has given way to the pattern

$$\tau = \tau^1 + \tau^2$$

Here the "time moments" need to be represented by the 1-simplices (such as) $\langle XT \rangle$, whilst the "time intervals" become the corresponding 2-simplices (such as) $\langle OTX \rangle$. This is because, to a general observer - such as S' it is not possible to keep separate the two measures of the classical notions of x and t.

But in the more general discussion of Special Relativity, in which no one of the apparent space axes, x,y, or z is allowed any [illegible] role, the time-pattern must clearly be of the form

$$\tau = \tau^3 + \tau^4$$

This is illustrated below in Figure 4. Students of Relativity will appreciate that this structure is also what underlies the General Theory.

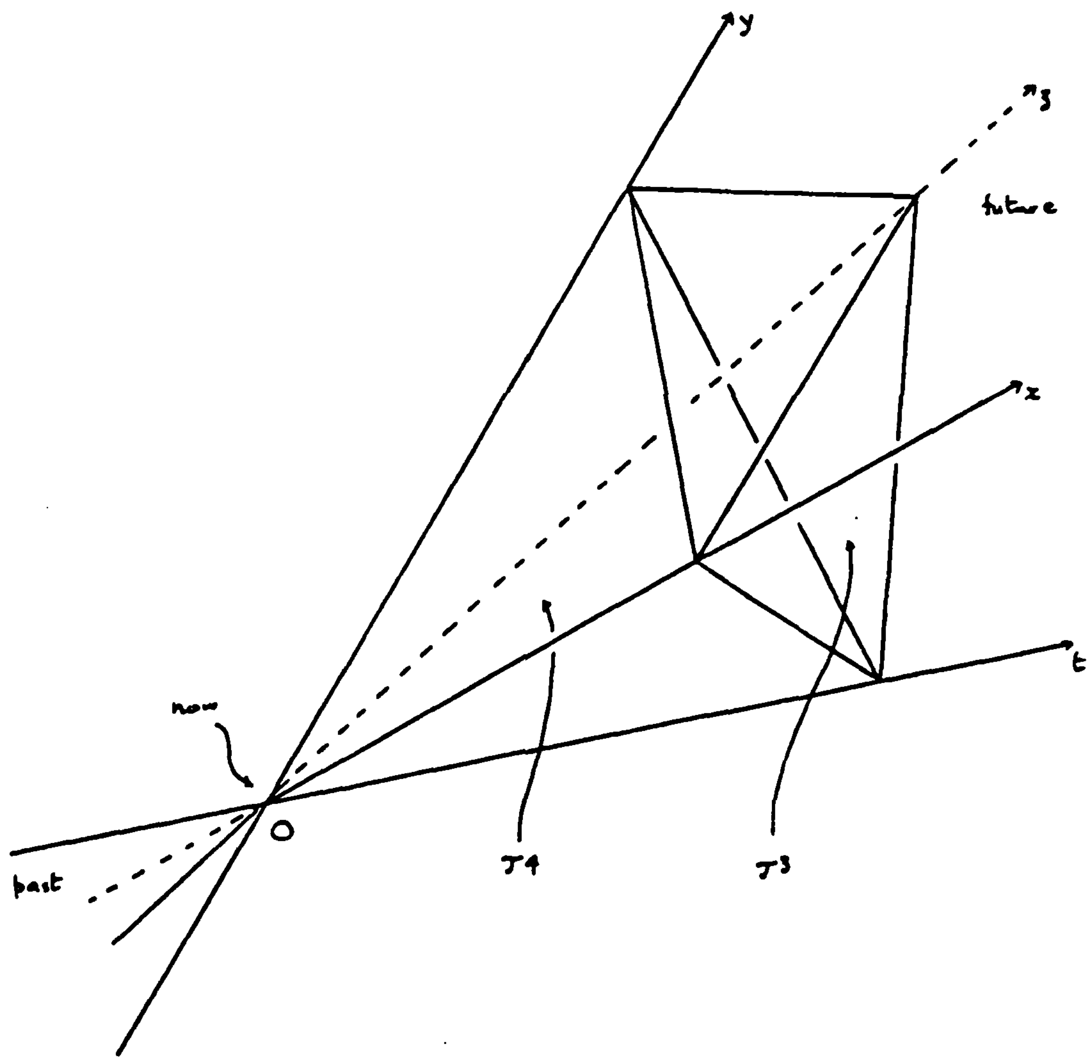


Figure 4 : Relativity $\tau = \tau^3 + \tau^4$

It is because of this shift in the dimensions attributed to time-measured that Einstein (following Minkowski) asserted the predominance of "proper-time" (given by the 4-dimensional interval ds, where $ds^2 = c^2dt^2 - dx^2 - dy^2 - dx^2$). This proper-time is eligible to be a thing like or τ^4 above, being a mapping on the typical 4-simplex $\langle O\ T\ X\ Y\ Z \rangle$ of Figure 4.

Time as a t-force

It is not an exaggeration to say that Relativity Theory is essentially a theory of time. It demanded that the classical Newtonian time-pattern be replaced by a new and different one. This difference arose because of the requirement for a new structure of physical observations (of the point-particle kind). This new structure was itself a consequence of the physical role played by the light signal. It is reasonable therefore to see this scientific advance as strong evidence *for time being* a multi-dimensional pattern on some structure. If that structure is the relatively simple one required for the point-particle dynamics of conventional physics then the time-pattern is the simple

$$\tau = \tau^3 + \tau^4$$

But when we move into the backcloth S(N) associated with the social and structures discussed in this research we would expect the dimensions of the graded time-patterns to be varied as a consequence.

All this would suggest that it is compatible with the physical sciences to introduce the following structural definition of time.

Definition : On any given backcloth S(N) time is a specific traffic, viz.,

(i) the traffic which consists of a total ordering of all p-simplices in S(N)

(ii) the time-pattern of this traffic is of the form

$$\tau = \sum_{p=0}^{n} (\tau^p + \tau^{p+1}) = \sum_{p} T^p$$

(iii) each T^p contains the pattern τ^p and τ^{p+1}, where

τ^p is the now-pattern

and τ^{p+1} is the interval-pattern.

It is because of this shift in the dimensions attributed to time-measures that Einstein (following Minkowski) asserted the predominance of "proper-time" (given by the 4-dimensional interval ds, where $ds^2 = c^2dt^2 - dx^2 - dy^2 - dx^2$). This proper-time is eligible to be a thing like our τ^4 above, being a mapping on the typical 4-simplex $\langle O\ T\ X\ Y\ Z\rangle$ of Figure 4.

In the Newtonian case we are supposed to accept that S(N) is a 1-dimensional structure, and so the time-pattern can only be $\tau = \tau^0 + \tau^1$. In the Einstein case we are supposed to extend this so that S(N) is of maximum dimension 4. This would give

$$\tau = T^0 + T^1 + T^2 + T^3$$

where $T^0 = \tau^0 + \tau^1$ (the Newtonian special case)
and where T^1, T^2, T^3 are various versions of the Relativity case. In this particular instance we already see that the representation of the now-concept is a graded pattern

$$\nu = \nu^0 + \nu^1 + \nu^2 + \nu^3$$

whilst the interval-concept is a graded pattern

$$\pi = \pi^1 + \pi^2 + \pi^3 + \pi^4$$

Our definition clearly gives us a genuine generalisation of the orthodox scientific view of time-coordinatisation and of time-interval. But now we can begin to analyse its significance in other areas of experience. If each individual can be regarded as traffic on some $S(N)$ then his sense of time is linked to that particular structure. In addition it is likely that $S(N)$ contains a classical physics-type structure which provides him with a Newtonian time $\tau^0 \to \tau^1$. The culture of our civilization presently assumes that whenever we use the word "time" we are in fact referring to this Newtonian τ^1. But although this convention is a most useful one, by providing us with a common frame of reference, yet it nevertheless disguises the essential nature of our structurally-based experience of time.

To be precise, we consider the consequence of this cultural assumption. It consists in drawing the conclusions from a change in the time-pattern which "referring to Newtonian time" involves. Thus suppose an individual experiences a time traffic of dimension p, where $p > 1$. Then he finds it socially and culturally necessary to replace this by the Newtonian τ^1 , as follows.

time-pattern $\to$ $\tau^p \to \tau^{p+1}$ (the "now" is on p-simplices)

changes to $\to$ $\tau^0 \to \tau^1$ (Newtonian pattern)

But this means that the individual experiences a t-force of repulsion (where $t = p+1$), as far as the interval-pattern is concerned. This t-force is variously expressed by such phrases as "time flies" or "time drags" - an indication of the fact that τ^1 and τ^{p+1} are out of step.

This means that, because each of us is associated with some structure $S(N)$, we feel that time (that is to say, Newtonian τ^1) is experienced as a set of t-forces in that structure. The concept of "event" is a peciliarly structural concept. Some events are 0-simplices, some will be 3-simplices; generally an event will be the "recognition of a p-simplex" in $S(N)$. Now if we assume (and we stress that this can only be an assumption, without more research data) that, in $S(N)$, there is a (1-1) correspondence between its 1-simplices and those of the Newtonian reference frame, then the time-intervals for the gap between one p-simplex σ_p and the next σ_p will bear a numerical relationship to the τ^1, viz.,

$$\text{time-ratio of} \quad \text{"}\binom{p+2}{2} \text{ to } 1\text{"}$$

because this $\binom{p+2}{2}$ of the number of edges in the least connection (a σ_{p+1}) between σ_p and σ_p'.

This would suggest that a p-event (recognition of a σ_p in $S(N)$) "takes longer to arrive" than does a 0-event by a factor of

$$(p+2)(p+1)/2$$

As an example, if the τ^1 unit is 1 day, then an event which needs 28 days to mature (to arrive) must be a 6-event, because of the equation

$$(p+2)(p+1) = 56$$

reducing to

$$(p-6)(p+9) = 0$$

Although we do not really know (without a detailed research programme to investigate it) at this stage whether or not the τ^1-on-$S(N)$ is identical with the Newtonian-τ^1 (based as that is on some external point-particle-dynamics structure, such as the standard clock or pendulum) nevertheless it is of interest to list the time-intervals associated with various p-events, if that were in fact the case. In this list the interval-pattern is the corresponding τ^{p+1}.

p-event	ratio of $\tau^{p+1} : \tau^1$	
0-event	1	1 day
1-event	3	
4-event	15	
6-event	28	1 month
10-event	66	
18-event	190	
25-event	351	$\sim$ 1 year

If the unit of τ^1 is 1 day then a 25-event requires almost 1 year to "arrive" - that is to say, the interval between successive 25-events is 351 days.

The calculations assume that a p-event (recognition of a σ_p is $S(N)$) occurs by way of the "edges" which go to make up the σ_{p+1} which bridges the previous-σ_p and the present-σ_p. If, on the other hand, we assume that the situation is the worst possible then we might not be able to recognise a σ_p until <u>all its faces</u> have been separately recognised. But this would involve recognising all the intervals between the successive $\sigma_o, \sigma_1, \sigma_2, \ldots \sigma_p$. This number is the sum

$$\sum_{t=1}^{p+1} \binom{p+2}{t+1} \binom{t+1}{2}$$

since the number of t-faces of a σ_{p+1} is $\binom{p+2}{t+1}$, and the number of edges in each σ_t is $\binom{t+1}{2}$. This gives us a time ratio of structural-interval to Newtonian-interval, for any successive p-events in $S(N)$, as

$$(p+2)\ (p+1)\ .\ 2^{p-1}$$

This worst-case ratio gives us the following list:

p-event	ratio of $\tau^{p+1} : \tau^1$	(worst-case)
0-event	1	1 day
1-event	6	
4-event	240	
5-event	672	$\sim$ 2 years

We notice too that the ratio contains the factor 2^{p-1} and this means that there is a natural log-scale relation between the two times (c.f. discussion of MOM and MOH in section [8] above).

Of course, if we move up the hierarchy to S(N+1) then it will generally be the case that a σ_p at this new level will correspond to a much greater σ_q at the N-level ($q >> p$). This would imply that, on any assumption about the ratio of time-intervals, a 1-event in S(N+1) requires a considerable time ratio when observed in S(N). If, for example, each vertex at (N+1) covers 6 vertices at N - and assuming a partition, then

a 0-event in S(N+1) $\equiv$ a 6-event in S(N)

and so the time-interval between successive 0-events in S(N+1) will be 28 units in S(N) - not using the worst-case. But an argument can be made out for associating this "worst-case" with the question of <u>moving up the hierarchy from</u> N <u>to</u> (N+1). This rests on the fact that at the (N+L)-level <u>all</u> the subsets of an N-level set can be seen as vertices (at (N+1)). The number of non-empty subsets of the N-level, (p+1) vertices, is (2^p-1) suggesting a log-relation once again. <u>If we therefore move from an N-level time-pattern to an (N+1)-level pattern we would be inclined to experience what we have called this worst-case correspondence</u>.

In either event we see that this idea of a graded time, associated with ideas of abstract geometrical structure, matches well our experiences of when events actually occur (in Newtonian time). Most events are not "points" and so cannot be fixed on a simple axis. At what precise point, for example, does one "grow up"; when "precisely" is a business successful ? These p-events are not 0-events and so Newtonian time is not applicable. Recent studies of the so-called "body-time" would seem to support this analysis. If bodily functions at certain physiological N-levels are characterised by the progression from one p-event to another p-event, then we would expect the time-intervals associated with these rhythms to be correspondingly proportional. If the pulse rate is used as a match with the Newtonian τ^1 of 1 second then the body's sense of an 8-hour day corresponds to the experience of a 240-event (approximately). Does this have physiological implications at that N-level defined by the "heart beat" . What is the set of 241 vertices (give or take a vertex) which constitute this 240-event ?

By using this kind of approach we can see that some new numerical relations can be calculated for time-intervals between different kinds of events (say, between one p-event and a second p-event). This has consequences for the standard time-series (dynamic modelling) analysis in predictive modelling[20].

But apart from the attempt to connect τ^{p+1} and τ^1 there is a general problem of time factors involved in the change of any pattern on a structure S(N).

If $\delta\pi = \delta\pi^0 \oplus \delta\pi^1 \oplus \ldots \oplus \delta\pi^p \oplus \ldots$

then the change $\delta\pi^p$ is idenfied with some $\Delta\mu^p$ and so is associated with the set of (p+1)-events (simplices) in S(N). If we <u>observe</u> these changes in Newtonian clock-time then we shall find that the ratio of clock-times needed to observe $\delta\pi^0 : \delta\pi^p$ is likely to be the ratio of $1 : \binom{p+2}{1}$ or $1 : (p+2)$. Does this give us an idea of predicting (in terms of clock-time) the consequences of planning decisions ?

[13] Connectivity of events - likelihood and probability

In this section I want to raise the question of the role of probabilistic statistics in our social sciences. To what extent is the use of mathematical probabilities justified as an expression of our intuitive sense of likelihood ? After all, that is what the use of distribution theories is really all about ? By taking refuge behind a standard statistical formulation are we too readily abandoning the ancient scientific rationale which searches for a determinism in our universe ?

To look at this problem we examine the basic notions of the probabilist's "space of events" (sample space) and search for its intrinsic structure (in our terms). This leads us to a specific conjecture, viz.,

> probabilistic interpretation of events is justified if and only if the corresponding structure of those events is described by a zero obstruction vector.

Interestingly enough we shall see that the concept of the two conjugate complexes KY(X) and KX(Y) play an interpretable role in the elementary algebra of probabilities.

Whether or not it is reasonable to apply probability theory (its analytic/algebraic results) to social problems via, for example, distribution theories (c.f. the entropy-maximising arguments discussed in section [10] is an important methodological question. The mathematical distribution theories which purports to describe how people do their shopping, choose their homes or marriage partners, are subject to the spread of disease, etc., leave many of us with a sense of lilliputian insignificance. But perhaps these theories are relevant only to an event structure KY(P) where the people, P, become the vertices. The conjugate structure KP(Y) naturally represents each P_i as a significant structure in its own right and how does probability theory deal with that situation ?

1. Structure of the Space of Events

In the first place we consider the set of events, E, as the finite set generated by the n elementary events $\{X_1, X_2, \ldots, X_n\} = X$, and suppose that $n > 2$. The sample space is then regarded as the power set X, viz. P(X),

$$E = P(X)$$

and this means that, omitting the empty set $\emptyset$, the contemplated set of events number $N = 2^n - 1$. If we write

$$X = \{X_i \mid i = 1, \ldots, n\}$$

and

$$E = \{E_j \mid j = 1, \ldots, N\}$$

the conventional view of this probabilistic sample space is equivalent to the postulate of a mathematical relation

$$\lambda \subset E \times X$$

where $(E_j, X_i) \in \lambda$ means that X_i occurs as an element in the event E_j.

Since $E = P(X)$ every subset of X is an event in E and this means that the simplicial complex

$$K_E(X)$$

must be a single simplex σ_{n-1}, together with all its faces. Thus any particular event E_k denotes a face of σ_{n-1}, and in the geometrical representation of $K_E(X)$ (in the euclidean space $\mathcal{E}^{n-1}$) this means it is a subpolyhedron of the (n-1)-dimensional polyhedron σ_{n-1}. This is why, in the Binomial Distribution, the number of p-dimensional events must be $\binom{n}{p+1}$, this being the number of combinations of n vertices taken (p+1) at a time.

<u>Example 1</u> When X has 4 members $\{X_1\ X_2\ X_3\ X_4\}$ they become the vertices of a tetrahedron (σ_3) in E^3 and the sample space E is the set of $15 = 2^4 - 1$ events represented by the faces of σ_3, viz.,

4 vertices, 6 edges, 4 triangles, 1 tetrahedron.

These events (members of E) can be denoted by the following notation, in which (e.g.) E_{123} means the event represented by the "triangle" $\langle X_1\ X_2\ X_3\rangle$.

	X_1	X_2	X_3	X_4
E_1	1			
E_2		1		
E_3			1	
E_4				1
E_{12}	1	1		
E_{13}	1		1	
E_{14}	1			1
E_{23}		1	1	
E_{24}		1		1
E_{34}			1	1
E_{123}	1	1	1	
E_{124}	1	1		1
E_{134}	1		1	1
E_{234}		1	1	1
E_{1234}		1	1	1

By inspection of this matrix we obtain the following structure vectors.

In KE(X), $\quad Q = \{\overset{3}{1}\ \ 1\ \ 1\ \ \overset{0}{1}\}$

In KX(E) $\quad Q^{-1} = \{\overset{7}{4}\ \ 4\ \ 4\ \ \overset{4}{4}\ \ \overset{3}{1}\ \ 1\ \ 1\ \ \overset{0}{1}\}$

The sample space, E, from this structural point of view is therefore a <u>graded space of events</u>, conveniently written as

$$E = E^1 \cup E^2 \cup E^3 \cup \ldots \cup E^n$$

when (e.g.) event $E_{\alpha_1\alpha_2 \ldots \alpha_p}$ is in the space E^p and the conventional binomial distribution of probabilities must then correspond to the <u>pattern</u>

$$\pi = \pi^0 \oplus \pi^1 \oplus \ldots \oplus \pi^p \oplus \ldots \oplus \pi^{n-1}$$

where $\pi^p : E^{p+1} \to \mathbb{R}$ and $\pi^p(E^{p+1}) = \frac{1}{N}\cdot\binom{n}{p+1}$. This means that π takes the <u>constant</u> <u>value</u> N^{-1} on each event in E.

Furthermore, the Q-Analysis of KE(X) immediately gives the <u>structure</u> <u>vector</u> as

$$\underset{\sim}{Q} = \{\overset{n-1}{1} \; 1 \; \ldots \; 1 \; \overset{0}{1}\}$$

with a corresponding <u>obstruction vector</u>

$$\hat{\underset{\sim}{Q}} = \{\overset{n-1}{1} \; 0 \; \ldots \; 0 \; \overset{0}{0}\}$$

Thus, between $q = 0$ and $q = (n-2)$ there is <u>zero obstruction</u>, in KE(X), to changes $\delta\pi$.

Since the Poisson and Normal distributions may be derived from the Binomial they too are based on the case that $E = P(X)$ so that we see that this simplicial complex structure plays a fundamental role in that general statistical theory which relies on probability distribution functions.

The conjugate complex KX(E), defined by λ^{-1}, is such that each X_i is λ^{-1}-related to m events, E_j, where

$$m = \binom{n-1}{0} + \binom{n-1}{1} + \binom{n-1}{2} + \ldots + \binom{n-1}{r-1} + \ldots + \binom{n-1}{n-1}$$

Since (e.g.) X_i appears in events like $\langle X_1 \ldots X_i \ldots X_p \rangle$ $\binom{n-1}{p-1}$ times (fixing X_i and selecting the remaining $(p-1)$ elements), this gives us $m = (1+1)^{n-1} = 2^{n-1}$, and so each X_i in $KX(E)$ is a $(2^{n-1} - 1)$-simplex.

How are these simplices connected?

Well, if we consider X_1 and X_2 as a typical pair we can easily evaluate the q-values of their shared faces, in the following way. X_1 and X_2 share a vertex when that vertex is any event with a name like $E_{12\ldots}$. This means that

in E^2 they share 1 vertex (the event E_{12}),

in E^3 they share $\binom{n-2}{1}$ vertices, because they share E_{12i} for all i,

and in E^p they will share $\binom{n-2}{p-2}$ vertices - every event $E_{12\ldots}$ which possesses $(p-2)$ subscripts (other than 1, 2).

It follows that X_1 and X_2 (and similarly every pair X_i, X_j, where $i \neq j$), share a q-simplex (are q-connected) where

$$q + 1 = \binom{n-2}{0} + \binom{n-2}{1} + \ldots + \binom{n-2}{n-2} = 2^{n-2} \qquad (n > 2)$$

The structure vector $\underline{Q}^{-1}$ for $KX(E)$ must therefore be

$$\begin{array}{lllllll} \underline{Q}^{-1} = \{n & n & \ldots & n & 1 & \ldots & 1\} \\ & \downarrow & & \downarrow & & & \downarrow \\ \text{where} \quad q = & 2^{n-1}-1 & & 2^{n-2} & & & 0 \end{array}$$

This vector contains an equal number of 1's and n's, viz.,

2^{n-2} entries of 1

and $2^{n-1} - 2^{n-2} = 2^{n-2}$ entries of n.

The obstruction vector $\hat{\underset{\sim}{Q}}^{-1}$ contains zero's only for $q = 0$ to $q = 2^{n-2}-1$. inclusive.

If $|\underset{\sim}{Q}|$ denotes the standard norm of $\underset{\sim}{Q}$ we get, for KE(X):

$$|\underset{\sim}{Q}|^2 = 1^2 + 1^2 + \ldots + 1^2 \qquad (2^n \text{ terms})$$
$$= 2^n$$

whilst KX(F): $|\underset{\sim}{Q}^{-1}|^2 = 2^{n-2}(n^2) + 2^{n-2}(1^2)$
$$= 2^{n-2}(n^2 + 1)$$

The <u>first structure coefficient</u> h[15] is now given by

$$|\underset{\sim}{Q}|^2 . |\underset{\sim}{Q}^{-1}|^2 . h^2 = \underset{\sim}{Q} \cdot \underset{\sim}{Q}^{-1} = 2^{n-2}(n) + 2^{n-2}(1)$$

giving

$$h^2 = \frac{2^{n-2}(n+1)}{2^{n-1}\sqrt{n^2+1}} \quad \text{and} \quad h = \frac{1}{\sqrt{2}} \left\{ \frac{n+1}{\sqrt{n^2+1}} \right\}^{1/2}$$

If we now regard the space of events E as the power set, P(X), of a <u>countable</u> set X then we must regard the structure vectors as corresponding to the case $n \to \infty$. This means that $\underset{\sim}{Q}^{-1}$ tends towards $\underset{\sim}{Q}$ but that the angle between these vectors approaches from above the limiting value of $\pi/4$, since $\lim_{n \to \infty} h = \frac{1}{\sqrt{2}}$.

2. <u>Difficulties in Interpreting probability as likelihood</u>

If we consider the probability distribution as a pattern π on the events E we are actually dealing with a graded pattern on the structure KE(X). When we write down the orthodox condition, viz.,

$$\sum_{E} \text{prob } (e \in E) = 1$$

where the summation is over all the events in E, we are in effect ignoring the grading of the pattern π. For, in fact,

$$\text{prob } (e \in E) = \pi^t(e) \quad \text{whenever } e \in E^{t+1}$$

But if we overlook this feature, for the moment, we can move on to the notion that prob (e) = 1 means that the event "e" is certain to be observed. The assumed connection with the intuitive idea of likelihood (the "chance" of an event being observed, on the occasion of a measurement or trial) is that the values of this probability set function constitute a legitimate and plausible measure of that idea.

In terms of the methodology of Q-Analysis this needs to be formulated in the following way.

(i) The notion of likelihood is an example of <u>traffic</u> on the structure KE(X) of the space-of-events. This is legitimate because it meets the requirements[15] that traffic must be defined in terms of the vertex set X (the set of elementary events).

(ii) The representation of this traffic-of-likelihood by a suitable pattern π on KE(X) can also be always arranged so that

$$0 \leq \pi(e) \leq 1 \quad \text{and} \quad \sum_{E} \pi(e) = 1, \text{ for all } e \in E$$

since these are trivial arithmetrical constraints.

(iii) But π must behave in such a way that it represents the traffic <u>both</u> <u>before</u> <u>and</u> <u>after</u> the observation of an event. So whatever the initial distribution (over KE(X)) we assume for π, the process of observation which results in an event e_o must be manifest by a change $\delta\pi$ via which the new pattern π' satisfies

$$\pi' = \pi + \delta\pi$$

and $\pi'(e_o) = 1$ whilst $\pi'(e) = 0$, $e \neq e_o$

(iv) The above discussion means that the traffic (sense of likelihood) experiences a whole set of t-forces characterised by $\delta\pi$, forces which evacuate some simplices (events) by repulsion and which attract towards other simplices (in fact, e_o). This would be the standard Newtonian interpretation on a rigid static backcloth S(N), viz., the complex KE(X) of events. But it is therefore important to stress that the *geometry* of KE(X) must be capable of carrying the t-forces represented by $\delta\pi$. In the usual case which we are now considering this geometry is adequate because it has a zero obstruction vector $\hat{\underline{Q}}$.

This suggests that we have already found, via consideration of the geometrical structure of KE(X), a criterion for interpreting a probability function as a pattern appropriate to the traffic of likelihood (expectation, chance).

<u>If KE(X) is the structure of the event-space then likelihood (as traffic) can be represented by a probability function in which the grading is ignored if and only if the obstruction vector $\hat{\underline{Q}}$ is zero</u>.

We see that this condition is certainly fulfilled whenever the set of events is of the form E = P(X). But this situation is actually uncommon. Whereas it is plausible to guess at a zero obstruction vector in traditional gambling activities (throwing dice, drawing cards) the study of data in many other areas (such as social communities, urban studies, psychology, medical diagnosis, design, etc.) points strongly to the breakdown of this criterion. A zero obstruction vector for a complex KE(X) seems to be the exception rather than the rule. It is of course confused by a confusion of hierarchical levels of data[5] because if we increase N sufficiently (in considering $KE(X)$ as the backcloth $S(N)$) then we can usually simplify the structure so much as to make it dimensionally trivial and so arrange for $\hat{\underline{Q}}$ to approach zero. But this process inevitably trivialises (in the colloquial sense) the data so that its content evaporates. Making N large enough is like ending up, in the extreme case, with $S(N+k)$, where the data set at the $(N+k)$-level contains one element and this will be a cover of all lower level data sets. Seen from this $(N+k)$-level the data at any lower level will be one simplex and $\hat{\underline{Q}} = \underline{0}$, but the price to be paid for this will be that there will be only one event - the "whole" data set.

3. <u>What is the role of the conjugate complex $KX(E)$?</u>

The structure of $KX(E)$ is what affects changes in traffic on $KX(E)$, traffic which is defined by the events E and which is manifest on the set X (the elementary events). If this traffic is represented by a pattern

$$\theta : KX(E) \to R$$

then θ could be (e.g.) a sort of inverse probability distribution. The values of θ (regarding it, for the moment, as ungraded) would be probabilities of vertices $\{X_i\}$ which are found in events $\{E_j\}$. This situation is well known in applications of probability theory; it is particularly manifest in the appeal to Bayes' Theorem, and illustrated by some such pattern as the following.

There are m people in a set P and n social groups in a set G; given a probability function π = prob (person P_i is in group G_j), how can we find a probability function θ = prob (group G_r contains a person P_s)?

Here we are appealing to the events in two conjugate structures. If π is a pattern on $KY(X)$ then θ is a pattern on $KX(Y)$. A theorem such as Bayes' Theorem is a means of relating the (ungraded) patterns π and Θ , in special circumstances, These circumstances involve a partitioning of the set (the group G), which is equivalent to identifying G as the formal set X of elementary events. Thus the "problem" quoted corresponds to identifying $G \equiv X$ and $P \equiv E$.

But we have seen that even when KE(X) possesses zero obstruction it does not follow that KX(E) possesses zero obstruction. This means that even in situations in which $\underline{Q} = \underline{O}$, and ungraded probability is the legitimate pattern for likelihood traffic, it does not follow that $\underline{Q}^{-1} = \underline{O}$. So even if the pattern π represents likelihood for KE(X) the pattern Θ cannot represent likelihood for KX(F).

This leaves us with a new research problem, viz., how are wo to find a new pattern on a "structure of events KE(X)" which can more faithfully represent the traffic of likelihood and which, in the special case in which $\underline{Q} = O$, coincides with the standard probabilistic theory ?

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Fred CHAMP, positional-chess analyst

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In this paper we give the results of further research into the computer simulation of positional play in chess. A well-defined hierarchical approach is used to produce a vector mapping for the positional evaluation. It is illustrated by an analysis of a grandmaster game, Karpov *vs.* Spassky.

Introduction

In earlier papers (Atkin, 1972; Atkin & Witten, 1975) the idea that the game of chess corresponds to certain well-defined structures in multidimensional space has been developed. Precisely, if S, W, B denote the sets {Squares of the Board}, {White men (pawns and pieces)}, {Black men (pawns and pieces)} respectively, then the rules of the game induce a (typical) relation $\Gamma = \Gamma_W \subset W \times S$. Such a relation defines two simplicial complexes KW(S) and its conjugate KS(W), where dim KW(S) = 26, and which can be represented by suitably connected convex polyhedra in E^{53}. The move-properties of the individual pieces {P, N, B, R, Q, K} are expressed in Γ by saying that $(W_i, S_j) \in \Gamma_W$ if and only if W_i "attacks" S_j. Slight variations in Γ can be introduced to allow for specific tactical or positional approaches, for example, we can allow a piece W_i to "see through" other pieces or pawns (to varying extents) when deciding on the relation Γ.

When the game is in mode (I, J), White having made I moves and Black having made J moves, we characterize the state of affairs by the following mappings:

(i) on KW(S), stval : $S \to Z$ (Z = non-negative integers)
this to give the "strength value" of each square $S_i \in S$;
(ii) on KW(S), pval : $W \to Z$
this to give the "piece value" of each W_i x W;
(iii) on KS(W), cval : $W \to Z$
this to give the "control value" of each $W_i \in W$;
(iv) on KS(W), sval : $S \to Z$
this to give the "square value" of each $S_i \in S$.

Each such mapping is a graded pattern on the appropriate structure in E^{53}, viz.,

stval is a 0-pattern, π^0, on the vertices of KW(S),
pval(W_i) is a t-pattern, π^t, where t is the dimension of W_i in KW(S),
cval is a 0-pattern, π^0, on the vertices of KS(W),
and sval(S_i) is a t-pattern π^t, where t is the dimension of S_i in KS(W).

In addition we allow for the "inverse" relation between pairs of these by requiring

$$\begin{aligned} &\text{pval}(W)\,.\,\text{cval}(W) = h \text{ (a fixed constant)} \quad \text{(A)}\\ \text{and}\quad &\text{sval}(S)\,.\,\text{stval}(S) = k \text{ (a fixed constant)} \quad \text{(B)}\end{aligned}$$

remembering that values are rounded off to give integers.

Since, in KW(S), each W is a t-simplex (for some value of t) and is written

$$W = \langle S_{\alpha_0} S_{\alpha_1} \dots S_{\alpha_t} \rangle$$

we take

$$\text{pval}(W) = \sum_{S_i \in W} \text{stval}\,(S_i) \qquad \text{(C)}$$

and similarly,

$$\text{sval}(S) = \sum_{W_i \in S} \text{cval}\,(W_i). \qquad \text{(D)}$$

The mutual relation between these four mappings is represented by the diagram:

$$\begin{tikzcd} \text{stval} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow, "C"'] \arrow[dr, phantom, "B\quad A"] & \text{sval} \arrow[d, leftrightarrow, "D"] \\ \text{pval} \arrow[r, leftrightarrow] & \text{cval} \end{tikzcd}$$

The simplest way to enter this diagram, and the one we have used to-date in this research, is via the corner at "pval" by the classical values (notice we use the American "N" to represent Knight!):

$$\text{pval}\,\{P, B, N, R, Q, K\} = \{1, 3, 3, 5, 9, 100\}.$$

Notation: Fred CHAMP is the name of our computer program because he is a *CH*ess-*A*dapted *M*ulti-dimensional *P*layer (known as Fred when he is wearing his cloth cap).

A hierarchical approach to the structure

We pursue the idea (Atkin, 1976) that the concepts associated with *positional judgement* in chess can be contained in, but require, a hierarchical view of the game—and therefore of the structures KW(S) etc. and mappings $\{\pi_N\}$ already introduced. We denote this hierarchy (which uses the notion of *cover sets*, rather than partitioning) by H and refer to its possible levels via the following scheme.

H : N-level	squares S_i; pieces W_i, B_j
(N+1)-level	sets of squares; sets of pieces
(N+2)-level	sets of sets of squares; sets of sets of pieces
.	
.	
.	

In the first instance we can get an intuitive idea of the significance of H by associating the N-level with the level of *tactical* decisions, the (N+1)-level with the level of (first order) *positional* judgements, and the (N+2)-level with the level of (second order) positional judgements. This is because tactics is concerned with the level of precise moves, of deciding to place a piece on a single square, whilst positional play is concerned with the control of a file (a set of squares) or of the centre, or of the whole King-side (an (N+2)-level matter), and so forth.

Precisely, we use the following elements (which can always be altered if required) at the various levels.

N-level: the set of squares S; the set of White/Black men W/B.
(N+1)-level: a set $S' \subset P(S)$; a set $W' \subset P(W)$ (similarly for B) where the elements are as follows:

Elements of S'

Name of element	Symbol	Subset of S
centre	S_1'	{d4, d5, e4, e5}
QR-file	S_2'	{a1, a2, a3, a4, a5, a7, a7, a8}
QN-file	S_3'	{b1, b2, b3, b4, b5, b6, b7, b8}
QB-file	S_4'	{c1, c2, c3, c4, c5, c6, c7, c8}
Q-file	S_5'	{d1, d2, d3, d4, d5, d6, d7, d8}
K-file	S_6'	{e1, e2, e3, e4, e5, e6, e7, e8}
KB-file	S_7'	{f1, f2, f3, f4, f5, f6, f7, f8}
KN-file	S_8'	{g1, g2, g3, g4, g5, g6, g7, g8}
KR-file	S_9'	{h1, h2, h3, h4, h5, h6, h7, h8}
diagonal W1	S_{10}'	{b1, a2}
diagonal B1	S_{11}'	{c1, b2, a3}
diagonal W2	S_{12}'	{d1, c2, b3, a4}
diagonal B2	S_{13}'	{e1, d2, c3, b4, a5}
diagonal W3	S_{14}'	{f1, e2, d3, c4, b5, a6}
diagonal B3	S_{15}'	{g1, f2, e3, d4, c5, b6, a7}
diagonal W4	S_{16}'	{h1, g2, f3, e4, d5, c6, b7, a8}
diagonal B4	S_{17}'	{h2, g3, f4, e5, d6, c7, b8}
diagonal W5	S_{18}'	{h3, g4, f5, e6, d7, c8}
diagonal B5	S_{19}'	{h4, g5, f6, e7, d8}
diagonal W6	S_{20}'	{h5, g6, f7, e8}
diagonal B6	S_{21}'	{h6, g7, f8}
diagonal W7	S_{22}'	{h7, g8}
diagonal B7	S_{23}'	{g1, h2}
diagonal W8	S_{24}'	{f1, g2, h3}
diagonal B8	S_{25}'	{e1, f2, g3, h4}
diagonal W9	S_{26}'	{d1, e2, f3, g4, h5}
diagonal B9	S_{27}'	{c1, d2, e3, f4, g5, h6}
diagonal W10	S_{28}'	{b1, c2, d3, e4, f5, g6, h7}
diagonal B10	S_{29}'	{a1, b2, c3, d4, e5, f6, g7, h8}
diagonal W11	S_{30}'	{a2, b3, c4, d5, e6, f7, g8}
diagonal B11	S_{31}'	{a3, b4, c5, d6, e7, f8}
diagonal W12	S_{32}'	{a4, b5, c6, d7, e8}
diagonal B12	S_{33}'	{a5, b6, c7, d8}
diagonal W13	S_{34}'	{a6, b7, c8}
diagonal B13	S_{35}'	{a7, b8}
enemy K-simplex	S_{36}'	block of 5×5 squares around enemy K with inner 3×3 block given triple weighting
own K-simplex	S_{37}'	block of 5×5 squares around own K with inner 3×3 block given triple weighting

Name of element	Symbol	Subset of S
weak enemy P-control	S_{28}'	squares of ranks, 3, 4, 5, 6 which cannot be defended by an enemy pawn (unless that Pawn should change files by a capture)
weak own P-control	S_{29}'	squares of ranks, 3, 4, 5, 6 which cannot be defended by an own pawn (unless that pawn should change files by a capture)

Note: the triple weighting in S_{30}', S_{31}' is incorporated into the mapping "stval", which otherwise takes a value unity on each square.

Elements of W′ (B′ defined similarly)

Name of element	Symbol	Subset of W
King	W_1'	{WK}
Queen	W_2'	{WQ}
Rooks	W_3'	{WQR, WKR, WK}
Black-square Bishops	W_4'	{WQB}
White-square Bishops	W_5'	{WKB}
Knights	W_6'	{WQN, WKN}
Q-side pawns	W_7'	{pawns on Q-side}
K-side pawns	W_8'	{pawns on K-side}
centre pawns	W_9'	{pawns on d4, d5, e4, e5}
central pawns	W_{10}'	{pawns on QB-, Q-, K-, KB-files}

(N+2)–level: a set $S'' \subset P^q(S)$; a set $W'' \subset P^q(W)$, as follows:

Element of S″

Name of element	Symbol	Composition in terms of sets of S′
K-files	S_1''	(S_7', S_8', S_9')
B-squares	S_2''	$(S_{11}', S_{13}', S_{15}', S_{17}', S_{19}', S_{21}', S_{23}', S_{25}', S_{27}', S_{29}', S_{31}', S_{33}', S_{35}')$
Q-files	S_3''	(S_2', S_3', S_4')
w-squares	S_4''	$(S_{10}', S_{12}', S_{14}', S_{16}', S_{18}', S_{20}', S_{22}', S_{24}', S_{26}', S_{28}', S_{30}', S_{32}', S_{34}')$
centre	S_5''	(S_1', S_5', S_6')
weak enemy P-control	S_6''	(S_{28}')
enemy K-simplex	S_7''	(S_{30}')
own K-simplex	S_8''	(S_{31}')
weak own P-control	S_9''	(S_{29}')

Elements of W″ (B″ defined similarly)

Name of element	Symbol	Composition in terms of sets of W′
King and Knights	W_1''	(W_1', W_6')
heavy pieces	W_2''	(W_2', W_3', W_4', W_5')
pawn power	W_3''	$(W_7', W_8', W_9', W_{10}')$

Relations Γ, Λ, Σ

As we move from the N-level, at which Γ is defined, to the (N+1)-level we naturally induce a relation $\Lambda \subset W' \times S'$, defined as follows.

If $a \in W'$ and $b \in S'$ then $a \Lambda b$ iff there exists some $W_i \in a \cap W$ and some $S_j \in b \cap S$ such that $W_i \Gamma S_j$.

In the same way we naturally induce a relation $\Sigma \subset W'' \times S''$, defined as follows.

If $A \in W''$ and $B \in S''$ then $A \Sigma B$ iff there exists some $a \in A \cap W'$ and some $b \in B \cap S''$ such that $a \Lambda b$.

It now follows that we have the following structures, with their chess connotations:

Level	Structures	Significance	Relations
N	KW(S)	White's tactical view of Board	Γ_w
	KS(W)	Board's tactical view of White	Γ_w^{-1}
(N+1)	KW'(S')	White's 1st order positional view of Board	Λ_w
	KS'(W')	Board's 1st order positional view of White	Λ_w^{-1}
(N+2)	KW''(S'')	White's 2nd order positional view of Board	Σ_w
	KS''(W'')	Board's 2nd order positional view of White	Σ_w^{-1}

Hierarchical patterns

The set of patterns, denoted $\{\pi_N\}$, at the N-level give rise to induced patterns at the higher levels. We denote these by $\{\pi_{N+1}\}$ and $\{\pi_{N+2}\}$. This paper is concerned with positional judgement derived from a study of the "square-value" pattern, at different hierarchical levels.

N : sval $\in \{\pi_N\}$, on the structure defined by Γ, KW(S)
N+1 : sval $\in \{\pi_{N+1}\}$, on the structure defined by Λ, KS'(W')
N+2 : sval $\in \{\pi_{N+2}\}$, on the structure defined by Σ, KS''(W'').

The computer algorithms for the evaluation of these patterns are listed below.

At the N-level

```
(i) pval (player):=
      if player = K then 100
      else if player = Q then 9
      else if player = R then 55
      else if player = B then 3
      else if player = N then 3
      else 1;
(i)  cval (player) := 200/pval(player);
(iii) sval(square, side):= resultof
      begin
        result: = 0;
        for man ∈ side do
          if xrayattack(man,square) then result : = result+cval(man);
      end
```

where xrayattack (man, square) is *true* only if the man could legally capture an opposing man on that square, were one there, if all other pieces (but not pawns) were removed from the board.

At the $(N+1)$*-level*

```
super(i, side) : = resultof
  begin
    result : = 0;
    for S_j' ∈ S_i' do
      for square ∈ S_j' do result : = result+sval(square, side);
  end
```

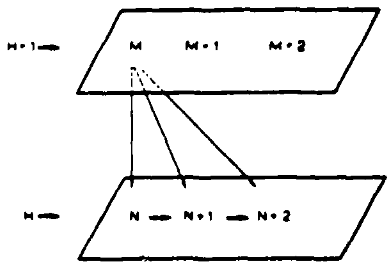
At the $(N+2)$-level

```
grand(i) : =
  if i = 1 then
100*(super(7,ownside)-super(8,enemyside)/(super(7,ownside)+super(8,enemyside))
  else if i = 2 then
100*(super(8,ownside)-super(7,enemyside))/(super(8,ownside)+super(7,enemyside))
  else if i = 3 then
100*(super(6,ownside)-super(9,enemyside))/(super(6,ownside)+super(9,enemyside)+1)
  else if i = 4 then
100*(super(9,ownside)-super(6,enemyside))/(super(9,ownside)+super(6,enemyside)+1)
  else if i = 5 then
100*(super(1,ownside)-super(1,enemyside))/(super(1,ownside)+super(1,enemyside))
  else if i = 6 then
100*(super(3,ownside)-super(3,enemyside))/(super(3,ownside)+super(3,enemyside))
  else if i = 7 then
100*(super(2,ownside)-super(2,enemyside))/(super(2,ownside)+super(2,enemyside))
  else if i = 8 then
100*(super(4,ownside)-super(4,enemyside))/(super(4,ownside)+super(4,enemyside))
  else if i = 9 then
100*(super(5,ownside)-super(5,enemyside))/(super(5,ownside)+super(5,enemyside))
```

[Note: this effectively forms the percentage ratio $(W-B)/(W+B)$ or $(W-B)/(W+B+1)$, where W and B are the sval($\sum$) values for White and Black respectively.]

A meta-hierarchy

In order to *play* a game of chess, or even to *analyse* a game already played, it is necessary to be able to *change the hierarchy* H. This is so because H is defined by Γ (and Λ, $\sum$) and Γ is itself determined by the mode (I, J), by the "state of the game". A move μ, by either White or Black, immediately alters $(I, J) \xrightarrow{\mu} (I' J')$ and so it alters H, which is therefore determined by the move μ, and should be written $H(\mu)$. In order to cope with this, Fred CHAMP (like all human players) must be able to exist in a condition equivalent to being "aware of all possible hierarchies $H(\mu)$, as μ varies". This suggests that we need another *level of hierarchy*, which we shall call $(H-1)$, from which $H(\mu)$ can be studied. If we allow possible levels of M, $(M+1)$, . . . in $(H-1)$ we get the following schema:



At the level (M, H+1) the hierarchy H (the state of the game) can be understood. The level (M+1, H+1) will contain sets of things found in (M, H+1)—that is to say, it will contain a *set* of hierarchies H(μ), or a set of moves or *a strategy*. The level (M+2, H+1) will contain a set of strategies, etc.

Only by moving into the (H+1) hierarchy can we hope to build Fred CHAMP into a Chess Master. But we can see that the level (M, H+1) is sufficient to make him into a Chess Analyst, since at this level we have the information needed to understand the hierarchy H(μ)—for any particular μ.

Positional analysis of a game is therefore associated with the (M, H+1)-level, briefly denoted as M-level. At this level Fred CHAMP examines the (N+1)-level pattern π_{N+1}, given by *super* (i, side) and the (N+2)-level pattern π_{N+2}, given by *grand*(i).

These patterns give vectors whose components have the following names:

ENY-K	(Enemy K-field)	K-SIDE	(Kind-Side)
OWN-K	(Own K-field)	Q-SIDE	(Queen-Side)
ENY-WK	(Enemy Weak squares)	BL-SQ	(Black Squares)
OWN-WK	(Own Weak squares)	WH-SQ	(White Squares)
		CENTRE	(Centre)

Illustrative example

In this study Fred CHAMP gives a positional analysis of the quiescent conditions in the game Karpov *vs.* Spassky (XIth Match Game, Moscow, 1974). The score of the game is as follows:

(1) d2-d4	: N-f6	See note:	A
(2) c2-c4	: e7-e6		B
(3) N-f3	: d7-d5		C
(4) N-c3	: B-e7;		D
(5) B-g5	: h7-h6		E
(6) B-h4	: 0-0		F
(7) e2-e3	: b7-b6		G
(8) B-e2	: B-b7		H
(9) B*f6	: B*f6		
(10) c4*d5	: e6*d5		
(11) 0-0	: Q-d6		I
(12) R-c1	: d7-a6		J
(13) a2-a3	: N-d7		K

(14) b2-b4	: b6-b5	
(15) N-e1	: c7-c6	
(16) N-d3	: N-b6	L
(17) a3-a4	: B-d8	M
(18) N-c5	: B-c8	
(19) a4-a5	: B-c7	N
(20) g2-g3	: N-c4	
(21) e3-e4	: B-h3	P
(22) R-e1	: d5*e4	
(23) N(c3)*e4	: Q-g6	
(24) B-h5	: Q-h7	Q
(25) Q-f3	: f7-f5	R
(26) N-c3	: g7-g6	
(27) Q*c6	: g6*h5	S
(28) N-d5	: f5-f4	
(29) R-e7	: Q-f5	T
(30) R*c7	: R(a8)-e8	
(31) Q*h6	: R-f7	
(32) R*f7	: K*f7	
(33) Q*f4	: R-e2	
(34) Q-c7+	: K-f8	U
(35) N-f4	: Resigns	

NOTES BY F. CHAMP

Each note is our interpretation of the 9-vectors π_{N+1} and π_{N+2}, where the vector components are ordered in the sequence,

π = {ENY-K, OWN-K, ENY-WK, OWN-WK, K-SIDE, Q-SIDE, BL-SQ, WH-SQ, CENTRE}

(Each note refers to the position after Black's move.)

(A) *Values of π_{N+1} for White and Black*

Wh	: 0	3918	0	0	1706	1526	4838	4314	1628
Bl	: 0	4428	0	0	1640	1526	3874	5322	1626

Neither side is exerting pressure on the other's K-field; there are no weak squares. White has a slight advantage on the K-side; the Q-side is even; White gains on the black-squares but this is balanced by Black on the white-squares; the centre is evenly contested.

Values of π_{N+2} (+ve for White, −ve for Black)

−99	100	0	0	2	0	11	−9	0

The vector shows what *percentage difference* now exists between the sides consequent on the previous two moves. In this case it shows the effect from the initial (zero-move) position. Thus White has gained an 11% advantage on the black-squares, compared with a gain of 9% for Black on the white-squares. Similarly we notice +2% (for White on the K-side. The −99% is a rounding-off "value" for −100%.

(B) π_{N+1}

Wh	: 0	3718	0	0	1706	1570	4838	4402	1828
Bl	: 0	4116	0	0	1706	1724	3734	6122	1892

Each K-field has been slightly weakened; Black has gained equality on the K-Side, increased his control on the white-squares, lost a little in control of the black-squares.

π_{N+2}

−99	100	0	0	0	−4	13	−15	−1

These are the percentage differences expressing the above remarks.

(C) π_{N+1}

Wh	: 0	3916	0	0	1838	1570	5894	4006	2158
Bl	: 0	3804	0	0	1706	1724	3778	6298	2224

The positional play now expresses the fact that White is increasing his control on the K-Side and the black-squares (so the intersection of these two sets provides him with tactical targets), whilst Black is trying to counter on the Q-Side and on the white-squares. The Centre control is almost evenly balanced.

π_{N+2}

−99	100	0	0	4	−4	22	−21	−1

White's percentage increment is now greater on the black-squares than is Black's on the white-squares.

(D) π_{N+1}

Wh	: 0	4048	0	0	1838	1702	5498	5062	2488
Bl	: 0	4002	0	0	1904	1724	4174	6298	2224

π_{N+2}

−99	100	0	0	−1	0	14	−10	6

Now Black is contesting control on the K-Side/white squares whilst White increases his score on the black-squares and centre.

(E) π_{N+1}

Wh	: 462	4114	0	0	1904	1702	5894	5062	2620
Bl	: 0	3802	0	0	1944	1724	4654	5898	2224

π_{N+2}

−77	100	0	0	0	0	12	17	8

White is using his positional advantage on the K-Side/black squares (by the move 5. B-g5 to weaken the Enemy-K-field, inducing 5 . . .h7-h6). White gains further advantage in the centre and on the black-squares whilst neither player acquires any percentage increment on their previous zones (K-Side and Q-Side). Neither side has any weak squares at this stage.

(F) π_{N+1}

Wh :	132	4048	0	0	1904	1636	5498	5062	2488
Bl :	0	2686	0	0	1910	1724	4532	5898	2218
π_{N+2}	−90	100	0	0	0	−2	10	−7	6

Black is now *fighting to recover his loss on his* OWN-K field by reducing White's score on it—but at the price of a severe loss of absolute score there (from 3802 to 2686). White continues to show a significant relative advantage (10%) on the black-squares. He has now traded his K-Side advantage for an ENY-K advantage.

(G) π_{N+1}

Wh :	132	3736	0	88	1970	1702	6298	4658	2754
Bl :	0	2686	0	198	1910	1790	5332	5230	2218
π_{N+2}	−90	100	−99	98	2	−2	8	−5	11

The first time in the game we see that each side has introduced weak-squares (which cannot be defended by a pawn), but that the opponent has not so far been able to score on them. White has increased his control of the centre (from 8% to 11%) whilst Black has not been able to reduce White's score on his (Black's) K-field.

(H) π_{N+1}

Wh :	132	3934	0	88	2168	1702	6298	5054	2754
Bl :	0	2620	0	264	1910	1922	5332	5296	2218
π_{N+2}	−89	100	−99	98	6	−5	8	−1	11

Black's position is deteriorating on his own K-field (2686 to 2620, without reducing White's score of 132 on it), on the K-Side, on the white-squares (5% to 1%). The only compensation *is a slight increase in control* on the Q-Side (from 2% to 5%), but the tactical danger is all on the K-Side. On the other hand Black has increased his own score on his own weak-squares (from 198 to 264), *showing that he is well aware of the danger.*

(I) π_{N+1}

Wh :	0	2886	66	88	1910	1634	5506	4476	2216
Bl :	88	2070	0	286	1402	1768	5310	3418	1730
π_{N+2}	−99	94	−62	98	15	−3	2	13	12

After the exchanges initiated by White (9. B*f6, etc.) we see that Black has gained some attack on White's K-field (100% dropped to 94%), but lost positional ground on the white-squares (where he *previously* had an advantage) and on the Q-Side *(from 5% to* 3% advantage). White has lost his score on Black's K-field (132 to 0) but has increased his control on the K-Side plus black-squares plus centre, and now has a 13% advantage on the white-squares. Since White also shows a score of 66 on Black's weak-squares (against Black's score of 286 there) the results of the exchanges are positionally favourable for White.

(J) π_{N+1}

Wh :	0	2886	106	88	1910	1834	5706	4636	2216
Bl :	88	2070	0	326	1402	1808	4910	3898	1730
π_{N+2}	−99	94	−50	98	15	1	7	9	12

Now White has an advantage on all fronts, except that Black has a score of 88 on White's K-field (action of the Queen)—but White's control there is 94 % (instead of 100 %). White has increased his score on Black's weak-squares (66 to 106) and Black has countered by defending these (286 to 326). The weak-squares in question are c6 (attacked indirectly by White's Rook and defended by Bishop, Knight and Queen).

(K) π_{N+1}

Wh :	0	2886	106	110	1910	1834	6106	4234	2216
Bl :	88	2334	0	194	1534	1874	5702	3502	1796
π_{N+2}	−99	94	−29	99	11	0	3	9	10

Black has reduced his score on c6 (weak-square) by N-d7, although this increased his score on the black-squares (White's drop from 7% to 3%) and the centre. White's overall positional advantage is maintained.

(L) π_{N+1}

Wh :	0	2688	746	344	1778	1992	6026	4104	2018
Bl :	88	2070	732	216	1402	1918	3266	5696	1930
π_{N+2}	−99	94	55	−36	12	2	30	−15	2

White's control on the black-squares is a large 30% advantage (due to Pawn and Knight moves by both sides). White now has 55% advantage on Black's weak-squares (the set b6, c6, c7, d6) whilst Black enjoys a 36% advantage on White's weak-squares (the set a3, a4, b3, c3, c4). The positional struggle is now about weak-black-squares (White's pressure) and weak-white-squares (Black's counter pressure).

(M) π_{N+1}

Wh :	0	2688	746	410	1778	1992	5626	4504	2018
Bl :	88	1938	754	348	1402	2116	3266	5696	1600
π_{N+2}	−99	94	36	−29	12	−2	27	−11	12

The struggle continues around the weak-squares. Notice that if Black captures the Pawn on a4 he loses score on White's weak-square c4. The Bishop move, B-d8, has reduced Black's score on his own K-field. Both Black and White have reduced their advantage on the other's weak-squares.

(N) π_{N+1}

Wh :	66	2424	946	674	1646	1992	4970	5160	2150
Bl :	418	1872	1154	348	1600	1984	3530	6158	1930

π_{N+2}

	−92	71	46	−26	1	0	17	−8	5

Apart from the threatened tactical mate (which Fred CHAMP ignores) we see that White is winning the positional struggle for control of the weak-squares, there being a continuing trade-off between K-Side, Q-Side, etc. scores and that control.

(O) π_{N+1}

Wh :	66	2024	946	784	1646	1992	4970	5160	2150
Bl :	550	1872	1088	480	1578	1918	4410	5432	2062
π_{N+2}	−92	57	32	−16	2	2	6	−2	2

Black's attack on White's K-field has resulted in an increase in white-weak-square scores (squares f3 and h3). Black's attack has reduced White's control on his own K-field (from 71% to 57%), but Black's advantage is still restricted to ENY-WK and WH-SQ.

(Q) π_{N+1}

Wh :	516	2130	1078	586	1710	1728	6106	3440	1806
Bl :	528	1938	1220	436	1578	1652	3992	4720	1552
π_{N+2}	−57	60	42	−35	4	2	21	−15	8

White has now increased his control on ENY-K, OWN-K, ENY-WK, K-Side, BL-SQ, and CENTRE. Black has improved his position on ENY-K, ENY-WK, and WH-SQ.

(R) π_{N+1}

Wh :	582	2130	1512	586	1658	1684	6128	3616	1916
Bl :	528	1512	1198	590	1658	1542	4072	4228	1554
π_{N+2}	−43	60	43	−34	3	4	20	−7	10

Black's tactical counter-attack has not resulted in a decisive shift in positional control. Now White is steadily increasing or maintaining his advantage again. Black is slipping in control of ENY-K, ENY-WK and WH-SQ.

(S) π_{N+1}

Wh :	296	1690	2044	608	1182	2072	5240	3982	1982
Bl :	528	1200	798	846	1140	1408	3404	3960	1198
π_{N+2}	−59	52	41	−13	−10	19	21	0	25

White has sacrificed a piece to produce a large increase in absolute score on ENY-WK (now 2044), on Q-Side (now 2072). Black is left with an advantage on the K-Side (for the first time in the game)—due chiefly to the advanced Pawns there. White's attack on ENY-WK is now immediately on the sixth rank and (via the Rook) threatens the K-field with a possible R-e7.

(T) π_{N+1}

Wh :	748	1370	2304	542	1314	2072	6336	2966	1718
Bl :	884	1200	886	1128	1520	1518	4174	3952	1284
π_{N+1}									
	−22	22	34	−24	−6	15	21	−13	14

The struggle is now very sharp, each side trying to take tactical advantage of his positional plus—and ignoring the rest. But White's absolute score on the ENY-WK is 2304 (against 2044) and on BL-SQ it is 6336 (against 5240). The largest improvement for White occurs in ENY-K (748 against 296) and a consequent drop for Black from 59% to 22%. The rest is a tactical exploitation for White.

(U) π_{N+1}

Wh :	682	1118	2362	476	1172	1952	6120	2018	1274
Bl :	772	750	842	908	774	1160	1932	3640	1064
π_{N+1}									
	−4	18	44	−27	20	25	52	−28	9

White's tactical exploitation has resulted in a material advantage of three Pawns as well as a large positional advantage across most of the vector; Black is left with some control over the Q-Side and WH-SQ (a theme throughout the game) whilst White's massive 44% advantage on ENY-WK, together with the tactical mobility of his Knights is too much for Black—who has inadequate defensive resources for his K-field after 35. N-f4 (due to the positions of Knight and Bishop).

Can Fred CHAMP become a player?

We fully anticipate that the answer to this is Yes. As indicated above this will require a Strategy at the level (M+1, H+1), and preliminary results so far tested are encouraging. In addition it might be worth exploring a set of such strategies, a selection from which would be made by appealing to a super-strategy located in (M+2, H+1). In any event we must also require a tactical move selector at level (N, H), based on the positional assessment and selected strategy. The chief requirement is probably a search for captures, since the quiescent condition is always submitted to the *positional* analysis.

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VOLUME 3

DECISION: FOUNDATION AND PRACTICE

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DECISION: FOUNDATION AND PRACTICE

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Decision: (L. decidere) A cutting away, a separation, the making of a distinction.

"Human knowledge and human power meet in one; for where the cause is not known the effect cannot be produced. Nature to be commanded must be obeyed; and that which in contemplation is as the cause is in operation as the rule" (Bacon, Novum Organum Book 1, III)

Contents

1 PREAMBLE

In these notes I have attempted to bring together in uneasy synthesis several strands of my own studies. They are confluent but it would be premature to present them as an integral whole. They are best regarded *scaffolding for an* architecture of decision - taken with other contributions and the discussion of this conference, they may yield further glimpses of the structures for which we are all striving.

The key points made here are:

1.1 Foundational

(1.1.1) That the problems of a science of decision are truly foundational and impinge strongly on the foundational problems of a range of other sciences.

(1.1.2) That different approaches and answers to the problems are not only possible but also vital to progress. There is no unitary foundation for a theory of decision and the differing pre-suppositions of disparate approaches each generate their own dialectical tensions that lead to the synthesis of key theories.

(1.1.3) Attempts to reduce the variety of approaches to a unified theory are essentially doomed to failure, and yet are not to be discouraged since they themselves reflect important preceonceptions at a meta-theoretic level that are a further important source of conflict and progress.

1.2 Practical

(1.2.1) The significance of the advent of low-cost digital computers providing through interactive graphic interfaces the capabilities of "man-machine symbiosis" and the "amplification of intelligence" is of fundamental importance to the development of decision theories and systems.

(1.2.2) Popper's "3 worlds" model is very useful in determining the essential role of the computer. It provides a new dynamic for world 3 (the world of statements in themselves, of books and libraries) just as did steam, internal combustion and jet engines for world 1 (the world of physical objects).

(1.2.3) Man-machine symbiosis will only come to fruition when all the factors are right, technological, psychological and conceptual. The next few years will see the technology required become available in primitive, but usable, form. The concepts necessary are currently only in nascent form and their development cannot take place except through actual experimentation and experience.

1.3 Technical

(1.3.1) Klir's "epistemological hierarchy", of "source", "data", "generative", "structure" and "meta", systems provides a useful framework in which to analyse the onotological pre-suppositions necessary to decision systems.

(1.3.2) The epistemological problem can thereafter be 'solved' in a very general sense in terms of an "admissible space" of models derived from orderings of complexity on models and approximation on the relation between models and data.

(1.3.3) Analogy between systems may be analysed formally in a category-theoretic framework. This analysis may be used to allow new systems to be explored by analogy with known systems.

(1.3.4) The Wittgensteinian argument that if I know something then I know all its consequences is false psychologically. The computer, however, as a "consequence-generator" is a tool to provide this facility - to make us aware of the consequences of our pre-suppositions, axioms and hypotheses. In this sense computer systems can provide 'completion' of the human thought process.

(1.3.5) The concept of a 'database' may be generalized to include a very wide class of systems for the acquisition, storage and manipulation of information. The role of a database in man-machine symbiosis is to provide a "possible world" simulation from which to derive consequences of actions.

2 FOUNDATIONS

The study of decision is central to systems technology in that it requires foundational access to all the key system sciences: sociology, psychology, linguistics, philosophy, computation, statistics, logic,..... Not only is each of these individual sciences stretched to its limits through the requirements of the study of decision, but also the inter-disciplinary combinations required generate what are effectively new sciences such as computational logic, philosophical linguistics, artificial intelligence, and so on.

The key significance of decision is that it involves action and hence also interaction with the world, with an environment. This is why I term the study of decision a "systems technology" - the circumstantial purification possible (and necessary) in a science becomes clearly artificial in the context of decision. 'Pure decision' is a myth in practice and a syntactic contrivance in theory - we decide only in order to act - we do not know what it is to make a decision except as we may see it lead to action, and we evaluate that decision in terms of its effects on action, and the effects of action on the world.

This key role played by action and its evaluation means that concepts of value are intrinsic to decision and its study. If we are not concerned about the outcomes of action then we have no basis for decision. The minimum structure with which we can express our concern is at least some relation of preference, forcing the display of some degree of committment. Neutrality is inconsistent with decision.

Having commenced with a polemic that emphasizes the central role of decision as a systems technology, I will now switch to the converse view that emphasizes the foundational role of decision at the heart of all the sciences. We cannot have knowledge acquisition without decision. There is the decision to make a distinction and define a domain about which to acquire information. There is the decision to use certain observational methods and terminology. There is the decision to use certain bases of explanation. There is the decision that a particular explanation best fits the observations. There is the decision that further data gathering is necessary to secure agreement as to the explanation.

And so on. Decision is at the heart of every science and decisions have to be made at every stage in the generation of the knowledge that is that science. Yet these decisions are outside the sciences that they generate. They form a meta-science with its own structure of decisions. And so on ad infinitum. We have iteration, recursion, and also paradox in that what appeared to be the foundations of decision also have decision at their foundations.

If this line of argument is accepted what conclusions may we draw ? Certainly that no single-aspect, or single-level approach to the study of decision can possibly capture more than a small part of its overall complexity and dynamics. Also that a unified model of decision is impossible in a strict sense since we have essential circularity - to study decision we have to break into a dynamic process of which the study itself is part - the only possible model of decision is decision itself.

2.1 Three Positions

The argument becomes most pointed if we summarize it in terms of the "rule of natural numbers" - that zero, one and infinity are the only numbers that can arise in nature. All three possibilities have been proposed in terms of models of decision:

[ZERO] There is no foundation for decision.

This has been the sceptical position throughout the ages, leading in its most extreme forms to total nihilism. The first clear statement of nihilist scepticism is attributed to Gorgias, but probably pre-dating him and many times re-discovered. Gorgias held:

(1) Nothing exists;

(2) Even if something did exist it could not be known;

(3) Even if were known this knowledge could not be communicated.

Hume is the best-known proponent of scepticism in recent times but new defences of the position are being published to the present day (Unger 1975). The clearest, and most convincing, statement of the sceptical position is still that of the Roman philosopher Sextus Empiricus in his "Outlines of Pyrrhonism" (trans. Bury 1933), who discusses a form of non-nihilist scepticism originating with Pyrrho of Ellis but substantially developed by many subsequent philosophers into a methodology of thought and decision based on the "suspension of judgement". Pyrrhonists based their suspension on some 10 rules which seem fresh and cogent today, e.g. the necessary of regression ad infinitum in any form of explanation not based on dogmatism.

It is easy to dismiss the [ZERO] hypothesis as being absurd and offensive to common-sense - Hume has often been villified but never answered:

"To refute him has been, ever since he wrote, a favourite pastime among metaphysicians. For my part, I find none of their refutations convincing; nevertheless, I cannot but hope that something less sceptical than Hume's system may be discoverable." (Russell 1946 Ch.XVII)

"I found Hume's refutation of inductive inference clear and conclusive." (Popper 1963 Ch.1 IV)

Popper's reply to Hume is based not on answer but acceptance - he re-establishes an empiricist epistemology on the possibility of "laws" being falsified but accepts the Humean position that they cannot be verified:

"we must regard all laws or theories as hypothetical or conjectural; that is, as guesses" (Popper 1972 Ch.1 6.)

Popper's reply exemplifies the dialectical significance of scepticism. Whilst the sceptical position itself it seems to offer only disillusionment, that we:

"sit down in forlorn Scepticism" because we have departed "from sense and instinct to follow the light of a superior Principle" and "a thousand scruples spring up in our minds concerning those things which before we seemed fully to comprehend" (Berkeley 1710 1.)

and Russell terms Hume's scepticism:

"the bankruptcy of eighteenth century reasonableness" (Russell 1946 p.645)

- it is, however, this same dissolution of illusion, the ripping of the veil of maya, the dynamic bankruptcy that leaves us with all false currency spent and only new beginnings before us, that is the vital force of scepticism as a genesis for knowledge.

Popper answered scepticism with a new basis for the acquisition of knowledge. Descartes re-discovered scepticism as the tool of ultimate doubt that removes all but the essence of reality. Sartre continues in the Cartesian tradition with his emphasis on "neantisation" (usually translated as "nihilation") as the force behind the transcendent upsurge of consciousness that makes knowledge possible. As Catalano remarks in his commentary on "L'Etre et Neant" (Satre 1943):

"when I ask, 'What is a tree ?' I remove, or negate, the tree from the totality of nature in order to question it as a distinct entity. Also, when I question the nature of a tree, I must have a certain 'distance' within myself that allows the tree to reveal itself to me. It is this 'nothingness' within myself that both separates the tree as this thing within nature and allows me to be aware of the tree. <u>It is this break with a causal series, which would tie being in with being in a fullness of being, that is the nothingness within man and the source of nothingness within the world.</u>" (Catalano 1974 p.66)

I could develop and exemplify this line of argument further but enough has been said here to illustrate the role of what Margaret Wiley (1966) has termed "Creative Scepticism" (and illustrates with literary examples as well as those from Eastern and Western philosophy). It is not the nihilist scepticism of Gorgias that became the dogmatic scepticism of later many later philosphers - this is self-defeating because the positive affirmation of non-existence is itself subject to scepticism. It is rather the Pyrrhonism propounded by Sextus Empiricus that suspends belief, searches out opposites, quests for truth through balance rather than dogma, and holds the manner of quest itself subject to doubt at the very moment that truth appears to have been found.

In practical decision-making the [ZERO] hypothesis has a key role in allowing us to break out of self-consistent systems that somehow do not work or, more insidiously, that do work but not as well as they could. In general it is the "tried and trusted" rule which generates the biggest explosion of novelty under the fuse of doubt - it is the "strong point" of an argument that yields most under a sceptical attack. We should doubt that which we find most efficacious, and disbelieve that which seems most obvious.

In this day and age Kuhn's (1970) "normal science" proceeds at such a rapid pace that the consequences of an argument, its verification through a wealth of exemplars, and its practical utilization through implementation in systems, are as good as over once begun. We consolidate innovation to form dogma at a pace that allows little scope for contemplative imagination - the circle is no longer open than it is complete again. With the advent of the computer this tendency becomes amplified since computers are generally programmed to be the ultimate dogmatist, propounding incessantly and without variation those dogmas that have been set into them through software. Bremmerman, Rogson and Salaff

(1965) has shown that the fortuitous processes of evolution cannot be used to break out of such algorithmic dogma. It is active scepticism that must, in Popper's words:

"replace routine more and more by a critical approach" (Popper 1974a)

and somehow we have to find ways to embed it in our decision-making systems.

[ONE] There is <u>one</u> correct foundation for decision.

The [ONE] hypothesis has its dynamics and its dangers fully equal to those of [ZERO]. The great significance of existence hypotheses and existence proofs and the key role they play in mathematics is always something of a surprise to those who meet it for the first time. To go from knowing nothing about A to knowing that A exists may seem a very small step on the path to those who wish to know what A actually <u>is</u>. However, an existence proof is often sufficient in its own right to lead to a derivation of the properties of A and even a construction of A itself.

The line of argument involved is of the form:

(i) A exists.

(ii) Any A must P.

(iii) B does P.

(iv) No other entity does P.

(v) Hence B is A.

It is interesting to note that the obvious temptation to put this into symbolic logic in the form of the classical predicate calculus must be resisted. This is because step (ii) is not adequately captured by the statement:

(ii') $\forall A\ P(A)$

since we have the standard result:

$$\forall A\ P(A) \supset \exists A\ P(A)$$

that is, (ii') pre-supposes (i), whereas (ii) itself is intended to be independent of the truth of (i). We can state that "all unicorns have horns" without having claimed that "a unicorn exists". It is clearly desirable that this pattern of reasoning be adequately formalized, and Schock (1968) has given a very clear exposition of the problems involved and some of the solutions developed. The incapacity to express arguments about existence is one of the major defects of the classical predicate calculus.

Returning to the argument sequence stated above, we can see that its significance lies in the fact that given only that A exists, and that A has the property P, we may find out under some circumstances precisely what A actually is. Somehow the necessity of existence of A has generated a complete ontology of A. The danger is that a false hypothesis of existence can lead through a weak and obvious property to a strong ontological result. The strength of such fallacious reasoning is that the existence hypothesis itself appears to have little content - certainly too little to be responsible for that of the result derived from it.

The classic example of the mis-application of the argument above is:

(I) There esists a largest positive integer.

(II) The square of any integer is greater than or equal to it. The square of the largest integer cannot be greater than it so that it must be equal to it.

(III) 1 squared equals 1.

(IV) No other positive integer squared equals itself.

(V) Hence the largest positive integer is 1.

Only the first step, (I) the existence hypothesis, is false in this line of argument. From the supposition that a largest positive integer exists we have managed to determine precisely what it must be.

Note also the key role of step (iv). In the example given step (IV) may be proved explicitly. However the [ONE] hypothesis gives us both existence and unicity without any further requirement for proof - steps (i) and (iv) in the argument are available for free. Essentially, the [ONE] hypothesis says that *if we can find an agreed property that A must have to be called A and we can then find an actual entity B that has that property, then there is no need to perform any further tests of B to verify that it is A, not any need to look for alternatives to B to falsify that it is A. Without further activity we may say that B necessarily is A.*

The [ONE] pre-supposition often turns up in technical literature as an assumption of the existence of a <u>unique</u> <u>optimum</u> <u>solution</u> to a problem, e.g. "We will determine the best linear classifier in this decision space". There may be no such best entity because the decision criterion cannot be uniformly satisfied, and even if there is one it may not be unique. These various possibilities show up as an ambiguity in the use of the word "optimum":

Def: Opt1 - an optimum solution is one such that no other is better;

Def: Opt2 - an optimum solution is one that is better than all others;

Def: Opt3 - an optimum solution is one that is better or equal to any other.

The three definitions coincide under conditions of unicity but not necessarily otherwise. To differentiate between them we have to enlarge our vocabulary and call Opt1 "admissible" (Gaines 1977) rather than optimum - the key factor being that the non-existence of better solutions may be due to incomparability. Opt2 would be called a "unique optimum", leaving Opt3 as the correct precisiation of "optimum" (reading "correct" here as "agreed by convention" since any of these definitions may be taken as precisifying the colloquial term "optimum").

In the control literature lack of appreciation of these distinctions nas several time resulted in the publication of extensions to the Pontryagin maximum principle which purport to show that it is applicable to discontinuous decision spaces also. Such forms of "discrete maximum principle" are however incorrect with the "proofs" incorporating tacit assumptions of false results that do not carry over to the discrete case. One of the most powerful features of continuity is the well-ordering it establishes in solution neighbourhoods, and this is what allows Pontryagin's formulation but no discrete equivalent.

Categorial adjunctions may be seen as arising essentially through the unicity of a pair of reciprocal functors. The Goguen/Arbib/Ehrig behaviour/structure adjunction (Gaines 1978a) encompassing a wide range of system 'identification' schemes is dependent on the existence of a unique structure ascribable to an observed behaviour. Attempts to determine a similar adjunction for stochastic systems were doomed to failure because no comparable unique solution was definable. However, the meta-systemic move to define the 'solution' in terms of the "admissible space" of structures (Gaines 1977) has allowed Ralescu (1977) to express behaviour/structure transformations in the stochastic and fuzzy cases as adjunctions because the "admissible space" is itself unique.

It is the pre-supposition of [ONE] that most often leads to fruitless searches for solutions that result in the conclusion that "the problem is insufficiently well-defined". What we mean by "well-defined" seems to be the existence of a unique solution. However, it should be clear that problems can be 'solved' in some sense without necessitating unicity of solution, and thus that some problems may be solved even though they are "ill-defined". Indeed, requiring them to be precisified to a state of well-definition in this sense may destroy the essence of the problem.

However, although one may point to the problems that [ONE] causes, one should not be blind to its virtues. In particular cases, the defence of a false theory against a powerful attack on its strong points can generate precisely the environment in which new ideas are generated. Certainly many good ideas are not developed as early as they might be because their originators drop them prematurely, only to see others re-generate them later and show that superficial weaknesses overlay great strength. Defending weak positions is often infinitely more rewarding than buttressing up strong ones - as Kenneth Boulding (1964) has noted one must be "willing to make a fool of oneself".

In the general case also, [ONE] has its virtues - even if we are dis-satisfied with all existent theories and prepared to defend none, it is the belief that there is [ONE] that keeps us looking - the "unified field theory" for gravitational and electromagnetic forces, an organic basis for schizophrenia, controlled energy from thermonuclear reactions, and so on - all of them unsolved problems but where the belief that a solution exists makes them inspirations of major fields of endeavour.

[INFINITY] There are an <u>indefinite</u> <u>variety</u> of foundations for decision.

This pluralist hypothesis is that which best summarizes actual decision-making practice. The decisions of everyday life are usually highly over-determined and skill in practical decision-making comes from the ability to balance and make most effective use a variety of bases for decision. This is not necessarily a problem of multi-criteria, but most often one of multiple information sources each of which, in theory, provides sufficient information for decision.

A good example of this is the long-standing controversy over distance perception, "What are the cues that people use in determining the distance of an object ?". Experimenter X claims that phenomenon A is the prime determinant and demonstrates this by removing all cues but A - sure enough distance perception remains and is highly accurate. Experimenter Y claims that phenomenon B is the prime determinant and demonstrates this by removing all cues but B - sure enough distance perception remains and is highly accurate. Sooner or later, after the most refined experimental designs to ensure that no cues of type B are slipping in to confound those of type A, or vice versa, it is realized that not only are A and B each individually completely adequate distance cues, but the people subconsciously switch from one to another depending on which is available. At this point the excitement of controversy dies down, perhaps even the scientific research (the [ONE] hypothesis is highly important as a social dynamic !), and a few patient researchers are left determining <u>all</u> the different, interchangeable, bases for distance perception (Gibson 1950).

The real problem, once a pluralist basis for some aspect of practical decision-making is found, it to determine how the many different bases for decision are brought together to determine a single decision when more than one is available, i.e. how is over-determination resolved ? This is a difficult point which is often missed - [INFINITY] seems to lack the dialectical strength of both [ZERO] and [ONE] because it allows for all possibilities and hence does not bring them into essential conflict. In terms of explanation this may be so - your explanation is consistent and adequate, so is mine - we are both good fellows who do not need to fight but can revel in mutual self-satisfaction. However, in terms of explanation even, a meta-problem immediately arises as to how two explanations can account for a single phenomena: are they ordered in that one can be derived from the other, but not vice versa ?; are they unrelated ? - in which case is there a deeper underlying explanation from which both may be derived ?; and so on. One of the rules of the scientific game is that, like acausality, plurality is not allowed except as a matter of short-term expediency.

In terms of decision, there is no rule of the game that says that a plurality of bases is not allowed. Moreover, there is no rule either that says that these bases cannot conflict - generally they do - over-determination in a precise theory leads to multiple values for essentially single-valued variables and hence conflict, paradox, and, if the rules of the theory are precisely applied, a total breakdown of the basis for decision. In distance perception the possibility of such conflicts between the multiple bases of perception leads to "optical illusions" (Gregory 1970). The related phenomenon of "reasoning illusions" in practical reasoning is neglected in work on formal logic because the classical predicate calculus has the formula:

$$P \& \sim P \supset Q$$

i.e., that a breakdown of the law of contradiction may be used to derive any conclusion, and hence there is nothing that can be usefully said about this (in the same way that nothing can be said about existence). However, in practical reasoning we seem able to avoid the Wittgensteinian trap of knowing all the consequences of our premises (I would suggest by using a more appropriate logic rather than just by not working out all possibilities), and the mechanisms for conflict resolution are a key component of our systems of practical reasoning.

Thus [INFINITY] does have its own means of generating dialectical conflict and it is the most subtle and important of all. We have to accept as a basis for practical reasoning that multiple accounts of equal standing will arise and can be in conflict. Decision-making under uncertainty is usually seen as leading to under-determination, but in practice it most often leads to over-determination. Conflict resolution because we are over-provided with conflicting advice is far more prevalent than the other forms of conflict where we have too little.

2.2 Role of Foundations

I have dramatized the foundations of decision because they are worthy of it. If we are unaware of the seething conflicts below any theories, methodologies and practical schemes that we erect then we are not only guilty of that false peace of mind that stems from ignorance, but we are also missing out on that major element of choice that comes through conflict. If there are different pre-suppositions possible even at a truly foundational level, all of which are of equal merit (in the sense that they can be defended one against the other), then we have freedom of action in moving between them. It is our choice to be sceptical, to defend a unifying theory, to give equal status to mutually contradictory schemes.

The realization of the extent of choice enables them to be taken lightly. Practical decision is sometimes a game against nature but most often a game against other decision-makers, and real games are most often "won" by changing the rules. Even in the 'hot-war' against nature itself, the rules under which we play are of our own contrivance - it was the decision to consider the 'impossible' concept of "action-at-a-distance" that enabled Isaac Newton to forecast the motions of apples and planets - it was the decision to place Mach's eyes in the 'impossible' vehicle of a photon that forced Albert Einstein to distort the 'certain' constancies of space.

If this seems more a prescription for rhetoric than for decision-science then so be it - if rhetoric were not so neglected a science the powerful analogy by which the whole of science is seen as the "persuasion of nature" would be more often used. In decision this becomes more than an analogy because it is the "persuasion of the world" through action that is the key to decision.

In summary, this section has pointed to two key dialectical conflicts in the foundations of decision. In terms of the form of argument outlined in (i) to (v) above, there is first a conflict over step (i), existence: the Gorgian sceptic denies it; the [ONE] and [INFINITY] hypotheses both affirm it; the Pyrrhonian sceptic transcends all of them by suspending judgement. The [ONE] and [INFINITY] hypotheses themselves come into conflict over step (iv), unicity - this is the classic conflict between the tendency to unify and that to disintegrate.

In any system of decision that we build all of these dialectical possibilities will be present, and in good systems they will be explicitly present. The greater the awareness that we have of them, the more control we have over the possible choices they give, the more versatile and powerful the decision system will be.

In the following sections, I will give some practical and some theoretical approaches to decision systems that have such versatility and breadth of approach as key objectives.

3 THE ROLE OF THE COMPUTER

The digital computer cannot be regarded as just a tool for work on decision. The tool is itself so significant that it will change our whole approach to both the theory and practice of decision. I say "will change" because the potential of the computer is far from being realized. As a 'stand-alone' calculation system it is already powerful and important. As a closely coupled complement to the mind of man, however, it generates a new kind of creature whose capabilities are yet beyond our imagination. Currently we are able to add the data-processing power of the computer to thought processes of man. At the next stage we shall effectively multiply the two together and generate a quantity with new dimensions.

In these notes I shall not duplicate the two papers:

"Man-computer communications: what next ?" (Gaines 1978d)

"Minicomputers in business applications in the next decade" (Gaines 1978b)

which are available. The first of these papers highlights informally the current trends in man-machine symbiosis emphasizing the role of the computer as a tool for the emancipation of cognition. The second details recent developments in low-cost interactive computer systems that are beginning to allow this to take place. That is, the first paper develops the principles of close man-computer interaction, and the second shows the current state-of-the-art of the relevant technology. The overall message is that we do not yet have either the communications interfaces, particularly voice-interaction, or the communications software, particularly 'world' models, that are necessary for full man-computer symbiosis. However, the message is also that we are moving rapidly in the right direction - raw processing power in small machines, colour graphic displays at low cost, and a variety of basic software modules to provide easy access to the power of the machine - these are already available to allow the individual to share some of his load in intellectual tasks with the computer. To a large extent we are already 'imagination-limited' - the "personal computer" can do far more for us now than is at all evident in current applications.

However, the lag of applications behind hardware/software technology is comparatively short - good systems have only been available for the last 3 years and the price is only now beginning to hit levels that are attractive to the 'non-professional' user (non-computer-professional, that is - many key first-time users are professionals in their own areas, doctors, accountants, clinicians). A pattern of use for the personal computer in the home and office is emerging from the dozen small business and hobbies magazines and journals now extant: text processing for correspondence and document production is one major application; tax returns for personal and business purposes another; there are also developments in "planning" packages that optimize delivery schedules, etc.

The most important feature of current forms of personal computing is that they are enabling key individuals to increase their own speed and flexibility of operation by decreasing their dependence on the availability and skills of others. It is the supporting services of a secretarial, clerical, acoounting, information-retrieval, nature that are being taken over by the personal computer. We seemed to have been entering an era of essential team-work where individual "acts of creation" were possible only through a mediating network of supporting staff. The personal computer is already changing this and the potential for further change is very great indeed. "Each man his own Leonardo" is now a realizable slogan in human terms - the 'team' is still there and still necessary but it is the 'record' of pooled human skill within the computer that forms the rest of the team not the people themselves.

3.1 Computers in World 3

In attempting to come to grips with the problem of understanding the new opportunities that computers create, I have found Popper's "3 worlds" model (Popper 1968) of great value. In his autobiography he introduces it (Popper 1974a p.143) by quoting Bolzano's notion of "truths in themselves" in contradistinction to "those thought processes by which a man may...grasp truths", proposing that:

> "thoughts in the sense of contents or statements in themselves and thoughts in the sense of thought processes belong to two entirely different 'worlds'."

and making the three-fold distinction:

> "If we call the world of 'things' - of physical objects - the <u>first world</u> and the world of subjective experience the <u>second world</u> we may call the world of statements in themselves the <u>third world</u> (...world 3)." (Popper 1974a p.144)

Popper notes:

> "I regard books and journals and letters as typically third-world objects, especially if they develop and discuss a theory." (Popper 1974 p.145)

and stresses the key role of world 3 in the development of human "civilisation", giving two gedanken experiments on the destruction of civilization to illustrate the status of world 3:

> "(1) all machines and tools are destroyed, also all our memories of science and technology, including our subjective knowledge of machines and tools, and how to use them. But <u>libraries and our capacity to learn from them</u> survive...our world civilization may be restored...from the World 3 that survives"

> "(2) in addition <u>all libraries are destroyed</u> ...men would be reduced to the barbarism of primitive man in early prehistory, and civilization could be restored only by the same slow and painful process that has characterized the story of man through Paleolithic times" (Popper 1968 p.334)

Popper emphasizes the distinct ontological status of world 3:

> "I regard the third world as being essentially the product of the human mind. It is we who create third-world objects. That these objects have their own inherent or autonomous laws which create unintended and unforeseeable consequences is only an instance (although a very interesting one) of a more general rule, the rule that all our actions have such consequences." (Popper 1974a p.148)

It seems to me that the computer provides a new dynamic for world 3 just as did the harnessing of energy in world 1. It brings world 3 into the demesne of man just as did the steam, internal combustion, and jet engines, world 1. That is we can move about in, conquer, control and fabricate to our needs the lands and materials of world 3 using computers in a way that makes our previous efforts, all but a few, look feeble. Those few we shall shall look back upon in wonder as we do the construction in world 1 of the Egyptian pyramids, equalled in world 3 by Greek philosophy. However, such 'impossible' achievements prior to the harnessing of inhuman energy and inhuman intellect will be surpassed in achievement, if not in wonder, through our control of mechanisms that give us control of the worlds in which they exist: the energetic engines of world 1 and the informatic engines of world 3.

The role of computers in world 3 can be seen most clearly by contrasting information within a library with that in a computer database. The library itself is passive, waiting for scholars and technicians to tap its stored information, but powerless to process that information in any way, to classify it, extend it, and correlate it, except through human mediation. The database contains the same information as the library but may also itself be active through processes that interact with that information without necessary human mediation, sifting through the stored data structures, analysing and comparing information, and building new structures to enhance and extend those already present. The library is like a museum of preserved flowers, a static record of unchanging knowledge, whereas the database can be a living garden subject to growth and evolution, changing even as we study it.

The contrast becomes most pointed if we look at the exceptions that prove the rule. Librarianship is the art of preserving the static garden and also of cultivating it as much as is possible by preparing indexes, concordances, and so on, and also making it maximally available and attractive to scholars who will cross-pollinate and extend it - a good library is one where much effort is put into overcoming its intrinsic stasis. On the other hand, current databases are going through that usual stage of computer-based systems where they are designed primarily to mimic that which exists, the static library. Research on inference-based "knowledge structures" in artificial intelligence research is moving in the right direction, but we still have a long way to go before the potential of the computer database begins to be realized (Gaines 1978c).

However, the important thing is that the potential exists and that we have mechnisms capable of doing routinely and on a large scale what good librarians and scholars are currently only able to do with difficulty and on a small scale. Speeding up the processes of scholarship will have the same effect on world 3 that speeding up the processes of transport has had on world 1. Our physical world is a very different place from that of 100 years ago - we are able to take actions in it that disregard boundaries of size and distance because we have, to a very large extent, conquered both. A similar phenomenon has taken place in world 2 where modern communication techniques allow us to share subjective experiences through film and television - to live out vicariously the pleasures and horrors of the lives of others without actually doing so. Popper has pointed to the independent existence and impact of world 3 even as a static store - the ideas of Plato, Hume, Hitler and the Beatles, are active today in so much as they reach into world 2 and germinate in the minds of men. How much more active they will become as world 3 itself becomes energized with its own power sources such that the small energies of world 2 are able to control far greater powers in world 3 as they do in world 1. Power has its dangers - we may yet destroy world 1 and there are no doubt similar possibilities in world 3 - but danger is a necessary face of humanity and the last aspect of any world that should make us wish to avoid it.

3.2 The First Faltering Steps

The next section outlines some of the technical developments necessary to exploit the potential of computers for the conquest of world 3. However, it is interesting first to review briefly how this might come about and where the first steps are being taken. I commenced this section by emphasizing the role of the personal computer and in the long term this is the key to the next stage of development. Personal computers provide a bridge from world 2 into world 3 that the previous generation of centralized off-line machines did not. It will be the growing demands for this bridgehead to be be strengthened and extended that will lead us into new territories.

There are several uses of computers already that seem to me to point the way:

David Mulhall's (1977) use of data-processing in the treatment of disturbed inter-personal relationships is a clear example of the role of the computer in the "emancipation of cognition" to use Habermas' (1968 p.308) apt terminology. The graphical presentation of the social consequences of actions and interactions has proved to be both acceptable and effective. By seeing the world 2 situation that they have created for themselves in world 3 terms individuals are able to make changes through their own choice and decision. This is far removed from the "technical cognitive" approach of aversion therapy that treats people as objects in world 1.

Laurie Thomas and Mildred Shaw's work on the interactive construction and analysis of repertory grids (Shaw & Thomas 1978, Shaw 1978) again puts into the hands of the individual, and groups of individuals, a tool by which they may construct a world 3 model of their own views of worlds 1, 2 or 3. This is a particularly exciting development because it takes Kelley's (1955) insight into the world 2 nature of all our knowledge and applies it flexibly, but uniformly and rigorously, to that knowledge, whatever its source or content: the "Focus" system and its variants can be used to model and externalize (in world 3) the personal and group knowledge that we have (in world 2) of the physical world (world 1), inter-personal relations (in world 2) or scientific constructs (in world 3).

Systems that successfully anlalyse and encode professional skills and make them available to others less skilled are also pre-cursors of those which will conquer world 3. Mycin (Shortliffe 1976) is currently an outstanding example, but its success has lead to this area of development becoming one of major activity and we may expect to see more and more general systems for encoding and activating professional knowledge skills become available.

These examples are all ones whereby world 3 is built out of the material of world 2 and hence may be a world of fantasy just as easy as one of reality, i.e. the computer is not itself directly interacting with world 1. Popper himself notes the role of such fantasy in world 3:

"Even theories, products of our intellect, result from the criticism of myths, which are products of our imagination: they would not be possible without myths; nor would criticism be possible without the distinction between fact or fiction, or truth and falsity. This is why myths or fictions should not be excluded from the third world. So we are led to include art and, in fact, all human products into which we have injected some of our ideas, and which incorporate the result of criticism, in a sense wider than a merely intellectual one." (Popper 1974a p.155)

It is this acquisition and processing of 'knowledge' by computer systems through interaction with world 2 that I would rate as having the most immediate importance and potential rather than any direct interaction with world 1. It is only after substantial developments in the computational structures necessary to fully interface with world 2 that we shall be able to fabricate systems that can be comparable to ourselves in having independent and meaningful access directly to world 1. That is, we shall build companions and colleagues long before we build robots, and the robots we eventually build will be nothing like those we currently envision since the conquest or world 3 will change our entire relationship with world 1.

I have blurred the distinction between the individual and the group in the above discussion and feel that this distinction will become increasingly blurred. World 3 has always been the creation of many minds and modern communications equipment may be seen as amplifying our capabilities to form 'group minds'. The computer takes this a step further in allowing us to communicate not just with passive records of the state of an individual's mind but with an active process that <u>replicates the activities of his mind</u>. However, the effect of this on individuals is to enhance their individualism not erode it - by being freed from dependence on the <u>availability</u> of others (i.e. their physical presence, attention, interest, co-operation, etc.) we are able to operate more effectively ourselves. We shall never be, nor ever wish to be, freed from our dependence as such on others - world 3 is the sum total of all our acts of creation, but once created it exists independently of us - the computer provides the means to faithfully record and make available far more of world 3, both its passive and active natures, than has any previous technology.

4 SOME TECHNIQUES

Part 2 of this paper has explored foundations and part 3 practical tools for decision science and applications. In this final section I shall outline a few of the techniques that have the potential to make fundamental changes to the basis of decision. This section is illustrative rather than comprehensive - it is here that we now have most work to do, but also the opportunities to do it.

Again I shall not duplicate presentations already available:

"Analogy categories, virtual machines and structured programming" (Gaines 1975)

"System identification, approximation and complexity" (Gaines 1977)

"General systems identification - fundamentals and results" (Gaines 1978a)

"Foundations of fuzzy reasonirg" (Gaines 1977)

"Nonstandard logics for database systems" (Gaines 1978c)

The first paper presents a formal analysis of analogies between systems in category-theoretic terms. The second and third papers present a review and up-to-date account of work on general system identification. The fourth paper presents a foundational account of uncertain reasoning, the necessity for its formalization, and the relationship between fuzzy and probabilistic accounts. The fifth paper presents the state-of-the-art of modern database systems, illustrates fundamental defects in the structures currently available, and suggests ways in which these may be remedied.

The following are brief notes on these topics.

4.1 Klir's Epistemological Hierarchy

I have found George Klir's (1976) notion of an epistemological hierarchy a very useful one in analysing, classifying, and developing techniques for decision systems. This is shown in Figure 1:

META-META SYSTEM, ETC.

META SYSTEM

STRUCTURE SYSTEM

GENERATIVE SYSTEM

DATA SYSTEM

SOURCE SYSTEM

Figure 1 Klir's Epistemological Hierarchy

This would be more fairly termed an ontological hierarchy since it represents the levels at which pre-suppositional decisions must be made before the epistemological process itself can begin.

The lowest level, of source systems, is effectively one of data definition wherby the way in which our experience of the world will be described is defined and agreed. The next level in the hierarchy is one of data systems, effectively one of system observation whereby our actual experience of some system is described in terms of the agreed domain of discourse at level zero. The next level is one of generative systems effectively one of a model for the system of which the experience is reported at the level below. The level above this is one of structure systems effectively one at which the models themselves are seen to have internal structure and hence to be analysable in other terms. The first meta level will be one at which this form of model analysis is itself seen as part of a class of possible approaches to model analysis, and so on.

This hierarchy is very useful in studying techniques for system analyis and clarifying debates in the literature on epistemology where apparent conflicts are often due to confusion between levels. In scientific work we usually appear to take the lowest level for granted, assuming that our vocabularly is already standardized and accepted. However, this is the level of phenomenological and existential debate, and it is not unreasonable to feel "dread" at the responsibility for ruthlessly encoding experience in languages which one knows lose the major part of its content and superimpose their own pre-conceptions upon it.

It is determining the relationship between data systems and generative systems that constitutes the epistemological problem of system identification, and this is a task well-suited to the patient power of the computer. This particular problem can be resolved in a very general sense that draws together much of the literature on modelling, identification, pattern recognition, etc., and the basis for this is outlined in the next section.

4.2 Identification, Complexity and Approximation

Let E be a set of possible exemplars of system behaviour, e.g. the set of all possible patterns on a grid, or the set of all possible strings of input/output behaviour of an automaton. Let $i: I \rightarrow E$ be a sub-object monomorphism from an 'index-set' I to E that represents the 'observed', or 'classified', sub-set of E. Let $b: I \rightarrow V$ be a mapping from I to a 'truth-set' V which represent possible observational assignments of values to elements of E. Goguen (1974) calls such a mapping a 'V-set' with I as carrier, and V will normally be some ordered algebraic structure such as a lattice, or the interval [0,1] with max/min, add/multiply, or logical operations (Gaines & Kohout 1976). For example, V might be a set of possible pattern classes, or V might be the interval [0,1] with the mapping b corresponding to the observed relative frequencies of the exemplars in E. I have previously termed b the (observed) "behaviour" of a system, and taken the problem of system identification to be one of inferring a "model" that accounts for b in some well-defined way.

Let M be a set of mappings from E to V such that any $m \in M$ represents a possible 'model' of behaviours such as b. Figure 2 shows the mappings under consideration and it is apparent that a basic view of the problem would be to require m to be an extension of b from the sub-object of E, I, to the whole of E, so that: $im=b$.

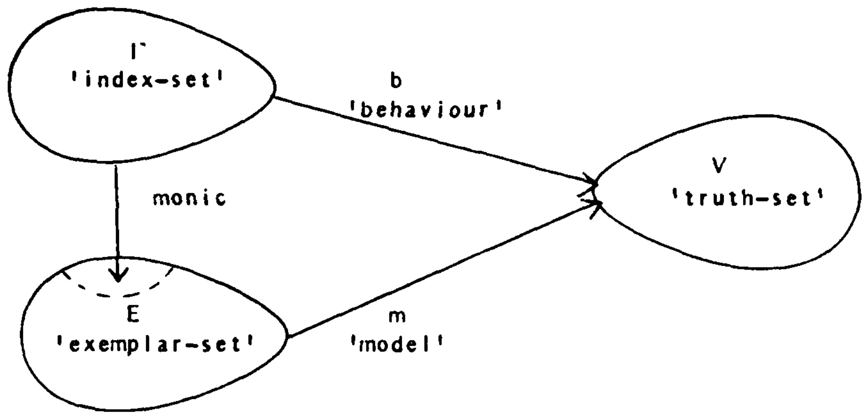

Figure 2 Modelling as Extension of a Mapping

Consider this to be the requirement for the moment, that m should be chosen such that $im=b$, and consider the situation where many m satisfy the requirement. How do we select amongst the many possible extensions of b ? In particular situations, we will usually have some criterion that, ceteris paribus, allows us to select one model as being preferable to another, e.g. that it has the least 'parameters', or the least 'states', or some other Ockham's razor type of criterion. In general, all such criteria can be subsumed under the requirement that there is some pre-order on M, $\leq$, such that the lesser models in the pre-order are intrinsically preferred to those greater, i.e. if $m,n \in M$ are two models that are both valid extensions of some behaviour b, and $m \leq n$, then we will not prefer n to m as a model of b. This preference order is not necessarily of Ockham's razor type, for example it could be one based on elegance or the party-line, but it is so often one of simplicity it is convenient to use this terminology and say that if $m \leq n$ then n is not 'simpler' than m. There has been a great deal of discussion in the philosophical literature (Bunge 1963, Kemeny 1953, Sober 1975) of our justification for preferring 'simpler' models in a variety of circumstances, but for present purposes any underlying rationale does not matter.

At this stage our formulation of the problem of system identification might seem complete - "out of all models that are valid extensions of the observed behaviour chose the simplest". However, whilst this is a reasonable formulation for the deterministic case where the behavioural mapping b is well-defined and should be precisely extended, it is not an adequate formulation of the non-deterministic case. For example, if b represents an observed distribution of behaviour of a stochastic system then determining a model that precisely generates the observed relative frequencies is a number-theoretic problem whose solution has little relevance to actual requirements. What we really want is a model that approximates the observed frequencies to an extent that residual differences between behaviour and model are statistically insignificant. Equally, if b represents a given 'degree of membership' assigned to the possible behaviours of a fuzzy system, then what we require is a model that places the same fuzzy restriction on behaviour within a reasonable tolerance. In general, we do not require m to be an exact extension of b but rather a reasonable approximation to such an extension - the diagram of Figure 2 need not commute, only 'almost commute'.

In a particular situation one will have highly specific concepts of what one means by 'almost commutes', i.e. to what extent im approximates b. However, in general the key notion is again one of order, that it should be possible to say that one model is a better, or worse, approximation to b than is another. To do this one needs to define there to be a mapping from the set of behaviours, to the set of pre-orders on models, Ord_M, $f: B \rightarrow Ord_M$, such that if $\leq_b$ is the image of b under f, then, for $m,n \in M$, $m \leq_b n$ means that n is not a better approximation to b than m is.

Having defined the pre-orders of simplicity and approximation, we are in a position to define a solution to the identification problem in terms of their product, $\overset{*}{\leq}_b$, defined as:

$$\forall\, m,n \in M, \quad m \overset{*}{\leq}_b n \Leftrightarrow \quad m \leq n \text{ and } m \leq_b n$$

i.e. $m \overset{*}{\leq}_b n$ if and only if n is both neither simpler nor a better approximation to b than m. The minimal elements in this order are all admissible solutions to the identification problem because they cannot be bettered in simplicity without worsening the approximation, and they cannot be bettered in approximation without worsening the simplicity. They form the admissible subspace of models determined by b, $M_b \subset M$, such that:

$$M_b \equiv \{m : \forall\, n \in M, \ n \overset{*}{\leq}_b n \Leftrightarrow m \overset{*}{\leq}_b n\}$$

i.e. if any model is better than one in M. then it is equivalent to it.

One interesting side-effect of requiring the diagram of Figure 2 only to 'almost-commute' is that the expression of the problem may now be simplified by the artifice of extending the set V with an additional element representing 'unknown', or 'unobserved', assignments to elements of E. This removes the need for the sub-object monomorphism i, which is now subsumed into the semantics of the approximation ordering $\leq_b$ in that models that differ only in their assignments of values to elements of E to which b assigns the 'unknown' assignment must be equivalent in approximation. 'Don't-care' assignments clearly also have the same semantics.

This formulation of system modelling may be used to draw a number of important conclusions about the actual implementation of identification systems:

First, consider the set M of possible models. It is clear that the selection of M alone is but a small step in setting up the modelling problem. The simplicity ordering over the models must also be specified and variation of this affects the solution to the problem as much as does that of the choice of the models themselves. For example, even the basic class of finite automata may be ordered by number of states, number of links between states, or by some more complex function that takes into account hierarchical structure, etc. Zeigler (1976) has given several examples of such order relations on automata and we have found that varying among them has a very profound effect in practical modelling situations, both on the speed of computation and the solutions found.

Secondly, consider the set E the set of possible exemplars. The 'language' in which these are described is arbitrary and can be varied without affecting the essential modelling problem. However, it is clear that changes in E will be reflected by changes in the admissible sub-set of models, not only in the trivial sense that the models themselves must change to reflect the variations in E, but also in that the simplicity ordering on models must also change if the situation is to remain invariant. This is unlikely to be so since this ordering will generally be derived from intensional, extra-linguistic, considerations. Variations in the language in which exemplars are described and in the class of models and simplicity orderings are outside the framework of the modelling situation and yet can have a very profound effect on it. I have previously suggested that, whilst the process of modelling described here is very Carnapian in its search for the models best confirmed by the observed behaviour, the great significance of changes in the actual terms of reference of the modelling process itself corresponds to Kuhnian "scientific revolutions" - changes in the underying concepts and assumptions on which the modelling itself is based.

Thirdly, the implementation of a modelling scheme once the various ordered sets required have been defined is basically simple. One needs only an algorithm that generates possible models in order of decreasing simplicity. As each m is generated it is checked against b for its position in the approximation order. If this is is worse than that of simpler models previously found the model is rejected, but otherwise it is added to the admissible set. The key trick here is the derivation of a production algorithm for models for complete enumeration of models of decreasing simplicity, and Wharton (1977) has recently given some techniques for doing this for very wide classes of models.

Fourthly, the measures of approximation used are clearly very significant and it is interesting that these seem to have a high degree of problem independence, largely because the set V is basically problem independent, a classification, or a distribution, or a fuzzy restriction. For example, consider V to be the interval [0,1] with the mappings b and m giving normalized relative frequency distributions. A suitable measure of approximation between b and m is then one that has a minimum when . e E, me=be (neglecting now the sub-object mapping i), and varies above this minimum to the extent that they differ. Examples of such measures are:

$$SE = \sum_{e \in E} (me-be)^2 \qquad (1)$$

$$CE = \sum_{e \in E} (me-be)^2/me \qquad (2)$$

$$LE = \sum_{e \in E} -be\ \ln(me) \qquad (3)$$

Measure (1) is a standard least-mean-squares test used extensively in the literature on modelling and pattern recogntion, and by de Finetti (1972) as the foundation for his theory of probability. Measure (2) is the chi-square statistic used as a statistical test of similarity of distributions and by Maryanski and Booth (1977) in their grammatical inference scheme. Measure (3) is a Gibb's function giving rise to a Shannon entropy measure that has been widely used in grammatical inference and in Savage's foundations of subjective probability (1970). Pearl (1978) has pulled together these and other such functions into a single economic model of such approximation.

Fifthly, the determination of the admissible set is often not in itself the final solution to a modelling problem. Pattern recognition, system indentification, etc., are generally carried out for a purpose, to give us a basis for decision, and we have to decide which model to actually use out of the admissible set. Often this will involve action to obtain further data on further exemplars to provide for better discrimination between admissible models. Clearly this can itself conflict with the use of the models giving rise to a 'two-armed bandit' form of problem (Witten 1976). In terms of the formulation here it may well be appropriate to change the measures of approximation to ones of 'achievement' when the overall task of the system is action rather than prediction, i.e. to set up a pragmatic epistemology in which 'truth' is evaluated in terms of 'usefulness'.

Finally, these last remarks highlight the extent to which the order relations of simplicity and approximation are an operational definition of our axiology. I have stressed that no epistemology can be free of arbitrary ontological decisions, and I will emphasize also that it cannot be free of axiological ones either. We take on a great deal of responsibility in our decisions as to both ontological and axiological pre-suppositions before we even begin to acquire the information for action.

The implementation of modelling schema based on the framework of simplicity and approximation outlined here seems to be an excellent task to allocate to a computer. The machine can take over the routine drudgery of "normal science", effectively using a Carnapian paradigm of incremental confirmation of hypotheses. Clearly, there is nothing within the framework I have proposed that allows the "revolutionary" components of epistemological advance to be automated. This is one of the roles for man in complementing the computer. When the modelling process seems to be going badly then we intervene to make changes in the space of models, simplicity ordering, approximation measures, etc. This will be possible and effective to the extent that these are expressed in a mutually comprehensible form to man and machine.

4.3 The Relation of Analogy

The concept of there being an <u>analogy</u> between two systems such that one can "reason by analogy" and make statements about one system based on knowledge about the other is very important to any form of decision system. It is the analogy of a system at time t to itself at time t' that is crucial to our being able to acquire knowledge at all. It is the analogy of one person to another that gives us the concept of a human race and a science of psychology. Korzybski (1958) has emphasized the semantic problems of the concept of "identity", and it is clear that in practice this is an abstraction for there being an ultimate level of analogy.

Whilst there have been many studies of the role of analogy in science (Hesse 1966, Leatherdale 1974) and it forms a major component of "plausible reasoning" (Polya 1954), there appears to have been little attempt to formalize it systematically. In Gaines (1968) I introduced a category-theoretic formulation of what we mean by an analogue computer and in Gaines (1975) . extended it to systems in general.

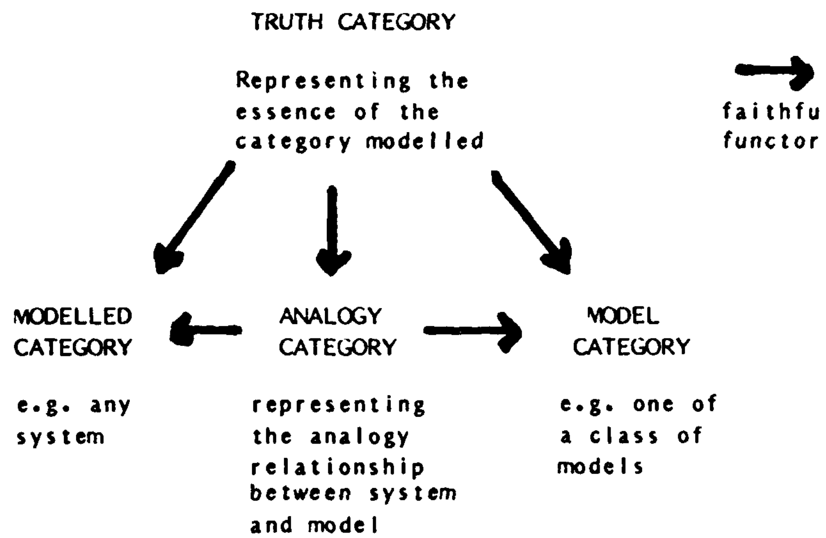

Figure 3 The Analogy Category Between A System and Its Model

The basis of this formalization of analogy is shown in Figure 3: the "category to be modelled" and the "model category" are such that there is no functor of interest from one to the other, i.e. both contain 'structure' that the other does not; however, there is an "analogy category" that maps into both model, and modelled, categories through faithful functors and hence represents mutual structure; there are many such possible analogy categories and to ensure that they are non-trivial it is necessary to introduce a "truth-category" such that the diagram of Figure 3 commutes; all possible analogy categories satisfying Figure 3 then form a semi-lattice.

Gaines (1975) gives 3 postulates on which to base a theory of analogy:

I A system can be represented by a category.

II A truth category having a faithful functor to each of a category and its model can adequately represent all that we mean by a "correct", or "significant", or "adequate", or "true", analogy.

III The semi-lattice ordering of analogy categories represents what we mean by one analogy being "more comprehensive", "closer", or "more detailed", than another.

This theory is attractive in being operational - it is possible to compute the entire semi-lattice of analogies between any two systems. However, the computation gives rise to a combinatorial explosion in all but the most trivial cases, and this again seems the type of task ideally suited to a computer. In practice it will not be the calculation of the analogy structure between to well-defined systems that is of importance, but the _exploration of an ill-defined system using the analogy between it and a known system_.

4.4 The Logic of Decision

All of the techniques outlined in this section involve the use of a variety on standard and non-standard logics and the manipulation of logical structures, theorem-proving, etc. As we refine a science of decision these logics are formalized to sets of axioms and rules of inference that enable us to derive all deductive consequences of statements in appropriate formal languages. However, we find in practice that such a 'logical science of decision' falters at an early and trivial stage - trivial in that it has nothing to do with the content of the facts and theories being used but only with the syntax of the logics. We, unaided, are unable to cope with the task of formal deduction to all but a virtually trivial extent.

Thus, for a person, to know a proposition is _not_ to know all its consequences - we do not "know" in the sense that Wittgenstein defines:

> "If I know an object I also know all its possible occurrences in states of affairs. (Every one of these possibilities must be part of the nature of the object.) A new possibility cannot be discovered later." (Wittgenstein, _Tractatus_ 2.0123)

Even if such "knowing" is physically impossible, it is clearly important that the potential is there - that given a proposition, x, and a body of propositions, A, we should be able to determine of any proposition, y, in the deductive closure of A under some logic, whether x implies y, y implies x, neither, or both.

The role of the computer in providing the deductive reasoning power to expand human "knowing" to the Wittgenstieian ideal is one of the most important aspects of man-machine symbiosis. For major areas of logic, such as the varieties of modal logic, it is claimed that we are already at this stage:

> "The formal techniques to be used are essentially mechanical. Proving theorems within a given system of logic involves following a straightforward mechnical procedure with paper and pencil that could as well be done by computer." (Snyder 1971 p.12)

However, despite advances in theorem-proving for many-valued and modal logics (Gaines & Kohout 1977) we have not yet done as Snyder suggests. There seems to be no reason why we should not do so in the near future, however, and set up "possible world" simulations of the consequence structures of major sets of real-world premises.

The significance of this is world 3 terms is illustrated by Popper in his demonstration that the currently encoded part of world 3 (which he terms world 3.1) is transcended by the consequence structure that may be generated from it:

> "world 3 transcends essentially its own encoded section. There are lots of

examples, but I will take a simple one. There can be no more than a finite number of numbers in world 3.1. Neither a library nor a human brain incorporates an infinite series of natural numbers. But world 3 possesses the lot, because of the theorem (or axiom): every number has a successor." (Popper 1974b p.1050)

In emphasizing the logical role of the computer in man-machine symbiosis we must not lose sight of the converse role essentially provided by man. The lack of rigorous foundations to any theory of decision, and the necessary lack of the possibility of such foundations, means that any system based on a logical approach to the expansion of knowledge is inherently restricted and bound ultimately to fail. The human component of a man-machine symbiosis complements the machine part by providing the possibility of meta-systemic branches and deviations from the programmed system. If particular "deviations" prove valuable then they will be programmed into the repertoire of the machine, but it seems reasonable to conjecture that whatever the level of formalization and rigour there will be a meta-level at which the human component provides necessary informality resulting in lack of rigour and the possibility of acts of creation. The human component breaks the 'closure' of the machine. Between man and conputer there is a battle of imcompletion and completion - for man completion is intellectual death - for the computer incompletion is intolerable ignorance.

4.5 The Database as a General System

Database systems originated as mechanisms for storing and retrieving information and their later development has largely concentrated on immediate commercial requirements (Nolan 1973). Currently a wide range of practical and cost-effective systems are in use, and there have been notable successes in more advanced applications involving natural language interaction with commercial databases by naive users. There are however severe limitations in the extension of databases into practical "management information systems" due to the rigidity of the data-structures imposed by current implementations and the system-theoretic concepts that underly them. Recently there have been a variety of attempts to liberalize the underlying logical structure of database systems to allow a wider range of more realistic data structures to be encompassed. Gaines 1978c illustrates the practical need for such extensions, surveys recent developments, gives examples of how a variety of classical logical and systemic problems arise in database systems, and shows the underlying structure common to a wide variety of extensions.

Codd's (1970) relational formulation of databases was the first step in the direction developed here, in that it greatly generalized and made far more flexible the forms of data structure and retrieval specification allowed. However, available implementations of databases are in terms of "hard", static, deterministic relations, whereas in real-world applications data is often imprecise, inherently dynamic and non-deterministic. In recent years there has been a range of developments concerned with representing and using data that can only be analysed in these "softer" terms. Some of the work has been explicitly concerned with database systems, but much of it, whilst highly relevant, has been in other application areas.

Gaines (1978c) develops a systemic account of databases that relates their dynamics to topological automata theory and enables normalization to be viewed as a constraint on the category of database automata. Within this framework the extension of databases to "softer" structures may be seen as the introduction of nonstandard logical structures. Some of the real-world defects of standard relational databases are shown to correspond to the occurrence of paradoxes in the corresponding logics. This paper also discusses the roles of modal, multi-valued and fuzzy logics in databases and a common computational framework for their representation in relational databases is outlined.

I believe this concept of a "database" generalized as above is quite fundamental to system and decision theory. All the classical system-theoretic constructs which we use, dynamical systems, automata, etc., are special cases of the general "database". The "possible worlds" treatent of logical structures that has proved so fruitful in giving firm foundations to a variety of non-classical logics may be regarded as using a "database" model. The database model itself seems to be a natural tool with which to investigate practical reasoning (Zadeh 1978b) of a fuzzy (Gaines 1976) or possibilistic (Zadeh 1978a) nature. If we see the role of the computer in man-machine symbiosis to provide a generalized database facility allowing man to aggregate information and from it explore possible worlds then this is probably as far as we yet can, or need, to see. Currently, as Gaines (1978c) shows, we are far from this possibility - however, the technology exists, many developments are underway, and tomorrow may see the potential of man-machine symbiosis become actualized.

5 CONCLUSIONS

After section 2 it would be foolish to attempt to draw conclusions about decision itself. In terms of the study of decision however, there is a clear prescription not to place one's trust in foundations - certainly not to become over-enthusiastic about optimizing particular approaches based on particular foundations. Robustness, rather than optimality, is what we have to seek - robustness not just against uncertainty in our data but also against essential uncertainties in the foundations of the subject itself.

Robustness in many key structures comes from disparate entities working together in a complementary fashion to enhance the other's strengths and eradicate the other's weaknesses: the combination of iron and concrete to form ferro-concrete is of just this nature. This world 1 example has its parallel in world 3 through the combination of man and computer. We do not yet know how to weld these two together. We cannot yet foresee the properties of the new system that we shall have created. We, as individuals, will probably never comprehend its use, since that level of comprehension will only be open to the new creature, the creation of symbiosis, itself.

In the near future, it is the movement towards operationalism, towards the embedding of theories in an applicable form within the computer, that has most to offer us. The basic concepts of ontology, epistemology, axiology, and their associated logical calculi, may now be actualized as effective algorithms. These will give us access to systems of plausible reasoning that extend and complement our own.

The scope for innovation in the operation of the mind itself is now limitless - as closing parentheses let me quote again one who mastered all 3 worlds as much as was possible in his time:

> "It would be unsound fancy and self-contradictory to expect that things which have never yet been done can be done except by means that have never yet been tried." (Bacon, Novum Organum Book 1, VI)

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VOLUME 4

COMPETING MODES OF COGNITION AND COMMUNICATION
IN SIMULATED AND SELF-REFLECTIVE SYSTEMS

Competing Modes of Cognition and Communication in Simulated and Self-Reflective Systems

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1. ON COMPETING MODES AND THE STUDY OF THEIR RELATIONS

1.1 Introduction

From some empirical and computer simulation studies of cognition and communication in human dyads and networks (9-13), has emerged a growing concern (a) with competitive tendencies or modes of behavior revealed by such systems, and (b) with approaches for generating images of relations between (models of) such competitive patterns. This paper offers some snapshots of (a) and (b). The latter comprises computer simulation and self-reflective techniques as means of studying patterns of cognition and interaction in psychosocial systems from their "outside" and "inside". Assumed to be of relevance to decision theory and praxis, the snapshots exemplify ways of studying man in the context of his cognitive and group activity, and concentrate on

(i) some competing patterns of cognition and communication as modes of "resolving" affective-cognitive inconsistency, explored through computer simulation, partly with reference to a referendum controversy, and

(ii) some possible competing modes of referencing that seem to be indicated in two studies of symbolic interaction in which self-reflective (video-playback) method has been used.

1.2 Preliminary definitions

"Cognitive inconsistency" is the label for a state of imbalance in cognitive and/or attitudinal organization, involving intra- or inter-personal "psycho-logical" contradictions between affective, cognitive- and sometimes behavioral - elements of a psychosocial system (a person, a conversational unit, a small group)(4).

"Mode of inconsistency resolution" is the term for a strategy resorted to in order to resolve or reduce a state of cognitive inconsistency. According to the family of theories that share the postulate of balance-tendencies in cognitive and attidudinal organizations (4), a variety of modes is available: The actor in a state of intolerable inconsistency may be expected to resor to change in information processing and communication activity - intake and output, change in attitude - cognitive and affective, change of interest level, or other strategies, but without any guarantee of succeeding in resolution his inconsistency.

Some space will be devoted to one precis of an effective cognitive consistency definition, and to computer-operational implementation of some of these modes.

"Symbolic interaction" may perhaps most easily be defined in terms of what it is not. While social interaction qua transaction between two parties involves the exchange of goods and bads (25) in which the participant cannot both retain and part with that which is offered to the other, symbolic interaction is the expression of meaning through exchange of symbols, involving the sharing of something expressed by one and understood by the other (that is, requiring what I term "meaning-tightness", which will be returned to). According to G.H. Mead (1934), who anticipated many cybernetic notions, symbolic interaction presupposes the capacity of the participants to mentally simulate the other (10,29,34).

"Referencing" is here used for the symbolic act of indicating something qua referent for labeling, conceptualization or conversation. In cases of symbolic interaction, there occurs referencing to myself, to you, and to some topic or theme which we discuss. In successful cases - not in terms of agreement, but in terms of some expressed meaning - "organizational closure"(32) and what I term "meaning-tightness" (14,16) seems to be a characteristic. But if there is competition between modes of being open and being closed, or being tight and loose, then challenges may be involved to calculi for self-referencing (37) that seeks to avoi paradoxes, such as the one involved in my uttering: "On this page I am not referring to myself", without resorting to the theory of logical types (35).

Competition between two tendencies or modes of behavior, p and q, are here said to exist if they are part of the availab action programs of a given system and compete for activation or for dominance in a given system context.

Competition for activation means that they cannot be concurrently in operation; the activation of p necessitates the passivation of q, and vice versa. This may be termed operational incompatibility. Thus, for example, the tendency to increased communication activity and the tendency to withdrawal from the field as a result of a state of cognitive inconsistency, may be said to be competitive in terms of incompatibility; both tendencies cannot be activated at the same instant in the same system.

Competition for dominance means that they are concurrently active or in parallel operation. In cases where p and q are concurrently active and opposed to each other or pulling in opposite directions, so that effects produced by p are counteracted by effects produced by q and vice versa, their mutual relation may be said to be one of contrariety. Thus, for example, the concurrent activation of motivation to agree and achieve consistency (required for some task solution) and to develop conflicting perspectives and thus inconsistency (required in the same task), may be said to constitute a competitive relation of contrariety. Indications of this kind of competitive pattern seem to be revealed in a follow-up analysis of empirical dialogue records from a study of moral dilemma processing dyads. Ten two person moral "juries", given the task of arriving at judgements on a moral dilemma, have been observed in a laboratory setting and constituted the referent systems for computer simulation. As the model, the DYAD SIMULATOR, constructed from a consistency-theoretical perspective, produced most mismatches in cases of initial strong imbalance. The protocols have been subjected to a renewed analysis in search of conditions for p- and q-patterns Results - presently being checked - seem to indicate oscillation in each of the dyads between tendencies towards close agreement and consistency and tendencies towards increased disagreement and inconsistency, irrespective of initial state. (I had hoped to include snapshots of this study in the present paper, but time and space prevent it).

However, while operating in parallel, two tendencies of behavior, p and q, may also be said to be competing for dominance in a less dramatic sense; when they are concurrently active, and invading each other, but not necessarily in opposition to each other. Thus, for example, to the extent that modes of self-reference may be said to compete for dominance with modes of other- or topic-reference during an event of symbolic interaction, while none of these modes prevents the implicit operation of the other mode, then they are competitive, but neither incompatible, nor contrary.

1.3 Systems theoretical context

Thus, in the case of competition between two tendencies or modes of behavior, p and q, the relation may be one of the following types:

(i) p and q are operationally incompatible, competing for activation

(ii) p and q are contrary, and operating in parallel, competing for dominance

(iii) p and q are parallel, but not contrary, competing for dominance.

Now, to the extent that such competitive or parallel tendencies are revealed by psychosocial systems, then models or theories for such systems are in trouble if they represent limited viewpoints that only allow for explanation of one of the relata in such competitive relations, and perhaps imply the unfruitfulness - or even untenability - of theories for the competitive relatum. Controversies between representatives of theories for cognitive consistency vs. theories for cognitive complexity, theories for social consensus vs. theories for social conflict, or between theories that rule out self-enclosure vs. theories that assert that any cognitive event is an event of self-enclosure, may appear somewhat artifical if in fact the source of the referent systems for such theories allows for the revelation of tendencies towards consistency <u>and</u> complexity, consensus <u>and</u> conflict, of self- <u>and</u> other- or it-reference.

If that is the case, then the <u>theoretical</u> competition is not resolved by making one side the winner, implying the untenability or unfruitfulness of models representing a supplementary, complementary, or even seemingly contradictory, standpoint. Nor should it be resolved in terms of a traditional theoretical synthesis, whereby the theory for $\underline{p}$, f_p, and the competing theory for $\underline{q}$, f_q, are destroyed. If both $\underline{p}$ and $\underline{q}$ are modes competing for activation or dominance, then there may be some higher-order program or mechanism present in the system field that takes care of shifts for dominance or activation. Bateson (6) posed the problem of shifts between competing interaction modes, such as assertiveness and submission, as early as 1936, and realized later that with the development of cybernetic thought (2), the problem of describing such relations may be solved in terms of circuits or systems of higher order . About the same time Angyal (5), gave examples of competitive, interfering or mutually invading, modes of personality organization. He, too, is concerned with higher-order change of setting, that is conditions for r(p,q).

The notion of dual modes competing in a particular system, may also be recognized in McCullogh's principle of redundancy of potential commands (28): The brain is capable of a mode (p) for non-hierarchical organization in terms of several potential posts of command, but at the instant of actual execution, this mode is replaced by a mode (q) of hierarchical organization whereby the actual command is asserted by one of the posts. Thus an image, r(p,q), is offered of the conditions for shifts between two competitive modes of organization: Preparation requires a p-mode, while actual execution a q-mode.

Obviously, the study of a p-pattern in a given real-world interaction situation would require different "spectacles" or systems view-points from those required in the study of competitive q-patterns. However, the same real-world data source may allow for parallel referent system to be selected from such different perspectives, given parallel "spectacles" or "field glasses", capable of reflecting r(p,q)- images, that is the conditions for shifts in terms of dominance or activation between p and q. Their application upon the same "territory" may provide a basis for acknowledging that different "maps", previously treated as theoretical competitors, may complement or supplement each other in providing valid images. The above examples of r(p,q) images may be seen as parts of a more comprehensive scheme of competitive modes of human organization, interaction, and cognition, outlined below.

Table 1 Disystems theoretical context. Only aspects marked * are touched upon in the present paper

	P-MODE	Q-MODE	R(P,Q) IMAGE
ORGANIZATION	non-hierarchical	hierarchical	shift as a function of time distance to execution
INTERACTION	symbolic I-You; self-referencing*	transactional I-it; it-referencing*	shift according to being or need-deficiency orientatio
COGNITION	conflict; assertion and negation	consistency;* unification	shift according to phase of dialectical movement

The relation between hierarchical and non-hierarchical modes of organization is seen as reflected in a higher-order image. The distinction between I-You units made possible in symbolic interaction and the I-it relations necessitated in instrumental transaction (17), is seen in the light of Maslow's <u>being</u> - and <u>deficiency</u>-orientations(27). Modes of cognitive complexity, conflict generation, and of cognitive consistency generation are seen as phases in dialectical reflection (Hegel,23).

The above disystemic scheme makes up the theoretical context for the snapshots to be presented in the following text, which only concerns those aspects marked "*" above.

1.4 Systems methodological context

To the extent that such rival tendencies are shown by man in the context of his organizational, interactional and cognitive activities, then complementarity seems to be called for - not only in terms of images, but in terms of approaches. Complementarity between images and between approaches that may generate images of referent systems capable of competitive modes of behavior, represents challenges of a kind that philosophy and methods of science offer no easy solution to. Pattee has advocated complementarity of approaches in the biological sciences (33), but without offering recipes for how to proceed. In the natural science, the famous discussion between Bohr and Einstein on complementarity *vs*. the possibility of unification (8) illustrates the epistemological difficulties involved. As for psychosocial sciences, concerned with systems in and of which the participants are capable of transcending and creating "laws" for modes of behavior, the difficulties may seem even greater. While not pretending to offer any ready-made recipes for the generation and handling of complementary images and building theories for competing modes, I shall briefly describe two approaches still in the process of becoming and being explored. One approach involves the usage of computer simulations with models or submodels, representing competitive viewpoints with reference to some common data source for referent systems. The other involves usage of self-reflective method, through which participants in small groups recall and reflect through interaction and video-playback on processes previously videotaped. Both are systems-oriented, but in a complementary manner.

Let $\mathcal{S}$ denote a real-world source of systems, $s_1, s_2 \ldots, s_n$, the behavior of which is referred to by a set $\mathcal{F}$ of models or model mechanisms, $f_1, f_2, \ldots, f_m$, representing a variety of viewpoints upon modes of behavior. Let $\mathcal{F}$ have some member that allows for modelling a mode p, and some other member modelling a competitive mode q. Let the members of $\mathcal{S}$ be allowed to interact, and the members of $\mathcal{F}$ be allowed to interact, and let records of these interactions be provided for. Let images generated through such interactions be denoted $R_{\mathcal{S}}$, when provided by (observations of) members of $\mathcal{S}$, and $R_{\mathcal{F}}$, when provided by the recording of the behavior of members of $\mathcal{F}$. Then the respective paradigms for exploration through computer simulations and through self-reflection may be depicted as in figures 1.1 and 1.2, where directed lines represent direction of reference, and broken ones influencing or generating relations.

(See Figs 1.1 and 1.2)

<u>The computer simulation paradigm</u> (Cf. Gullahorns in (21)) illustrated in fig.1.1 may involve the following rules and procedures:

(I) Given a real world data source, seek to formulate a model or model set, including mechanisms for p- and q-modes,

(II) allow for computer-operational interaction between such models or model mechanisms through simulation,

(III) establish bases for comparison between model behavior and referent systems behavior, that is between $R_{\mathcal{S}}$ and $R_{\mathcal{F}}$,

(IV) provide for a metasystem, capable of shifting between and adjusting models representing different viewpoints and levels, and capable of comparing images generated by simulation with these models and of comparing these images with those obtained through observation of referent systems(

(V) seek to differentiate between the models in terms of criteria concerning their respective boundaries for fruitfulness and tenability (13).

Part 2 of this paper reports on simulations of voter populations allowing for the exploration of images of relations between competive modes of resolving affective-cognitive inconsistency. Among such modes are (i) increase in the frequency of communication and attitudinal shifts, (ii) withdrawal from the field through loosing interest or staying at home, and (iii) selective attention and search for supportive messages. There is competition in a mutually exclusive sense between (i) and (ii), and a mode (iv) competitive to (iii), is revealed in the exploration.

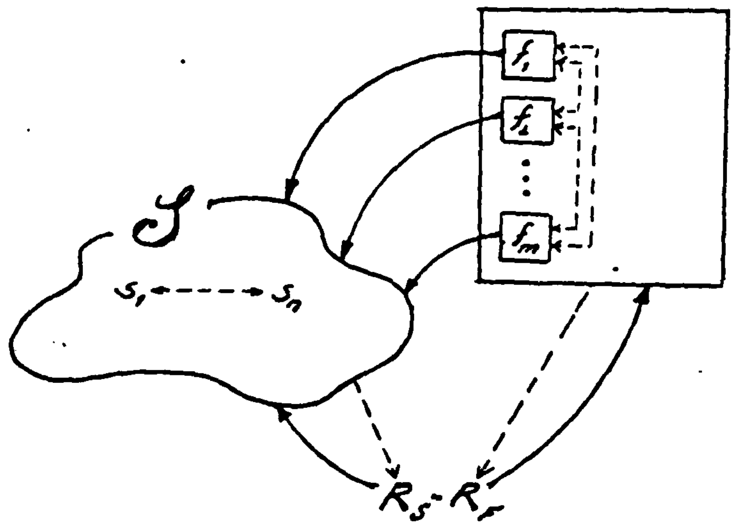

Figure 1.1 <u>The computer simulation paradigm</u>, allowing for generation of model systems behavior image, R_F, but with only indirect reference to a source of real-world systems, which may or may not allow for an image, R_S, to be empirically generated of their behavior. In this case, R_F and R_S may possibly be compared

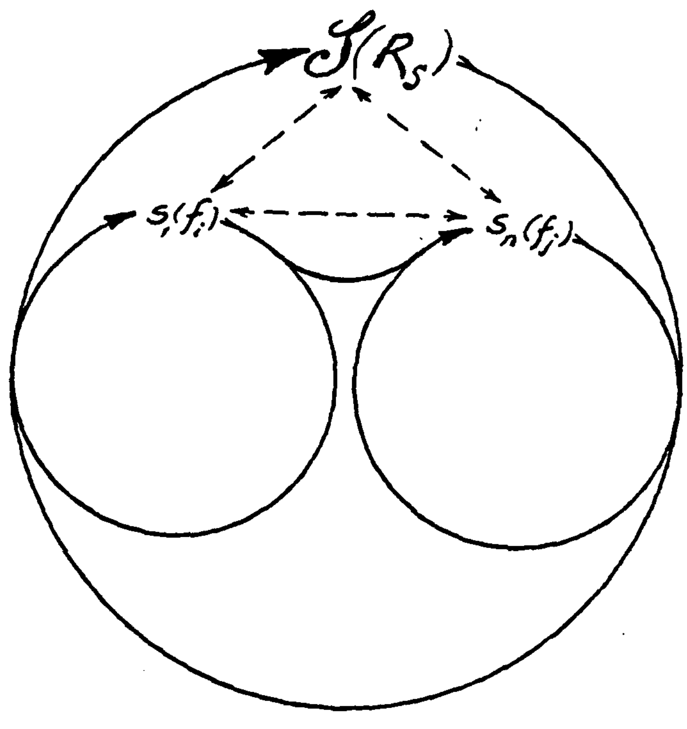

Figure 1.2 <u>The self-reflection paradigm</u>, allowing for the systems's generation of images of itself

Common symbols for figures 1.1 and 1.2: ⟶ reference direction
---- influence

The exploration is carried out in terms of a set of model mechanisms, allowed to interact computer-operationally, and compared with observed behavior in a referendum at the aggregate level and in a network panel at the community level. This network panel (22) has been subjected to post-dictory simulations in terms of a set of models, reported elsewhere (Bråten, Jahren & Jansen, 15). Models representing a consistency perspective compete with a model representing an economic perspective. A metastructure, labeled MULTISIM, is implemented for shifts, recordings, and comparison of model systems behavior and referent systems behavior in accordance with the above paradigm.

<u>The self-reflective paradigm</u> that is illustrated in figure 1.2, requires facilities for the real-world systems source's own generation of images of itself. Part 3 of this paper contains an introduction to the usage of <u>video-playback</u> as a facility for self-reflection in two studies by/of small groups. This method is seen in light of von Foerster's and Pask's approach to self-reflection (32,38), and is related to techniques employed in the approach to *conversation* by Pask (31) and to self-analytical groups by Mills (30), but has been initiated independently of these developments (10). One possible competitive relation indicated in my video-playback studies is that between (i) topic-referential and (ii) self-referential modes.

While being concerned with the generation of competing - and sometimes complementary - images, the two paradigms stand in a complementary relation: While the computer simulation - and associated empirical - studies are carried out from an <u>outside</u> observer's standpoint (although meaning-tightness* is involved), the self-reflective studies involve - at last in part - the standpoint of the <u>insider</u> and the capacity for mental self- and self-other simulation.

*The concept of 'meaning-tight systems' (14,16) will be returned in part 3. The statement
(U.1) "I see what you mean..."
is a declaration about a state of meaning-tightness in an event of symbolic interaction, irrespective of whether (U.1) is followed up by (U.2.1) or by (U.2.2):
(U.2.1)"...but I disagree with you."
(U.2.2)"...and I agree with you.".

2. COMPUTER SIMULATION: COMPETITIVE MODES OF INCONSISTENCY RESOLUTION

This part reports on explorations with a set of computer-operational mechanisms, representing modes of affective-cognitive inconsistency resolution of the following kind:

- Increased activity and selectivity in relation to message offers (attention, interpretation) and to network environment (partner selection)
- Increased probability of evaluative preference switch - if the above attempts turn out to be unsuccessful, given a certain period of uninterrupted inconsistency, and no choice commitment as yet
- Increased probability of withdrawal to a state of fixed disinterest (or non-participation if election time), if the above modes have not been successful.
  This mode is incompatible with those above; the actor cannot concurrently be more active and less active.

They make up part of a model (9) ** to be described below, and which has been used in a post-dictory referendum simulation at the national level (12), and as basis for a related model, belonging to a set of models, applied in a network panel simulation at the community level (15).

2.1 Competitive modes in social network context

Simulated actors are organizations, personal promotors called "agents", mass media, and persons.

Declaration of the latter actor class makes available procedures that contain the set of mechanisms explored in computer-operational form. The function of the other actor classes is merely to provide for adequate message offers and boundary conditions for the theory-explorational simulations.

The person actors make up a population that is the target of promotion of an A vs. B issue, allowing for incompatible choices by the actor persons.

Each mass medium is characterized by a coverage area and by the degree of coverage within that area (given by a coverage

** This model is closely related to the Abelson/Bernstein referendum simulation model (3) and shares the label "SIMCOM" with another model developed at the Geographical Dept. of Northwestern University. All three models have been developed independently of each other and are discussed in Guetzkow, Kotler and Schultz (21). My model was built in 1967, and implemented in SIMULA the following year in collaboration with one of the designers of that language, Kristen Nygaard, and with the assistance of Helge Klitzing, while Urban Norlén was assistant on the statistical side probability), and allows for message insertions of varied frequency. An agent may have the whole or a part of the population as his visiting district, and may vary his speed of operation according to the visiting frequency value assigned to him.

A specific actor person s(i,j) is identified according to his location in the symbolic population, where

(1) $i,j = 1,2\ldots,10, \text{ or } i,j, = 1,2\ldots,20,$

dependent upon his belonging to a population consisting of 100 or 400 persons.

Other persons that may enter into direct interaction with the person s(i,j), are those belonging to a set G, where

(2) $G = \{s(i,j+1),\ s(i,j-1),\ s(i+1,j),\ s(i-1,j)\}$

which together with s(i,j) constitutes the interpersonal communication network or "membership group" of s(i,j). Such nets are illustrated in the upper right part of fig.2.1.

(See Figs 2.1 and 2.2)

<u>Messages</u>, supportive of either <u>A</u> or <u>B</u> are being produced, exchanged and processed in the system. Besides carrying content that signifies the choice alternative supported, any message has both evaluative and informative contents, which vary according to the type and state of the message-producing actor. The evaluative content (of type real in the simulations) may be regarded as a reduced expression of the affective orientation held by the message-offering actor. The informative content (also of type real) may be regarded as expressive of the number of arguments <u>pro</u> the choice alternative that the message-offering actor is able to mobilize. The evaluative content and the argumentative content of a message may be drastically changed during processing by the recipient actor.

<u>Affective orientational state values</u> are defined according to allocated positions of <u>A</u> and <u>B</u> on an evaluative scale, ranging from 0.0 to 10.0, and to the distance between these two positions. V_A denotes the evaluative positions allocated to <u>A</u> by the actor person, and V_B the evaluative position associated with <u>B</u> (both treated as variables of type real). The strongest of the two positions determines his actual evaluation preference state.

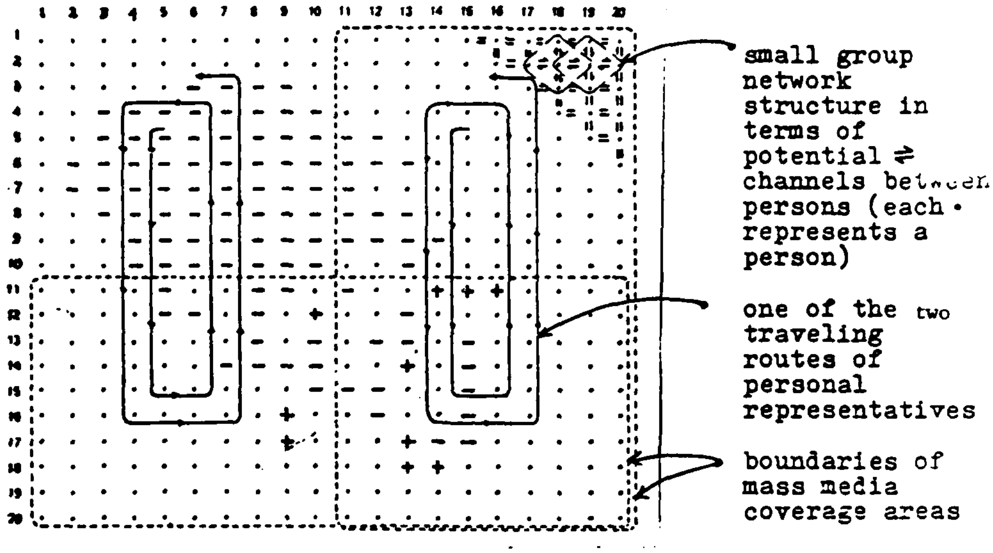

Figure 2.1 Map of a SIMCOM population of 400 (20 · 20) persons at a given point of simulated time ·

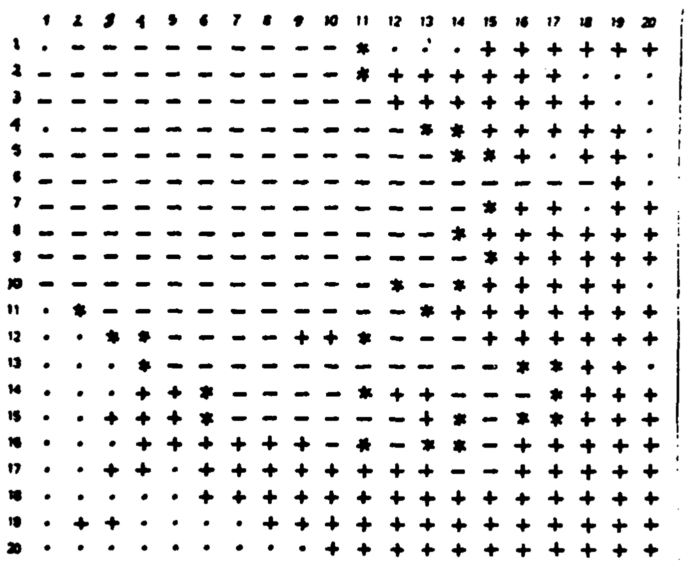

Figure 2.2 Map of the above population at a later point of simulated time

Figures 2.1 and 2.2 give snapshots of the simulation run used in an "explanation" of public opinion development and vote outcome at the national level in the EEC Referendum controversy in Norway (12), referred in section 2.3. Upper map represents initial state of simulation (time = 0), while lower map state at the date of referendum (simulated time = 65).

+ : Yes-person; − : No-person; . : Neutral, * : Fixed Neutral } : "homesitters"

The distance $|V_A - V_B|$ is here termed the evaluation difference (or "diffvalue").

In order that the person be defined as interested, his actual value on V_A and/or V_B must exceed a certain level, termed the interest treshold, L_i. (During the simulations reported here, the base value of L_i is fixed at 2.0).

Cognitive state of the person is defined by his accumulated knowledge in terms of number of stored arguments related to the choice alternatives. As A and B denote incompatible choice alternatives in relation to which the person actor can accumulate knowledge and take tentative evaluative stands, (the system is closed to other kinds of content), only the kind of knowledge which can be transformed into choice arguments are considered. All arguments contra one alternative are regarded as transformable into arguments pro the other alternative. The actual difference between accumulated arguments pro A, denoted I_A, and arguments pro B, denoted I_B, characterizing the actor person at a certain point of time, is his information difference ("diffinfo"), which is equal to $I_A - I_B$ when $V_A > V_B$, and equal to $I_B - I_A$ when $V_B > V_A$.

The person as a six-state system is described below (and in fig. 4). When transitions between states of inconsistency and consistency are disregarded (see below), the state $Z_t^6\,(i,j)$ of an actor $s(i,j)$ at time t may have one of the following six values:

(3) $Z_t^6\,(i,j)=$

1 : Neutral (N) or uninterested is true if $\max(V_A, V_B) < L_i$ and the system has not previously been in any other state (Cf. state 6).

2 : A-believer (Ab) is true if $V_A > V_B$ and the conditions for none of the five other states are satisfied.

3 : B-believer (Bb) is true if $V_A < V_B$, and the conditions for none of the five other states are satisfied.

4 : A-chooser (Ac) becomes true upon choice of A as an absorbing state.

5 : B-chooser (Bc) becomes true upon choice of B as an absorbing state.

6 : Fixed neutral (Fn) as an absorbing state becomes true upon withdrawal through reduction of the values of V_A and V_B, so that $\max(V_A, V_B) < L_i$ becomes true.

<u>Affective-cognitive inconsistency</u> is generated when an interested actor person associates positive evaluation with one of the choice alternatives, say <u>A</u>, while a weak evaluation is associated with <u>B</u>, and the difference in his accumulated arguments <u>pro</u> <u>A</u> and <u>pro</u> <u>B</u> is small and out of proportion to the evaluation difference. He will then enter a state of inconsistency. (This tendency is strengthened as a consequence of his making a choice which increases his actual evaluation difference). In general, the greater the actual evaluation distance, the greater information difference is required (in the same direction as his evaluative preference), in order to allow the actor to remain in a consistent state.

Thus inconsistency becomes true if

(4) $\max (V_A, V_B) > L_i \wedge \text{diffvalue} > (\text{diffinfo}.\ D)$

where D is a constant.

<u>Modes of inconsistency resolution</u> are given computer-operational form as part of a set of mechanisms, regulation (F1) action program selection, (F2) exposure, (F3) attention, (F4) interpretation, (F5) cognitive adaption, (F6) evaluative adaption, (F7) consistency transition, (F8) interest transition , (F9) act of choice, (F10) message content selection, and (F11) message offering to selected coactor. Gross Universe-Coupling Characteristic (24) of actor persons using these mechanisms is given in the lower part of fig.3(b).

(See Fig. 3)

According to this set, an occuring state of inconsistency (through the procedures illustrated in the upper part of fig.3(a)) first and foremost results in systematic change in the overt and covert communication and information processing behavior of the actor: He becomes more active in his communicative behavior (F1) and more discriminative in his coactor selection (F11), more selective in his message attention (F3) and more varied in his distortive interpretative behavior (F4). If he contiues to be in a state of inconsistency in spite of the operation of such

(a)

(b)

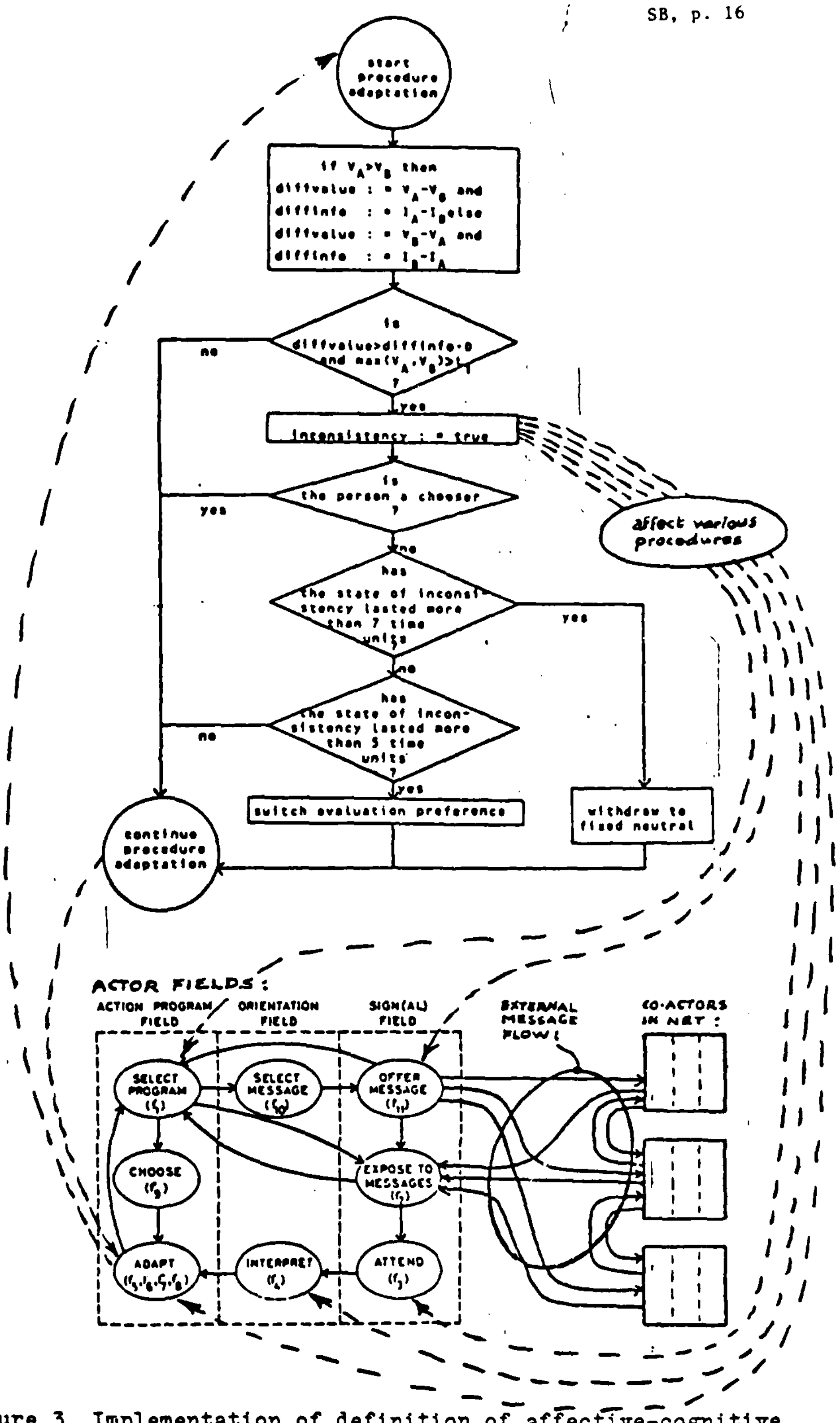

Figure 3 Implementation of definition of affective-cognitive inconsistency (a) and gross universe-coupling characteristic of the SIMCOM model (b). To the left processes and fields in the actor person

selective and defensive mechanisms, he may switch evaluation preferences as a first type of solution (F6), and finally withdraw to a fixed state of disinterest (F8), called fixed neutral, if none of the former modes succeed in solving his problems of inconsistency. Description of two of these mechanisms (F11,F4) is given below.

<u>Coactor selection of message offering (F11)</u> concerns the tendency of selective avoidance of discrepant message offers. Let S^j denote a selected potential coactor subset of G (Cf. (2)), used with priority j by the searching actor s(i,j) for random selection of a coactor for message offering and exchange. According to F11, the construction and priority of S^j as a function of the searching actor's consistency state and the (in)congruity between his evaluative preference and his group members' choice state, is such that

$$(5.1)\quad S^1 = \begin{cases} \text{G - incongruent choosers, if consistency} \\ \text{congruent choosers, if inconsistency} \end{cases}$$

$$(5.2)\quad S^2 = \begin{cases} \text{G - congruent choosers, if consistency} \\ \text{G - incongruent choosers, if inconsistency} \end{cases}$$

where S^2 is used in the random drawing if S^1 is empty. Otherwise S^1 is used.

<u>Distortive interpretation (F4)</u> means distortive transformation of a sender-defined message content into a recipient-processed content, where this transformation may involve a reduction of sender-defined content. This simplified notion means that content that may be added during interpretation is disregarded.

The degree of distortive reduction of a message attended to by an actor person is a function of (i) the type of message-producing medium or coactor, (ii) the (in)congruity between his evaluative state and the message content, (iii) his state of interest, and (iv) his state of consistency.

Let $d_j\ (M_i)$ denote degree of reduction of the content of kind i of M, such that the larger d_j, the more of M's content of kind i is reduced. According to F4 the following relations hold for d_j as a function of message-producing medium or coactor:

(6.1) $d_m(M_v) > d_a(M_v) > d_p(M_v)$

(6.2) $d_a(M_k) > d_m(M_k) < d_p(M_k)$

where M_v denotes evaluative message content, and M_k cognitive content, ar

$$d_j = \begin{cases} d_m \text{ if the sender is a mass medium} \\ d_a \text{ if the sender is an agent} \\ d_p \text{ if the sender is a group member person} \end{cases}$$

As for distortive reduction of cognitive content as a function of consistency state and (in)congruity between evaluative state and message content, the following applies:

(6.3) $d_{ii}(M_k) > d_{ic}(M_k) > d_{cc}(M_k) > d_{ci}(M_k)$

where

$$d_j = \begin{cases} d_{ci} \text{ if congruity and inconsistency} \\ d_{cc} \text{ if congruity and consistency} \\ d_{ic} \text{ if incongruity and consistency} \\ d_{ii} \text{ if incongruity and inconsistency} \end{cases}$$

As illustrated in fig.3(a), if such selective and defensive mechanisms, described above, fail in resolving inconsistency, the actor may resort to the more drastic mode of switching his evaluation preferences or even withdraw to the fixed state of disinterested neutral .

Computer experiments in the form of variations in the s[illegible] of pseudo-random numbers used in the random drawings employed in the mechanisms, provide the basis for the exploration. Populations of 400 persons were subjected to experimentation during fixed simulated periods of 120 time units, while the model-100 version was allowed to operate until the aggregated system approximated a stable system state with regard to distribution of neutrals and choosers ("voters"). These periods seldom surpassed 100 time units. In the transition diagram depicted in figure 4, mean person state transition frequency values are given for a series of 10 runs with each of the two populations and with variations only in "stream parameters". Both populations are subjected to agent promotion of B and mass media supporting A, and released from an initial state of a small percentage A-choosers in the populations, with the remaining being neutrals. The mean transition values are true for all conditions experienced by the' population members in these runs. Double arrows in the transition diagram mark <u>inconsistency resolution mode transitions</u>

(See Fig. 4)

<u>Competing modes of resolution</u> represented in the above diagram are that of switching evaluative preference (from *A*-believer to *B*-believer or vice versa) and withdrawal (from *A*- or *B*-believer to Fixed neutral). Obviously, the conditions and states that have released such drastic modes have remained so in spite of preceding operations of less drastic modes. In order to explore such conditions, we need access to simulation protocol data on the relation of state transition frequencies to inputs to, and conditions of the person. Let input variables and boundary conditions that vary with each actor person during a time period of unit length allow for a simple and proper categorization in terms of x (Cf. (7) below). The recorded time series of actor person statesmay then be described in terms of state transition frequencies, based on a calculation of

$$\frac{\text{the number of observed persons } S(i,j) \text{ with the boundary condition or input value } x, \text{ datable at the time point } t \text{ (or at some point in the period between } t \text{ and } t+1) \text{ that are in the state } u \text{ at } t \text{ and have moved to state } v \text{ at } t+1}{\text{the number of observed simulated persons } S(i,j) \text{ with the boundary condition or input value } x, \text{ datable at } t \text{ (or at some point between } t \text{ and } t+1) \text{ that are in the state } u \text{ at } t}$$

for u,v = 1,2,...,6 (Cf.(3)), and x = 0,1,2,...7, where

$$(7)\quad x = \begin{cases} 2 \text{ if only exposed to a coactor person offering an } \underline{A}\text{-message} \\ 3 \text{ if only exposed to a coactor person offering a } \underline{B}\text{-message} \\ 4 \text{ if only exposed to coactor persons offering both } \underline{A}\text{- and } \underline{B}\text{-messages} \\ 5 \text{ if } \underline{A}\text{-persons, but no } \underline{B}\text{-person, are within reach} \\ 6 \text{ if } \underline{B}\text{-persons, but no } \underline{A}\text{-person, are within reach} \\ 7 \text{ if both } \underline{A}\text{- and } \underline{B}\text{-persons are within reach (members of actor's group)} \\ 1 \text{ for any occuring condition and input value, including any of those specified above} \end{cases}$$

Past history of each person not represented in $Z^6(i,j)$ is disregarded (Cf.(3)). In order that a coactor person be within reach

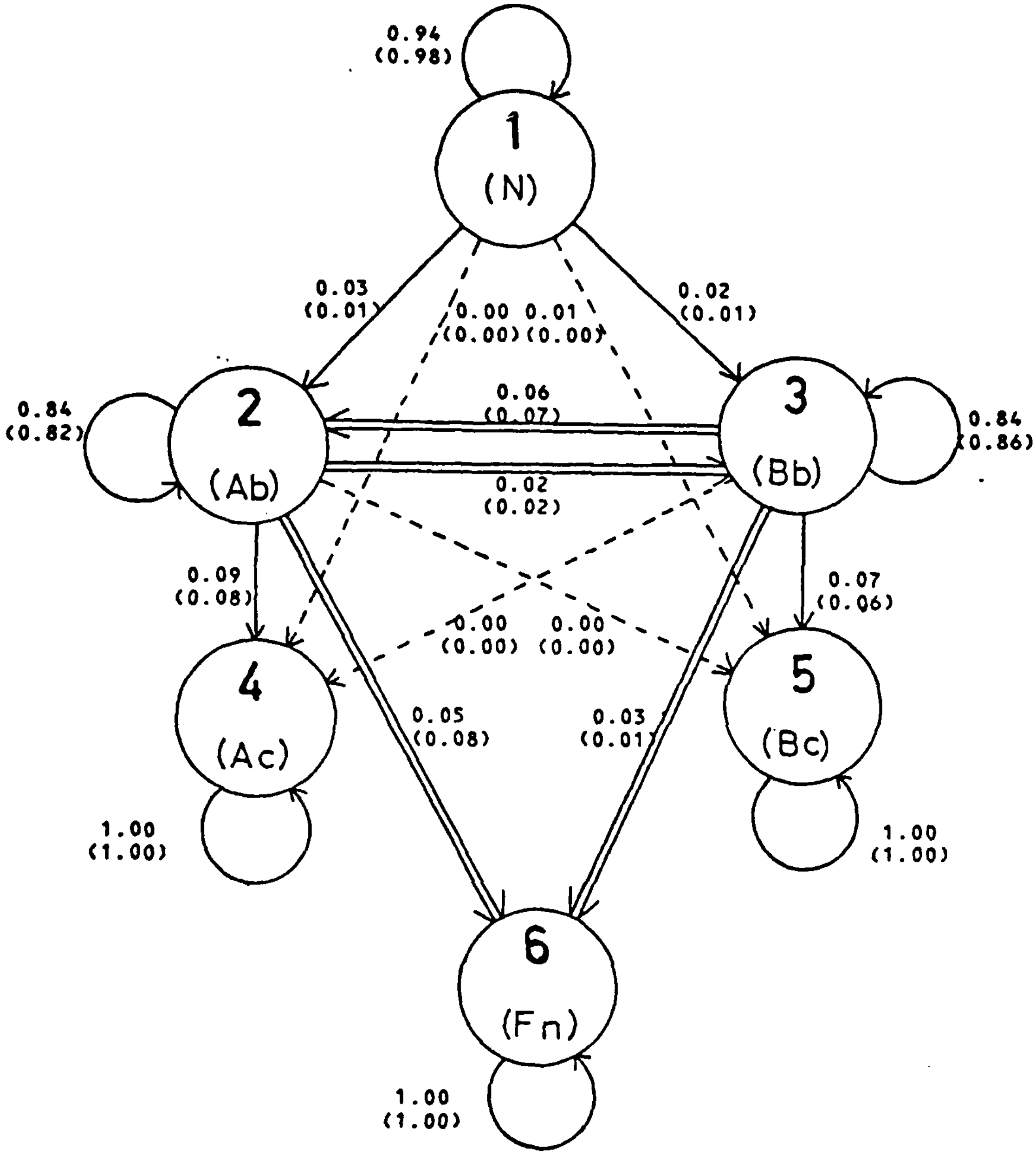

Figure 4 Person state transition diagram, giving the mean estimates for x=1 from 10 simulation runs with the SIMCOM-100 version. In parentheses are values generated by a run with the 400-version. Dotted arrows signify that while such transition values may be yielded, intermediate unrecorded transitions have occurred during the time period in which the in- and outgoing states are recorded. Double arrows mark inconsistency resolution modes.

of a person s(i,j), the former must belong to the set G which, together with s(i,j) defines the "membership group" (Cf. (2)). The values 5,6,7 are determined with reference to the time point t, while the other values are determined on the basis of events during the period between t and t+1, beginning at t. By "A-person" is meant an A-chooser or an A-believer, and by "B-person" a B-chooser or a B-believer.*** Estimates of transition probabilities are based on "observed" transition frequencies, obtained by recording the state change of each one-person member of the population at each time unit. Thus for each simulation run, estimates are given in the protocol of the values in the transition diagram of fig. 4 for the condition x. The values listed in the diagram, are true for all conditions experienced by the populations in the specified runs, for which x=1.

Relations between competing modes of inconsistency resolution may now be studied in terms of their variations with respect to "supportive" and "unsupportive" conditions. Table 2 gives values of transitions between state 2 and 3 (switching between A- and B-believer) and withdrawal from state 2 and 3 (to Fixed neutral) for x=1,2,...,7. It appears that switching or withdrawal occur not only under attitude-discrepant conditions, but also under attitude-supportive conditions. The transition rate for swithcing made by B-believers (Bb,Ab) is 0.11(0.16) under the condition of exposure to A-persons, and 0.7(0.7) when A-persons are group member. as compared to 0.09(0.10) upon exposure to supportive B-persons, and 0.09(0.11) when group members are B-persons. The transition rate of withdrawal by A-believers (Ab,Fn) is 0.33(0.17) and 0.11(0.15) under unsupportive conditions, as compared to 0.23(0.13 and 0.03(0.05) under supportive conditions.

Thus, the following image is generated in the above

*** In the EEC referendum simulation at the national level (12,13), in which these mechanisms are being used, No-persons correspond to A-persons, Yes-persons to B-persons, and uncommitted or home-sitting persons to neutrals.

Table 2 Some person state transition values, representing inconsistency resolution modes, generated on the basis of observations of a simulation run with a SIMCOM-400 and a SIMCOM-100 population

initial population system condition and type of promotion	conditions (x)		inconsistency resolution mode transitions: switching $\overrightarrow{\text{Ab,Bb}}$	switching $\overrightarrow{\text{Bb,Ab}}$	withdrawal $\overrightarrow{\text{Ab,Fn}}$	withdrawal $\overrightarrow{\text{Bb,Fn}}$
model 400 run: 5% initial A-chooser proportion, mass media promoting A and agents promoting B		undifferentiated (x=1)	0.02	0.07	0.08	0.01
	exposure to	A-persons (x=2)	0.00	0.11	0.23	0.00
	exposure to	B-persons (x=3)	0.14	0.09	0.33	0.02
	exposure to	A- & B-persons (x=4)	0.24	0.37	0.12	0.00
	group members	A-persons (x=8)	0.01	0.07	0.03	0.00
	group members	B-persons (x=9)	0.03	0.09	0.11	0.00
	group members	A- & B-persons (x=10)	0.03	0.13	0.16	0.01
model 100 run: 5% initial A-chooser proportion, mass media promoting A more heavily than B, agents promoting A and B		undifferentiated (x=1)	0.04	0.07	0.08	0.08
	exposure to	A-persons (x=2)	0.03	0.16	0.13	0.06
	exposure to	B-persons (x=3)	0.13	0.10	0.17	0.12
	exposure to	A- & B-persons (x=4)	0.21	0.12	0.00	0.12
	group members	A-persons (x=8)	0.05	0.07	0.05	0.11
	group members	B-persons (x=9)	0.07	0.11	0.15	0.00
	group members	A- & B-persons (x=10)	0.08	0.10	0.06	0.13

simulations:

(p_s) Selective search for, and attention to value-supportive messages from personal cocators need not weaken and may even strengthen the conditions for inconsistency, as messages from such sources may result in increased evaluation difference in the seeker, without necessarily allowing for a compensating adjustment of his information difference.

(q_s) Search for, and attention to value-discrepant messages from personal coactors may help to resolve a state of inconsistency, due to their potential for reducing the evaluation difference in the recipient, while having relatively low informational impact.

In spite of various defence mechanisms and weaker resolutions modes, the more drastic modes of transition to a switched state of evaluative preference or to a "fixed neutral" state are made even when the actor is exposed to a person who is supportive of his own evaluative position or where his group of available coactors consists only of supporters. Considering the implemented definition of affective-cognitive consistency (Cf.(4)), one realizes why consequences of this kind may follow: The only effective way of escaping inconsistency in the present model system context is an <u>increase of information difference or reduction of evaluation difference</u>, i.e. increase of $|I_A - I_B|$ or reduction of $|V_A - V_B|$. In other words: The inconsistent actor has to gain additional knowledge supporting his attitudinal preference (or repress knowledge in support of the unpreferred alternative), or modify his attitudinal position by strengthening his evaluation of the unpreferred alternative (or weakening his evaluation of the preferred one). The kind of messages most likely to offer help in decreasing evaluation difference in the present model system context are messages from personal coactors with an <u>opposed</u> attitudinal preference. According to the interpretation mechanism built into the system, <u>personal</u> coactor messages have relatively strong evaluative impact as compared to mass media messages, but relatively low informational impact. The informational impact is least if the recipient is in a state of inconsistency and there is incongruity between his attitudinal preference and that of the sender.

2.2 Relevance to politicians and prophets

The generated image may be assumed to be of relevance to decision-making. The way of getting yourself, qua decision-maker, into a vicious circle is to surround yourself with people thinking and feeling like you, and literally close the doors of your house to people with discrepant evaluations. If you are then forced into withdrawal, while decisions still have to be made, disastrous results may follow. Many such instances can be seen in modern political history.

Competition between the modes of

(p_t) being secret and closed to outsiders, and
(q_t) being open and actively proselyting

is revealed in a group of believers in a prophet. After a disconfirming experience a shift occurred from p_t to q_t. This can be explained in terms of dissonance theory and the above mechanism:

The case is reported by Festinger, Riecken and Schachter (19), who studied it in terms of Festinger's dissonance theory, - one of the most fruitful members of the family of consistency theories (4). A prophet, Mrs. Keech, had received messages from "superior beings" belonging to the planet 'Clarion' warning her that "Lake City" would be destroyed by a flood on December 21. The prophet and her believers would be picked up by a flying saucer and brought to safety prior to the flood. While the Lake City group of believers practiced a cautious policy and kept an air of secrecy towards outsiders before the date of the predicted events, the authors report that proselyting by those retaining their belief increased "meteorically" after that date, when it turned out that the prophet's predictions had failed completely. In sharp contrast to the previous activity pattern, intense communication was initiated with outsiders, including newspaper men and reporters, by almost every member of the group following disconfirmation.

The authors explain this change in terms of dissonance theory (19): When a painful dissonance is generated by the disconfirmation, which cannot be denied or rationalized, then those retaining their belief can reduce the dissonance through making the proposition 'More and more people believe my belief to be correct' come true. Hence the activation of the intense communication pattern directed at outsiders.

However, the activation of the intense communication pattern directed at outsiders, with the manifest function of "proselyting", could have latent functions explainable in terms of the modes implemented and explored above. For those not resorting to the mode of evalution preference switch, i.e. giving up their belief in the prophet, contact with sceptical non-believers would help to modify their evalution in the direction of a state more consistent with their changed knowledge state, brought about by the disconfirmation. In terms of the present set of implemented mechanisms, the disconfirming experience would mean that "own-messages" generated by the believer himself and referring to the prophecy failure would result in an increased amount of stored information in favour of the "our belief is wrong"-alternative, and thus in decreased diffinfo. Escape from intolerable inconsistency without resorting to switching, would lie in getting help to reduce the evaluation distance between the "our belief is right"-position and the "our belief is wrong"-position, ie. obtaining a reduction of diffvalue proportional to the newly reduced diffinfo (Cf.(4)). Non-supportive personal coactors and sceptical media correspondents would provide these kinds of message. Thus activation of an out-of-group symbolic action program may be seen as serving to adjust the evaluative state to the changed cognitive state. This does not prevent the program described by the actors themselves as well as by outside researchers, having the manifest function of acquiring new supporters. Its latent function may still be - if we accept the alternative explanation offered here - to adjust the evaluative state to one more consistent with the changed cognitive state, thereby allowing the evaluative preference to be retained.

2.3 Two simulation studies of an EEC-referendum

The above r(p,q) images generated through computer-operational implementations and simulations may be judged to be merely a twisted operationalization and precis of consistency theory. However, the application of the above set of mechanisms - in original and adapted form, respectively - in two post-dictory studies of empirically surveyed systems (the EEC referendum in Norway at national level (12,13) and community level (15),) has not produced sufficient mismatch between referent systems and model systems behavior to allow for statements of falsification.

In September 1972 Norwegian voters decided the issue 'Yes' or 'No' to Norway's entry into the European Common Market (EEC). In spite of a heavy 'establishment'-campaign in favour of membership during the year preceeding the referendum, 53.5 percent of the votes cast rejected the membership alternative.

A national level simulation of public opinion development, voting outcome, and participation has been carried out in a post-dictory application of a computer program with fixed mechanism parameter values, implemented in 1969, containing the above mechanisms $F_1, F_2, \ldots F_{11}$.

Conditions of influence and promotion are simplified in the simulation run as illustrated in fig.2.1. The Yes-side engages mass media and agents in their promotion campaign, while No-influence only occurs through spontaneous face-to-face contact in local networks. On the other hand, the initial No-share of the population is much higher than the Yes-share when the simulated campaign is started. Gradually, Yes-networks are also formed, and conditions leading to cross pressure and inconsistency produced. In spite of the simulated heavy Yes-campaign, the No-side gains most at the beginning of the simulation period. The establishment-campaign activates intra- and interpersonal processes in the persons and person networks, but does not control the direction of these processes as long as the No-side has strongholds in the population networks. However, there are still large neutral areas to draw from, and the gradually generated Yes-networks allow for a combination of spontaneous and preparatory communication to the advantage of the Yes-side. It is sufficient to gradually reduce the lead of the No-side, but insufficient to overcome it in time for the Referendum (Cf. state in fig.2.2 ,p.11_2).

Development of the No-to-EEC share in the simulated voter population allowed for comparison with vote intention shares according to polls by the two largest Norwegian agencies

(Fakta and Gallup), which were available for mutual comparison from October 1971 and onwards. Setting the time axis in a manner which made the population state match the October polls, the model produced a smoother development than was reflected in the polls (the impact of actual events other than the Yes-campaign was not included in the boundary conditions) and a vote distribution and participation level that closely fitted the referendum outcome (12,13). However, this fit was partly obtained through boundary conditions and time being interpreted in a manner that favoured the model.

In an independently carried out nationwide voter survey on the EEC-referendum, Valen (36) found that cross pressure generated by conflicting loyalties may have affected the level of voter participation. As compared to the previous parliamentary (Storting) election, the most significant decline in voting turnout occurred in communes where the Yes- and No-camps were about equal in strength. This fits grossly with the spatial pattern of distribution produced in the above postdictory simulation. At the referendum time, most of the withdrawn ("fixed neutral") persons were located in the boundary areas between No-camps and Yes-camps areas (Cf. fig.2.2).

A community network panel (22), designed and surveyed from the perspective of the above model, offers supplementary images of relations between competitive modes of cognition and interaction in the same referendum, and has been subjected to postdictory simulation in terms of a set of competing models (15).

The empirical network panel consisted of a set of household and acquaintance networks in the Oslo area, studied in two "waves" (before and after the Referendum.) The empirical data allows for processing at various structural resolution levels (24) - person, household dyad, network and aggregate -, and the relations between these levels. For inconsistent persons who resort to some kind of change, switch to the opposite position has higher priority than withdrawal. Resolution mode priorities seem to vary with the person-vote-intention-state and type of inconsistency - in relation to the various resolution levels: Withdrawal is preferred by No-persons who resort to resolution of inconsistency in relation to a Yes-dominated network or aggregate. However, most of the No-persons who seek resolution in relation to their household members, with opposed or neutral states, prefer switching. Some resort to withdrawal - but do not always remain in a neutral position. Here is an incongruity with the withdrawal mode, implemented in the simulation model, which makes withdrawal an absorbing state. However, the closer in time to the date of the referendum, the more attitudinal positions are "frozen". Withdrawal in the form of staying home and not participating in the election becomes an irreversible mode of resolution. Evidence of this mode is also indicated in the analysis by Valen (36) referred to above.

In order to allow for further exploration and generation of competitive mode images - not only of consistency resolution modes, but also of a predominant consistency (communication) pattern seen as competitive to a predominant economic (transaction) pattern, the network panel survey data were used. They served as a basis for implementation of models that vary with respect to level and theoretical perspective. They are local to a multiple simulation structure (MULTISIM), and serve in the exploration of problems associated with shifts of system definition, level, and time, with reference to systems within a common data source. Two of these models share a consistency perspective, and two others an economic perspective: so far the first three of the four models below have been implemented and subjected to a first run:

Model I is an adaption of the model reported above, and is constructed from an interpersonal communication and cognitive consistency viewpoint. Its mechanisms are activated at the (intra-)person level, utilizing person (dyad and network) and aggregate-state variable values, and directly and indirectly produce changes in these values.

Model II is constructed from a network balance viewpoint, influenced by a graph-theoretical version of cognitive consistency theory. The mechanisms are activated at the network level, utilizing the actual state of each net in terms of balanced and unbalanced dyadic relations, and produce state changes in nodes of the net.

<u>Model III</u> makes use of three postulated sets of state transition matrix values for the person entities as systems, conditional on the position of each person on a socio-economic index. The index is based on age, sex, education and income, and allows for low, medium and high position.

<u>Model IV</u> makes use of a socio-economic transaction perspective at the dyadic level. The "actor" nodes are seen as capable of offering goods and bads in terms of social rewa and punishment according to their position on the above socio-economic index. Thus state changes of the nodes are made in the direction of the states represented by those with higher index positions.

To date, the first three of these models have been implemented as part of a structure in line with the paradigm of fig.1.1, and a first run has been executed. So far problems - rather than images of competive relations - have been generated. Among these are

(a) problems associated with the fact that certain <u>masks</u>, i.e. fixed sets of sampling elements (24), need be chosen in the establishment of a data base. These masks and the applied systems definitions will affect not only the variety of the set of potential models allowed to be constructed, but their respective chances of emerging as explanatory vehicles

(b) problems raised by the necessity of adjusting definition of <u>levels</u> with shifts of model perspective, which then prevent a true comparison in terms of model system behavior,

(c) problems and challenges involved in relating model systems <u>time</u> definition(s) and referent system time,

(d) problems of <u>coordinating</u> rather than unifying the set of models, without the elements loosing their respective identities and viewpoints (15).

These appear to be problems that not only scientists oriented towards a multiple approach, but also a decision-makers face, whenever competition between perspectives is taken seriously. I believe it should be, and that many of these problems invite handling in terms of some general systems paradigm (24) - perhaps extended or adapted.

3. SELF-REFLECTION: POSSIBLY COMPETING MODES OF REFERENCING

3.1 Concepts and techniques for self-reflection

The notion of competitive dual modes may be traced far back in the history of ideas. It can be recognized in the taoistic *yang yin* principle, and in Western culture in the writing of Heraclitus. The latter develops a proposition about conflicting tendencies as a basis for his cosmology, according to which the Furies, representing *logos*, were ready to interfere (in terms of their r-image) in cases where one of the conflicting sides exceeded its "measure". In relatively modern times Hegel (23) makes use of similar notions in his dialectics. He adds an important quality, that of *self-reflection* and self-mediated negation, = the capacity of those involved in relations of contradiction and contrariety to grasp and transcend such relations through self-reflection. In his treatment of master-slave relations Hegel extends his notion of self-consciousness to encompass self-other consciousness. Master and slave are mutually bound and inter-locked. Through a self-reflecting and negating movement on the part of the slave the relationship may be transcended, - a movement in which the former master also has to gain a self-consciousness, in order that they may return to unification of a higher order, in which images are also formed, denied, and transcended through self-mediational movements. Thus, self-reflection may allow for the emergence of a higher-order potentially conscious psychosocial system.

But a peculiar complication arises with the emergence of self-reflection in higher organisms, as pointed out by von Foerster (1966):

> "In which sense reality indeed exists for a self-reflecting organism will become clear by the argument that defeats the solipsistic hypothesis. This argument proceeds by *reductio ad absurdum* of the thesis: "This world is only in my imagination; the only reality is the imaging 'I'."
> ....
> .... for if one assumes to be the sole reality, it turns out he is the imagination of someone else who, in turn, insists that *he* is the sole reality.
>
> The resolution of this paradox establishes the reality of environment through evidence of a second observer. Reality is that which can be witnessed; hence, rests on knowledge that can be shared, that is, "together-knowledge" or *con-scientia*". (p.47)

The development since the 1960s by Pask(31) of his "conv.. sa tional techniques" concerned with the exteriorising of man-compute circuits of mental events that are self-reflective, may be seen as consistent with the above resolving perspective. A consistent technique has also emerged from my own concern with dialogical systems. This is a self-reflective (video-playback) method, involving video-recording and playback directly to small-group participants, in order to elicit - and enforce recall and self-reflecting processes (10). While our memory may be said to serve the function of being a play-back device, it does so in a severely distorting fashion: As impressions are being formed and stored, repression and transformation in consistency directions are already at work. In therapeutic situations the psychoanalyst may serve as a sort of intermediate vehicle for self-reflection, but (s)he does so with a book of interpretation, e.g. Freud's theory, well tucked under the arm. In Mills' approach to self-analytical small groups, video-recording is used and researchers adopt a more passive role in the actual group interaction (33). But they act as intermediaries in interpreting the video-tapes, and their interpretations are fed back to the group, which continues to be self analytical - its sole task and topic. While concerned with self-reflection, Pask's "conversational techniques" and my video-playback technique, explicitly involve a topic or theme in addition to "self" as theme, and allow for direct feedback. Still, my self-reflective (video-playback) method cannot completely escape distortion and selective attention in the directing and focussing of camera lenses. But, as cameras from several angles are in concurrent operation, and an image mixer is used, much of the past is presented to the participants in a manner less distorted than if it were interpreted by some analyst or researcher.

3.2 On meaning-tight systems

In spite of these differences in techniques, there is a clear convergence between Mills' self-analytical approach to small groups, Pask's conversational approach and my dialogical one. They are all concerned with the genesis of what I term

<u>meaning-tight</u> systems (14,16), as a prerequisite for the establishment of knowledge that can be shared, or conscientia (Cf. von Foerster, 38), as well as for the emergence of psycho-logical contradictions and disagreements. The term means the capacity of psychosocial systems - a human individual, a small group or a more encompassing social system - to establish sufficient self-enclosure (in terms of meanings activated in the system) to allow communication - or in Pask's term conversation - inside my mind, or in a social group. Thereby at least two participants establish a sufficiently tight common referent ground for two or more perspectives, thus allowing for interaction, and proceed in terms of these perspectives. Even if disagreement on some issue is to be reached, a meaning-tight platform for that disagreement has to be established. Thus there is an aspect of the meaning-tight system that cannot be described in terms of an energy- or material viewpoint, not even if it is viewed as being variety- or "information-tight" in the sense of Ashby (1). In contradistinction to information quantities, which change their value with repetition and selection, and which do not presuppose two communing participants, the value of a meaning variable need not change with selection or repetition (Cf. Luhmann, 26), and requires at least a <u>dyad</u> of image and symbol processors inside one's head, in a human group, or between man and machine. If the declaration '<u>I see what you mean..</u>' is valid, then there is meaning-tightness, irrespective of whether the declaration be followed up by '<u>... and disagree</u>.' or by '<u>... and agree</u>.'

But why and how is it that such a system can be "<u>tight</u>" or "<u>closed</u>" with respect to meaning, when it may become loose the next instant? G.H.Mead offered an explanation in terms of one's capacity to assume the role of the other (29), that is to carry out mental self-other simulation (10). Clues are also provided by Frege (20), when making a distinction between <u>Sinn</u> and <u>Bedeutung</u>, and by the Socrates dialogues on virtue. Our dialogue may allow us to establish a common reference domain; to close in on and literally define - ie. to draw boundaries that link us together. My seeing the Morning Star and your seeing the Evening Star may lead to the joint discovery of a common domain. Your experience with virtue and my notion of virtue may be enclosed in a manner that allows us to discuss the highest possible virtue. Even though the dialogue proceeds in stages, where previous meanings may be discarded or transcended, enclosure is necessary for proceeding and transcending (Cf.Pask,3 While Ashby's law of requisite variety holds with respect to informational or organizational aspects, one may assert a principle of requisite meaning-tightness to hold for communication processes: A certain degree of closure is necessary in order to proceed - whether the achieved closeness is a building block in achieving understanding, or whether it is used to discard or transcend that which has been established.

3.3. Application of self-reflective video-playback to a "map design"****

Events in the break-down of meaning-tightness in a system of two interacting persons may be demonstrated by a "map design" developed by Blakar (7), which allows for usage of a self-reflective video-playback technique (10).

In the experimental map design for laboratory study of interpersonal communication dyads, the two subjects are each handed maps, which they are given to believe are identical, with theexception of a traveling route marked on one of them. The task is for the actor in possession of this information to communicate the route to his coactor. This defines their symbolic interaction game. The maps may be conceived as the respective coactor-model as they both initially use the situational definition "your map is identical to my map". As the maps actually are not identical, they are bound to fail in their task. That something is wrong is gradually realized in the observed dyads, but it usually takes a certain number of attempts at going through the map before the participants start attributing the causes for failure to themselves or to their coactor. Usually only at a later stage are questions raised as to the adequacy of the respective coactor in-game models, and as to the identity of the maps as possible sources for destroying meaning-tightness with respect to the route. Thus the fact that the maps are not identical, while being assumed to be so, creates a distortion of meaning-tightness in the dyad.

In a series of replication studies of such dyads, I have extended Blakar's map design into a self-reflective video playback

****Claes-Göran Brisendal has assisted me in the laboratory proceedings, which occurred 1973-74.

design. The experiments are conducted in our television studio, with the participants being fully aware of their being video-recorded. One camera follows (from above) their respective map tracing, while two others record their face-to-face contact and gestures. Through a mixer both of these records are concurrent parts of the image fed back to the participants, after they have realized that their communication attempts are futile. Before the video-playback, they are exposed to the following briefing:

> "......now, the really important part of this investigation begins, in which you qua participants may recall and study the conversation you just have been engaged in, and have been developing. Your task was that of communicating something from one of you to the other. You failed. But you were not the cause of that failure. It rests with the map you were handed , and which created the situation you were faced with. It could be depicted as something like this:
>
> (See Fig. 5 - which was shown to the participants at this point)
>
> ....
>
> One of you, who had the map route to communicate to the other, could be depicted as having a mental image of the form "my map is equal to your map, except for your lacking the route which I have on my map". And the other one had a mental image of the situation of the form "Your map is identical to my map". This was your initial definition of the situation. Gradually, as the communication proceeds, you start realizing that there is something wrong somewhere.
>
> ....
>
> and you may begin to ask yourselves certain questions, mentally or overtly: "Is there something wrong with my manner of explaining myself?" "Is it I who am incapable of following the explanation?" Gradually suspicion emerges: there may be something wrong with my map, with the map of the other. And here we are at a critical point at which I would be interested in your help, namely which modes of resolution are you mentally considering in the back of your mind, which ....
>
> ....
> We are daily involved in hundreds of conversations. The interesting aspects of these talks are not only what we are actually saying, concretely what we utters, but also what kind of thoughts, which contents, what kind of reflections we have concerning what we are saying, what we are hearing. Perhaps we may attempt to grasp these processes together. Thus, when you recall some of the things you experienced during this situation, and comment on it during the playback, it may be of value ...."

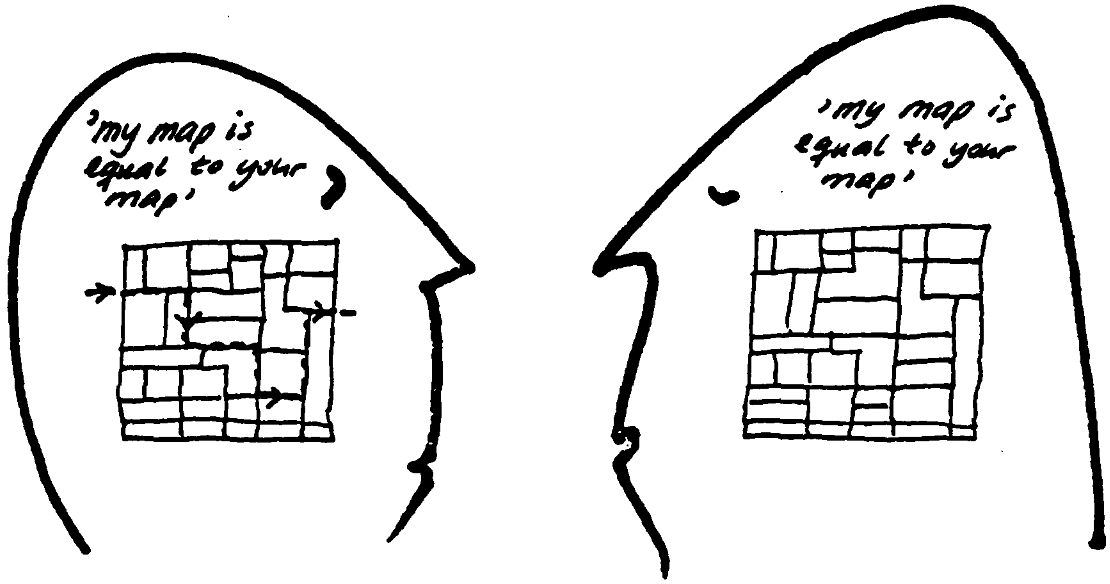

Figure 5 Diagram used in the briefing to participants of map-processing dyad after ~~break~~-down of communication task and before video-playback in order to elicit self-reflection in the dyads

Records of the self-reflecting comments of the participants, have not yet been analysed in depth. But first indications are of two kinds of shift between modes in these systems: There appear to be shifts between modes of map-referring (in the sense of the maps being external to the participants and associated with the experimentor) and modes of articulated self-referencing (in the true sense of self-enclosure) In the latter mode, there appears to be a shift between me-references and us-references, that is, between talking about the dyad qua unit in contradistinction to talking about a member of the dyad.

3.4 Self-reflective circuits in a discussion group

Competition of a similar kind seems to be revealed in a discussion group, that carried on self-reflection through the use of video-playback this spring. It consisted of researchers and graduate students, 6±1 in number. This time I did not have the role of experimentor, only initiator and participant. We ran five sessions on the theme "My computer milieu of yesterday, today and tomorrow". The talks occurred in our television studio. A video-cassett player was used by the group to feed back fragments of the discussion from the previous sessions, as "food" for self-reflection and further discussion. Thus, a situation is created by the group itself, whereby its past is made present, and filled out in a kind of quasi-interaction with the past conversation. The visual situation in one such session is depicted in figure 6.

The played-back sequences sometimes elicit self-reflection with reference to the group qua unit, sometimes with reference to a particular participant. Sometimes modes of self-reflecting interaction were seen to compete with task-instrumental interaction. Sometimes there were disagreements as to whether we should talk about and look at ourselves or go on with the theme. Some of the participants were more interested in the former, while others preferred the latter. But both sides realised that preconceptions about oneself, about the others, and about the actual

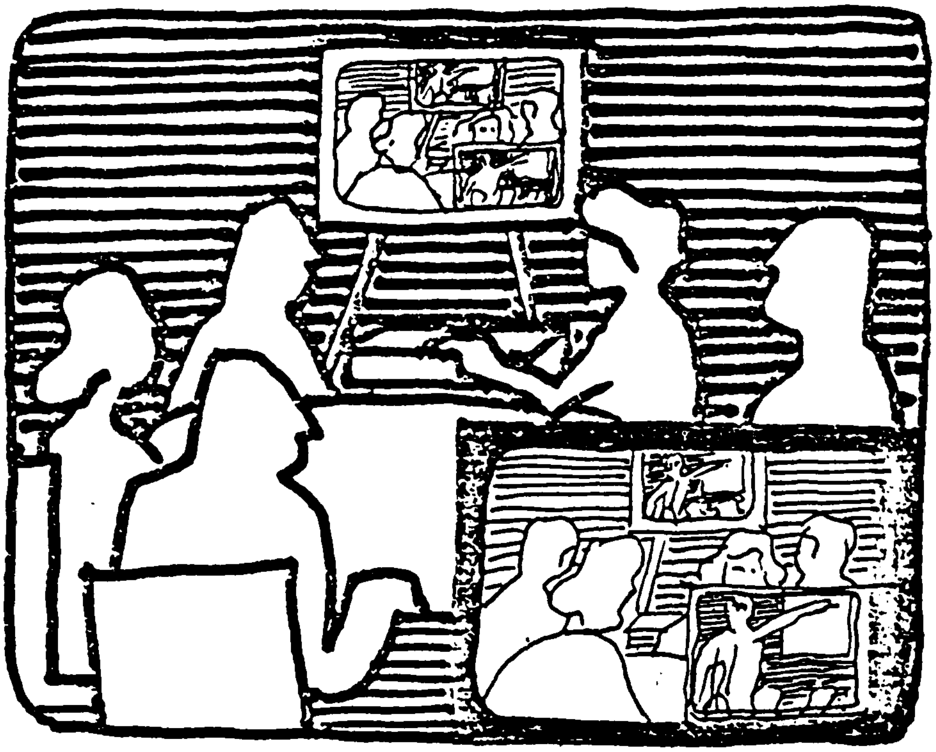

Figure 6 An extreme situation in the self-reflective discussion group, captured on the television screen. Lower right part of screen is used for a close-up of the video-playback image reflected upon in the group, which in turn pictures a self-reflection situation, which in turn....

topic ("Do we really have concepts for comprehending the computerized milieu development?"), were being changed through our steadily being confronted with the immediate past of our proceeding

Sensitivity to competitive patterns of behavior was shown by the group. This is not surprising, as the participants had in part been "socialized" (by me) in thinking and observing in terms of competing patterns, and may thus be said to merely have taken part in realizing a self-fulfilling prophecy. In spite of this, there may be some basis for the following assertions:

(h.1) There may be a continuous competition between self(self)-referencing and topic(self)-referencing in task-processing groups, that is between focusing upon the theme and focusing upon the processor(s)

(h.2) Given the latter mode, there may be shifts between referencing to the group (ourselves-referencing) and referencing to a participant (myself- or yourself-referencing).

Needless to say, the above statements can be made into analytical - and thus tautological - ones, by a certain manner of definition. But questions about conditions for shifts cannot escape being empirical. Competitive tendencies of another kind were indicated in the group. Tension was sometimes seen to arise when meaning-tightness was at its lowest; when one was misunderstood and attacked on false premises. However, periods of self-reflection upon playback of such instances served the function of reducing tension and re-established some kind of meaning-tightness. Attack and criticism were more easily accepted when made upon a platform of meaning-tightness, and not on felt "misunderstanding". The latter image of a competitive relation was offered by one of the participants, and some agreed, while others disagreed.

Whether such kinds of image-generation through the self-reflective playback technique can be extended to groups reflecting other interests and having a different background, as a means of evolving self-consciousness, remains to be seen.

If there appears to be sharp competition of a more general kind between (p) a mode of self-referencing and (q) a mode of topic-referencing (although some topic is always hidden in the former, and self involved in the latter), then insights into conditions and mechanisms for such shifts between p and q, that is images of r(p,q), may perhaps be generated in self-reflective studies of the above kind.

It may well be that self-reflective video playback can be used as a technique for studying decision processes, and as a means for incrementing self-consciousness. This would have special value in situations (industrial democracy) where some participants (power-weak groupings) lack the independent (self) model necessary to reflect their (topic/material) interests. In such cases, exteriorised self-reflective techniques may be a prerequisite for meaningful participation in decision making.

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VOLUME 5

ON THE SPONTANEOUS EMERGENCE OF DECISION MAKING CONSTRAINTS IN COMMUNICATING HIERARCHICAL SYSTEMS

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by

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Contents

Abstract

In a self-organizing system a new constraint is emerging as a consequence of a bifurcation - which may give rise to the appearance of a higher hierarchical level in a multihierarchical organism.
In this paper the communication process (leading to the emergence of such constraints) between two hierarchical systems-each possessing initially two levels-is dealt with. The dynamics at all levels are modelled as finite state parametrized Markov chains. For the lower levels Q,Q′ the transitions are parametrized on control variables stemming from an underlying game-which simulates collectively all hierarchical levels below Q and Q′ respectively. The higher levels W,W′ are modelled by semi-Markov chains with holding times following a geometrical distribution. The control parameters upon which the transition probabilities at the levels W,W′ depend are:

1) Collective properties of the dynamic deliberations on levels Q,Q′ which measure (a) the percentage of occupancy of a state selected apriori as homeostatic and (b) the crosscorrelation(s) between the state sequences of the levels (Q ,W′) or (Q′,W) respectively and ,
2) The holding time statistics.

The feed forward control is exercised from the higher levels W and W′ towards the lower levels Q,Q′ correspondingly in order to modify the parameters of the underlying game with the objective of maximization of an additive (or multiplicative) "Figure of Merit" ensuring an optimum compromise between two conflicting drives: homeostatic tendency versus good crosscorrelations with the communicating partner.
Since an exhaustive search over all possible control laws (codes or "maps") between (W,Q) and (W′,Q′) is prohibitive and a search over

the members of a randomly selected small subset of codes only yields to an "existentialistic" approach, we examine in this paper the possibility of devising dynamics which will give rise to plausible heuristic control rules - in the form of feed forward negative control commands precipitated from a new, higher hierarchical level. The objective of this work then is a search for the adoption of an optimum common code which will map in a hierarchical fashion each partner's experience (Q,Q') and behavior (W,W'). Such a code would maximize the "Joint Figure of Merit" of the partners or it would optimise (i.e synchronize) their sequential behavioral mode turnover under the (conflicting) requirements of the specific situation.

I/. Introduction

In recent years the explosive accumulation of experimental results in various fields of behavioral sciences has started to challenge the General Systems-oriented researcher with appealing problems of Mathematical modeling. The central issue involves the search for codes which map levels of stored experience into behavioral levels and vice versa in multihierarchical systems (with emphasis on higher biological organisms). In particular the realm of psychopathology of human interactions offers many such opportunities.

One is reminded of the correlative ways in which "neurotic" experiences interrelated with misuse of symbolic language [due to the confusion between the hierarchical levels where contradictory statements and metastatements properly belong] are instrumental in further disorganising the levels of stored experiences, and altogether paralyse behavior. (Bateson et al.,1956, Sluzki, 1976).

The point worth noting here is the correlation (albeit weak) between physiological and psychological variables [e.g Hill, 1976, Lipowski et al. 1976, Serban, 1976].

In our work such stochastic dependencies are isomorphically incorporated in the model which follows i.e they determine the way that successive hierarchical levels in the individual systems ("organisms") interact.

The extent and significance of such modelling must be clearly indicated from the very start: The purpose of such Cybernetic models is the coherent and economical presentation of a rather extensive amount of phenomena - without attempting at the present time validation of any aspect of behavior.

In fact it is not clear at all what is to be "explained" in human behavior - characterised by given sequences of Decision Making acts : Humans - in contradistinction to most animals - do not in general possess preprogramed responses to externally impinging stimuli (e.g "instincts") and therefore cannot be treated as "black boxes"; that is they cannot be identified (i.e predicted and controlled) by the usual procedures of exploration and input - output modeling (Witten, 1978). What we are aiming at therefore is to put forward plausible isomorphisms or abstract metaphors concerning the dynamics of some modes of human communication taking place between homologous pairs of hierarchical levels of the partners concerned.

There exist two (coupled) prerequesites for meaningful* transactions that a hierarchical system undergoes vis - a - vis its "environment" (which may be just the communicating partner plus the "sociocultural milieu").

1) Reliable internal representations of the environment (mediated intraneuronally in ways discussed recently (e.g. Nicolis et al, 1976,1977)

2) Fast Decision Making processes - of the many possible behaviors activated by simultaneously triggered internal representations, these processes allow the organism to examine, in a sequential fashion, one behavioral model at a time.

In what follows we intend to develop a rather detailed model of this second aspect.

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* "Meaning" is defined here as the (positive) crosscorrelation between <u>symbol</u> and <u>message</u> or between Internal pattern representation and (external) stimulus.

II / Formulation of the problem

Inability to make fast and reliable Decisions underlies much of what is known today as psychopathology of (human) adaptation. Like all biological systems man is bombarded simultaneously by many external (or internal) stimuli. Each of these stimuli has to be cognized i.e matched with some preestablished dynamic pattern at the proper hierarchical level of the organism. The result of each elementary crosscorrelation-cognition- may lead to some efferent response (or action) of the organism concerned towards his environment-what we call observable behavior. Obviously,

in order to remain behaviorally coherent and survive the organism can only elicit one behavioral unit at a time. This means that, of the multitude of cognitions simultaneously processed at any given hierarchical level of the organism, the animal has to perform a non-linear filtering i.e he has to suppress all but one behavioral mode at any given time. This Decision Making process is based on objective criteria (or"values") aiming at the maximization of some "Figure of merit", which in an optimum way compromizes (or attributes in a preselected way different weights) between various conflicting and sometimes mutually exclusive drives or motivations (eg. fight v.s. flight, homeostasis v.s. Learning and evolution etc). Unlike all animals, man in general lacks "instincts" i.e preprogrammed responses, or "subroutines"- automatically selecting the appropriate behavior to specific stimulus. Against stimuli for which they are not preprogramed, animals display either indifference or inability to cope. In contradistinction humans have to learn everything from "scratch".

Out of a set of stimuli whose existence he cannot help but acknowledge man has to suppress a great deal- those which he feels impotant to respond to in a rewarding way. This is a first order selection. Out of a (rather small) remaining subset of stimuli man has to decide which one will be dealt with how, and when. It is obvious that the sharpness of the built - up internal representations or prestored dynamic patterns is the key issue underlying reliable crosscorrelations with cues impinging from the environment as well as viable Decision making. The quality of these internal representations in turn depends on the coding algorithm which has been used for their storage and retrieval.

The lower levels Q, Q' of the communication partners contain dynamically stored "internal representations" of some aspects of the environment, which for each system is just the dynamics of the higher level of the partner W', W respectively. The levels W, W' stand for behavior. The lower levels Q, Q' ("deep structures" with emotional endowment) send, via afferent channels, to the higher levels collective properties. These have to do with the degree of occupancy of an apriori selected as homeostatic state and the cross correlation between the state sequences of the levels (Q, W') or (Q',W) respectively.

At the levels W, W' decision making on which behavior to present to the partner is taking place.

Also, the higher levels W, W' act as stochastic controllers sending efferent feedforward signals to the lower levels.

In this paper (as will be described below) we are mainly interested in designing the Decision making process for behavioral turnover, as well as the feedforward controls, in such a way as to guarantee a delicate balance between conflicting parameters: Homeostasis and Learning. A (heuristic) constraint will be also imposed on the interaction: it has to do with the objective of Maximum Consonance (in conformity with the theory of Cognitive Dissonance Festinger, 1957) between the pairs (Q,W) and (Q', W') respectively. This objective requires that behavioral states at the levels W, W' which do not correspond to states at the levels Q,Q'

respectively should be progressively eliminated i.e. endowed with time decreasing probabilities of recurrence.

A question worth posing is whether or not the above heuristic constraint leads to codes corresponding to high values of joint figure of merit.

The communication process between the two hierarchical systems will be pursued as a bidirectional information transaction where the lower levels play the role of receivers and the higher levels play the role of transmitters-standing for "experience" and "behavior" respectively.

The evolution with time of the behavioral mode (state) switching at the levels W and W' will be displayed for specific feedforward control laws, conforming with the above mentioned objective and constraints

After these preliminaries let us now describe the model in some detail (Fig.1α,β).

At each hierarchical level we start by introducing a continuous state space description involving a number of dynamic variables and a set of parameters pertaining to the particular level.

III/ <u>Continuous Description</u>.

<u>The level Q of partner I</u>.

The dynamics at level Q are described by the Ito stochastic differential equation

$$d\underline{q}_1 = \Gamma_1(\underline{q}_1;\underline{v}_1;t)dt + \Gamma_1(\underline{q}_1,\underline{v}_1;t)d\underline{\mu}_1$$

where:

$\underline{q}_1$:is the state vector for the level Q

t:is the time

$\underline{v}_1$:is the control vector to be specified later

$\underline{\mu}_1(t)$: is a Wiener-Lévy process with autocorellation

$E\{\underline{\mu}_1^T(t_1)\underline{\mu}_1(t_2)\} = M\min(t_1,t_2)$

where M is a positive definite convariance matrix

$\varphi_1(\underline{q}_1;\underline{v}_1,t)$: is a vector function describing the dynamic structure of level Q parametrized on v_1, which incorporates all the underlying game parameters

$\Gamma_1(\underline{q};\underline{v}_1,t)$: is a matrix function parametrized as above

The evolution of the joint probability density function of the state vector components is described by the Fokker-Planck--Kolmogorov equation [Nicolis, Protonotarios and Lianos, 1975].

Similarly for the Q' of partner II we have the Itô stochastic differential equation.

$$d\underline{q}_2 = \varphi_2(\underline{q}_2;\underline{v}_2,t)dt + \Gamma_2(\underline{q}_2;\underline{v}_2,t)d\underline{\mu}_2$$

where $\underline{q}_2$ is the state vector for the level Q'.

Homeostasis and Cross-correlations.

In the phase space of level Q (or Q') we single out a region $H_1 (H_2)$ characterized as homeostatic: it is the region within which the activities at the level Q (Q') should evolve preferentially but not exclusively in order to achieve the intrinsic regulation of the system at the level concerned. On the other hand in order to ensure adaptability to the partner the organism should experience high cross-correlations between the incoming set of triggers $\underline{S}_2(t)$ induced by the higher level W' of the partner and the state vector $\underline{g}_1(t)$. For any given set of triggers emanating from the partner the above requirement for high crosscorrelation implies a rather extensive use of the trajectory at the level Q(Q') However such an extensive "*wandering*" *in the state space* jeopardizes the requirement for homeostasis.

In general therefore the two basic deliberations for each partner (namely the homeostatic necessity and the tendency for continuous adaptability to the partner's cues) seem to be in conflict. The higher levels W(W') act as stochastic controllers aiming at **ameliorating** the above conflict.

Let $u_1(t)$ be the probability that at time t the state $\underline{g}_1(t)$ lies within the homeostatic region H_1, i.e.

$$u_1(t) = P[\underline{g}_1(t) \in H_1] = \int_{H_1} p[\underline{g}_1;\ t\ ]d\underline{g}_1$$

where $p[\underline{g}_1;t]$ is the joint probability density function of the components of state vector $\underline{g}_1(t)$.

The cross-correlation between signals from the level W' and the state of level Q is given by:

$$r(t,\tau) = E\{\underline{s}_2^T(t+\tau)\underline{g}_1(t)\}$$

where T/is the transpose of a vector and τ is a delay parameter, fixed throughout the interaction of the two systems, e.g. $\tau=0$. Note that the vectors $\underline{s}_2$ and $\underline{q}_1$ have the same dimensionality. What are afferently reported from the level Q to the level W are time averages, approximating the above. Namely let T be the duration of the time window $I = \{t': t - T \leq t' < t\}$ then:

Homeostasis: $u(t)$ = percentage of time that $\underline{q}_1(t') \in H_1$ (homeostatic region) for $t' \in I$.

cross-correlation $$r(t,\tau) = \frac{1}{T} \int_{t-\tau-T}^{t-\tau} \underline{s}_2^T(\lambda+\tau)\underline{q}_1(\lambda)\,d\lambda$$

where we select either $\tau = 0$ or τ such that $\Sigma\, r(t,\tau)$ is maximum.

The level W of partner I

The dynamics at level W are described by the following stochastic differential equations.

$$d\underline{w}_1 = \underline{f}_1(\underline{w}_1;u,r)dt + G_1(\underline{w}_1;u,r)d\underline{\xi}_1$$

$$\underline{s}_1 = \underline{h}_1(\underline{w}_1;u,r)$$

where

$\underline{w}_1(t)$ is the state vector at level W.

$\underline{\xi}_1(t)$ is a Wiener-Levy process accounting for the intrinsic **spontaneity** of the system at the level W

$\underline{s}_1(t)$ is the observable behavior which acts as a trigger for the partner

The controller

The feed forward control mechanism for each system has as inputs the homeostasis and correlation afferent signals u and r as well as the state vector w_1 of the higher level W. So the state equations for the controller are:

$$\dot{\underline{v}}_1 = \underline{g}_1(\underline{v}_1, \underline{w}_1, u, r)$$

where $\underline{g}_1$ is a vector function to be specified so that the objective function (joint figure of merit) $\mathcal{J}$ (Nicolis, Protonotarios and Voulodemou 1977) is maximized: $\mathcal{J}$ is defined as follows:

$$\mathcal{J} = \mathcal{J}_1 \mathcal{J}_2 \qquad \text{(multiplication criterion)}$$

where,

$$\mathcal{J}_1 = \lambda_1 E[u_1] + (1-\lambda_1)E[r_1]$$

$$\mathcal{J}_2 = \lambda_2 E[u_2] + (1-\lambda_2)E[r_2]$$

$$\lambda_1, \lambda_2 \in (0,1).$$

or,

$$\mathcal{J} = \sigma_1 \mathcal{J}_1 + \sigma_2 \mathcal{J}_2$$

The characteristic of a multiplicative figure of merit is that it strongly favors parity between the partners while the additive criterion gives similar value for equal partners and for a fully dominating partner.

In the computer simulation (the results of which we present at the end of the paper) we use the additive criterion.

IV/. Discrete Model Description

In this work for reasons of simplicity we quantize the state space at each hierarchical level and admit only a finite number of states which stand for the salient sub-traits at the level involved. Thus the stochastic non-linear differential equations corresponding to the continuous state description are now replaced at all hierarchical levels by discrete time Markov chains characterized by appropriate transition matrices P_{ij} fully describing the transitions between the possible states of the systems at the levels involved.

The lower-levels Q, Q' are modelled as four-state Markov chains which are deduced as outcomes of respective antagonistic processes or "games" (J.S.Nicolis, E.N.Protonotarios and I.Voulodemos, 1976) going on between pairs of dynamic agents; these games simulate collectively all hierarchical levels below Q and Q' respectively.

At the levels Q, Q' one of the available four states will be considered as the "homeostatic" state(s); they are the states towards which the activities at the levels Q, Q' should evolve preferentially but not exclusively-in order to achieve the instrinsic regulation of the organism at the levels concerned. On the other hand in order to ensure adaptability via learning the organisms at the same level(s) Q, Q' should communicate adequately with each other i.e form high crosscorrelation(s) between the incoming set of triggers induced by the higher level(s) of the partner (W', W) and prestored dynamic patterns at the base levels(s) Q,Q'. For any given set of triggers emanating from the partner the above requirement for high crosscorrelation implies a rather extensive use of the repertoire of states available at the level(s) Q, Q'. However such an extensive "wandering" between individual states of the systems at the levels Q, Q' would certainly jeopardize the requirement for homeostasis. So, in general the two basic deliberations of each organism namely the homeostatic nesessity and the tendency for continuous adaptation to the partner's cues, seem to be in conflict.

The higher levels W, W' are modelled by semi-Markov chains of eight states. The reason for introducing the scheme displayed in Fig.1β has to do with the tacit assumption that the process of cognitive dissonance reduction has been already achieved, so that "words" belonging to Q, Q' and W, W' levels respectively are expressed by the same number of bits and therefore do not differ more than 3 bits. The transitional probabilities at the W, W' levels are functions of a) the percentage of occupancy of the homeo static state of underlying level (u), and b) the correlation (r) between the state sequences of the levels (Q,Q') or (Q',W) respectively. The holding time at the states of levels W, W' follows a geometrical distribution with mean value inversely proportional to the "degree of organization", or the Redundancy of the underlying levels (as will be discussed in the following sections). Upon receiving the above data u,r,u',r' the higher levels, hold the existing state for some time and then a transition occurs. The new state is emitted (as "behavior") to the partner in a three-bit binary symbolic code. On the other hand, depending on the received values r and u, r', u', efferent control commands are precipitated to the underlying game(s) which alter the amplitudes and phases of their (Rythmically varying) parameters respectively according to some control algorithm. The new game parameters will influence the transition probabilities at the level(s) Q, Q' thereby altering the homeostatic probability as well as the crosscorrelations (Q',W) and (Q,W'). This procedure repeats itself in successive cycles, asserting the communication between the partners.

There are many regimes of communication. In this paper we restrict ourselves to the cooperative regime of communication, whose goal is to achieve the intrasystemic code (or map) $W \rightarrow Q$, $W' \rightarrow Q'$ maximizing the joint "Figure of merit". This function weights in preselected proportions the homeostasis probabilities and the crosscorrelations (W,Q'), (W',Q). As a measure of the quality of communication we study the synchronization between the behavioral state turnover at the level W, W' as it evolves with time.

V/. The dynamics at the base levels Q, Q' and the underlying game.

Let us try to deduce the dynamic activity at the base levels Q,Q' -as an "off-shooting" of an underlying game. In each case we postulate the existence of two dynamic agents I, II involved in alternating plays (Fig. 1B). On every trial of the game each agent makes either a "cooperative" (C) or a "defective" (D) move. If both agents defect their probabilities of winning are zero.

If both agents cooperate, agent (II) "wins" with probability γ and agent (I) "wins" with probability $1-\gamma$, where "winning" corresponds to dominance of the agent concerned on the system's deliberations at the level Q.

If agent (I) cooperates and agent (II) defects the latter has a probability $\underline{a}$ of winning and the former a probability $\underline{1-a}$

of winning. If finally agent (I) defects and agent (II) cooperates the former wins with probability β and the latter with probability $1-\beta$.

We further assume that:

1) If an agent makes a winning move then, with probability one, he repeats that move on the next trial and,
2) If an agent makes a losing move then, with probability ρ, he will make the alternative move on the next trial.

We next intend to express the sixteen elements P_{ij} of the transition matrix at Q as functions of the parameters $\alpha, \beta, \gamma, \rho$. This has been done in a previous paper (Nicolis, Protonotarios and Voulodemou, 1977) and in Figs 2,3,4,5 we display the corresponding (self-explanatory) tree-structure diagrams.

From the knowledge of the transition matrix we can further compute the likelihood u_i of being in the state S_i $(i = 1,2,3,4)$

VI/. A semi-Markov chain model for the hierarchical levels W and W'.

The higher hierarchical levels W, W' are endowed with dynamics simulated for example by an 8-state Markov chain model. Each of the above dynamic processes-displaying the behavior of the first system-plays the role of "environment" for the partner-system.

We associate with each state W_i, W'_i of these levels a three-digit "word" i.e. 000, 001, 011, 111, 100, 010, 110 and 101. A (different) subset of four of these "words" is associated with the four states S_1, S_2, S_3, S_4 and S'_1, S'_2, S'_3, S'_4 of the two partners at the levels Q and Q' respectively. In this discrete form, the degree of "similarity" between synchronous individual states of each system at the level Q', or Q respectively is given by the expression

$$r_i' = 1 - \frac{D_i'}{3} \qquad \text{or,} \qquad r_i = 1 - \frac{D_i}{3}$$

where D_i, (or D_i') is the distance between the corresponding "words" i.e. the number of digits by which these two words differ.
The modelling processes at the levels W, W' concern the parametrization of the transition elements P_{ij}. We envisage them as follows: Let us consider the Markovian chain(s) at the level(s) W (or W') connected as in Fig 6.

Only successive states communicate from 1 to 8.

Let us to assign appropriate transitional probabilities as functions of u and $r = 1 - \frac{D}{3}$, so that certain intuitive postulates relative to the nature of the level(s) W (or W') are met.

For instance, in the human communication context, in case W, W' stand for "emotionally driven" behavioral levels, the string of states from 1 to 8 may emulate a scale ranging from "Depression" to "Euphoria" or, behaviorally speaking from Catatonia to Hyperactivity. In such a paradigm it seems appropriate that "ascending" transitional probabilities P(k,k+1) (k=1,....,7) (i.e sequential shifting 1 → 8 from "Catatonia" to Hyperactivity") should be increasing functions of the crosscorrelations (r) with the partner-environment and decreasing functions of the homeostasis level (u).
For example we may put $P(k,k+1) = P(r,u) = C_\varkappa\left[1-e^{-\nu_\varkappa r}\right]e^{-\varepsilon_\varkappa u} + \pi_\varkappa$ where $k = 1,2,\ldots,7$, $0 \le \pi_\varkappa \le 1$, $C_\varkappa \le 1-\pi_\varkappa$ and $\nu_\varkappa$, $\varepsilon_\varkappa$ are positive numbers. $\pi_\varkappa$ corresponds to an "intrinsic" spontaneity of jumping to the next state - in the absence of triggers transmitted from the partner.

Like wise, we may take for the "descending" transition probabilities the following r,u - parametrization.

$P(k,k-1) = P(r,u) = d_\varkappa\left[1-e^{-\mu_\varkappa u}\right]e^{-\lambda_\varkappa r} + q_\varkappa$ (where $q_\varkappa$ has a similar meaning as $\pi_\varkappa$ above) $k = 2,3,\ldots,8$, $0 \leq q_\varkappa \leq 1$, $0 \leq d_\varkappa \leq 1 - q_\varkappa$ and $\mu_\varkappa, \lambda_\varkappa$ are positive numbers.

Finally the probabilities of remaining in the same state are calculated as:

$$P(k,k) = 1 - P(k,k+1) - P(k,k-1) = (k = 2,\ldots\ldots,7)$$

$$= 1 - \pi_\varkappa - g_\varkappa - C_\varkappa\left[1 - e^{-\nu_\varkappa r}\right]e^{-\xi_\varkappa u} - d_\varkappa\left[1 - e^{-\mu_\varkappa u}\right]e^{-\lambda_\varkappa r}$$

$P(1,1) = 1-P(1,2)$ and $P(8,8) = 1-P(8,7)$.

Similar things hold for the deliberations on the level W'.

We now introduce a second kind of parametrization on the transition elements P_{ij} at the levels W and W' as follows: When the process at the level W enters a state i, we know that it determines the next state j to which it will move according to the state's i conditional probabilities $P_{ij}(r,u)$ - as described above.

However, after j has been selected, but before making this transition from state i to state j, the process, we imagine, "holds" or "gets stuck" for a time τ_{ij} in state i. The holding times are positive, integer - valued random variables each governed by probability mass function $h_{ij}(\tau)$ - called the "holding time density function".

The probability that the system at W will spend τ time units in state i if we do not know its successor state is

$\Lambda_i(\tau) = \sum_{j=1}^{N} P_{ij}h_{ij}(\tau)$ where N is the number of all successor states.

We call τ the "waiting" time in state i and $\Lambda_i(\tau)$ the waiting time probability mass function.

Let $P_{ij}(\tau)$ be the probability that a process which is now in state $\underline{i}$ and which will make a transition out of state i at time $\underline{\tau}$ will make that transition to state $\underline{j}$. Thus the $P_{ij}(\tau)$ are transitional probabilities <u>conditioned</u> by holding time; we call them "Conditional" transitional probabilities.

The waiting time distribution $\Lambda_i(\tau)$ and the set of conditional transition probabilities $P_{ij}(\tau)$ provide a complete alternative definition of the semi-Markov process.

So, if the process is originally described (as in the present case) in terms of P_{ij} and $h_{ij}(\tau)$ we compute

$$p_{ij}(\tau) = \frac{P_{ij}h_{ij}(\tau)}{\Lambda_i(\tau)} = \frac{P_{ij}h_{ij}(\tau)}{\sum_{j=1}^{N} P_{ij}h_{ij}(\tau)}$$

What is the reason for including in the dynamics of higher levels W, W' this holding time distribution-dependence?

The holding time-concept has to do, operationally speaking, with the "jerkyness" of the "clocking" mechanism activating the Markov chain(s) ie. the mechanism responsible for the turnover of vehabioral modes (states). We postulate that this clocking mechanism (some sort of a master pacemaker) becomes inactivated within time intervals (holding times) which increase with the degree of disorganization at the lower levels Q and Q' for the respective systems. More precisely, we postulate that any (random) imbalance among the concentrations of key substances (e.g. hormones, neurotransmitters) α, β, γ, α', β', γ' or differential concentrations ρ, ρ' externally or internally induced, may change the elements of the transitional probabilities P_{ij} at the lower levels Q or Q' in such a way that the redundancy (degree of organization) at this level will fluctuate concomitantly.

The redundancy at the level Q is defined as $R_Q = 1 - \frac{H_Q}{H_{max}}$ where $H_Q = \frac{1}{\tau} \sum_{\nu=1}^{\tau} H_{i\nu}$ is the time average entropy or uncertainty per state, at the level Q during the previous holding time, $H_i = - \sum_{k=1}^{4} P_{ik} \log_2 P_{ik}$ is the uncertainty of the state $i (i=1,2,3,4)$ and $H_{max} = \log_2 4$; i_ν stands for the index of the state at the particular moment ν.

The same imbalance mentioned above between the concentrations of key substances (or "game parameters") may be responsible for the temporal desynchronization or inactivation of the master pace-maker-thereby introducing holding times at the higher level(s) W, W' correlative with the drop of Reduncancy at the lower levels.

Let us now define the holding time probability mass function as a geometrical dustribution $h_{ij}(\tau) = \eta_{ij}(1-\eta_{ij})^{\tau-1}$ where $0 \leq \eta_{ij} \leq 1$ stands for the conditional probability of staying in the state i at the higher level W exactly one unit time before commuting, given that the transition is from state i to state j.

The mean value of the above distribution is $\frac{1}{\eta_{ij}}$ and its variance is $\frac{1-\eta_{ij}}{\eta_{ij}^2}$

We now calculate easily the elements $P_{ij}(\tau)$ of the Markovian dynamics at the level W (similar things hold for the level W').

It is reasonable to postulate - as far as the holding time dependence is concerned - that $P_{ij}(\tau)$ should be a decreasing function of τ as one moves from "Catatonia" to "hyperactivity" and an increasing function of τ as one moves from "hyperactivity" to "Catatonia". This means that the longer the system at W stays in a "Depressive" state the more probable this regime becomes; the longer the system stays in an "Euphoric" state, the more probable is the switch to a less Euphoric or more Depressive state. It is easy to check that the selected h_{ij} fulfill the above constaints.

To demonstrate this we write:

$$\eta_{i(i+1)} = \eta_d \quad \text{for} \quad i = 1,2,\ldots\ldots,7$$

$$\eta_{i(i-1)} = \eta_\mu \quad \text{for} \quad i = 2,3,\ldots\ldots,8$$

$$\text{and} \quad \eta_{ii} = \eta_0 \quad \text{for} \quad i = 1,2,\ldots\ldots,8$$

assuming $\eta_\mu < \eta_o < \eta_d$

So the relationship between the corresponding conditional holding times will be $\frac{1}{\eta_\mu} > \frac{1}{\eta_o} > \frac{1}{\eta_d}$

The holding time dependence on the Redundancy of the lower hierarchical level R_Ω is introduced in our example as follows :

$$\eta_\mu = \frac{1+R_\Omega}{36}$$

$$\eta_0 = \frac{1+R_\Omega}{30}$$

$$\eta_d = \frac{1+R_\Omega}{24}$$

We write now,

$$h_{i(i+1)}(\tau) = \eta_d(1-\eta_d)^{\tau-1} \quad \text{for } i = 1,2,\ldots.,7$$

$$h_{i(i-1)}(\tau) = \eta_\mu(1-\eta_\mu)^{\tau-1} \quad \text{for } i = 2,3,\ldots.,8$$

$$h_{ii}(\tau) = \eta_0(1-\eta_0)^{\tau-1} \quad \text{for } i = 1,2,\ldots.,8$$

$$h_{ij}(\tau) = 0 \quad \text{for} \quad (j-i)(j-i-1)(j-i+1) \neq 0$$

So, the following expressionsare deduced for the holding time-conditioned transitional probability elements:

$$p_{i(i+1)}(\tau) = \frac{P_{i(i+1)}h_{i(i+1)}(\tau)}{P_{i(i-1)}h_{i(i-1)}(\tau)+P_{ii}h_{ii}(\tau)+P_{i(i+1)}h_{i(i+1)}(\tau)}$$

$$= \frac{P_{i(i+1)}}{P_{i(i-1)}\frac{\eta_\mu}{\eta_d}\left[\frac{1-\eta_\mu}{1-\eta_d}\right]^{\tau-1} + P_{ii}\frac{\eta_0}{\eta_d}\left[\frac{1-\eta_0}{1-\eta_d}\right]^{\tau-1} + P_{i(i+1)}}$$

For $i = 2,3,\ldots\ldots,7$ and. $\tau = 1,2,3,\ldots\ldots$

This is as postulated above a decreasing function of τ for

$\eta_\mu < \eta_o < \eta_d$

We also have:

$$p_{12}(\tau) = \frac{P_{12}h_{12}(\tau)}{P_{11}h_{11}(\tau)+P_{12}h_{12}(\tau)} =$$

$$= \frac{P_{12}}{P_{11}\frac{\eta_0}{\eta_d}\left[\frac{1-\eta_0}{1-\eta_d}\right]^{\tau-1} + P_{12}}$$

Which is a decreasing function of τ.

For descending transitions we have:

$$p_{i(i-1)}(\tau) = \frac{P_{i(i-1)}h_{i(i-1)}(\tau)}{P_{i(i-1)}h_{i(i-1)}(\tau) + P_{ii}h_{ii}(\tau) + P_{i(i+1)}h_{i(1+1)}(\tau)}$$

$$= \frac{P_{i(i-1)}}{P_{i(i-1)} + P_{ii}\frac{\eta_0}{\eta_\mu}\left[\frac{1-\eta_0}{1-\eta_\mu}\right]^{\tau-1} + P_{i(i+1)}\frac{\eta_d}{\eta_\mu}\left[\frac{1-\eta_d}{1-\eta_\mu}\right]^{\tau-1}}$$

$i = 2,3,\ldots\ldots,7$

This is an increasing function of r for $\eta_\mu < \eta_o < \eta_d$

$$P_{87}(\tau) = \frac{P_{87}h_{87}(\tau)}{P_{87}h_{87}(\tau) + P_{88}h_{88}(\tau)} =$$

$$= \frac{P_{87}}{P_{87} + P_{88}\,\dfrac{\eta_0}{\eta_\mu}\left[\dfrac{1-\eta_0}{1-\eta_d}\right]^{\tau-1}}$$

Finally,

$$P_{ii}(\tau) = \frac{P_{ii}h_{ii}(\tau)}{P_{i(i-1)}h_{i(i-1)}(\tau) + P_{ii}h_{ii}(\tau) + P_{i(i+1)}h_{i(i+1)}(\tau)}$$

$$= \frac{P_{ii}}{P_{i(i-1)}\,\dfrac{\eta_\mu}{\eta_0}\left[\dfrac{1-\eta_\mu}{1-\eta_0}\right]^{\tau-1} + P_{ii} + P_{i(i+1)}\,\dfrac{\eta_d}{\eta_0}\left[\dfrac{1-\eta_d}{1-\eta_0}\right]^{\tau-1}}$$

for $i = 2,3,\ldots,7$

and $\tau = 1,2,3,\ldots$ and, finally we have:

$$P_{11}(\tau) = \frac{P_{11}h_{11}(\tau)}{P_{11}h_{11}(\tau) + P_{12}h_{12}(\tau)} = \frac{P_{11}}{P_{11} + P_{12}\,\dfrac{\eta_d}{\eta_0}\left[\dfrac{1-\eta_d}{1-\eta_0}\right]^{\tau-1}}$$

$$P_{88}(\tau) = \frac{P_{88}h_{88}(\tau)}{P_{87}h_{87}(\tau) + P_{88}h_{88}(\tau)} = \frac{P_{88}}{P_{87}\,\dfrac{\eta_\mu}{\eta_0}\left[\dfrac{1-\eta_\mu}{1-\eta_0}\right]^{\tau-1} + P_{88}}$$

for $\tau = 1,2,3,\ldots.$

In order to avoid excessive time in inactive or depressive state(s) (and especially the state (000)), we should allow the system to "flip over" from state (000) to state (111) with increasing probability whenever the disorganisation at the lower levels(s) Q(Q') exceed a certain thershold; or equivalently when the redundancy at the lower levels falls below a critical value R_{th}. This means that we provide for the activation of a transition pathway $1 \to 8$ with probability

$$P_{18} = \begin{cases} 1 - \frac{R}{R_{th}} & \text{for} \quad R < R_{th} \\ 0 & \text{for} \quad R > R_{th} \end{cases}$$

All transition probabilities remain intact with the exception of the element p(1,2) which becomes:

$$P(1,1) = 1-p(1,2)-p(1,8)$$

VII/<u>The control problem</u>.

1) <u>The biological rhythms (parameters α,β,γ,ρ $\alpha',\beta',\gamma',\rho'$) underlying the games.</u>

We take as expressions for the game parameters harmonic (circadian or ultradian) components of periodic functions simulating basic rhythms.

Let it be:

$$\alpha(t) = \frac{\alpha_{max} + \alpha_{min}}{2} + \frac{\alpha_{max} - \alpha_{min}}{2} \cos(\omega t+\varphi)$$

$$\beta(t) = \frac{\beta_{max} + \beta_{min}}{2} + \frac{\beta_{max} - \beta_{min}}{2} \cos(\omega t+\varphi)$$

$$\gamma(t) = \frac{\gamma_{max} + \gamma_{min}}{2} + \frac{\gamma_{max} - \gamma_{min}}{2} \cos(\omega t)$$

where ρ = const, $\alpha_{max} \geq \alpha_{min}$, $\beta_{max} \geq \beta_{min}$, $\gamma_{max} \geq \gamma_{min}$ are giver constants expressing the extremevalues of fluctuations concerned.

We take as one control variable the phase difference between $\alpha(t)$ and $\gamma(t)$ and we arrange $\alpha(t)$ and $\beta(t)$ to be 180^{o} out of phase. This emulates the case of phase rearrangement via control action among individual biological rhythms.
The other control parameter is ρ.
The control vector is $\underline{u} = [\rho,\varphi]$.

We also consider the less severe control mechanism which consist in changing ρ and the initial phase φ of the three rhythmsbeing otherwise in step i.e.

$\alpha(t) = \alpha_1+\alpha_2 \cos(\omega t+\varphi)$

$\beta(t) = \beta_1+\beta_2 \cos(\omega t+\varphi)$

$\gamma(t) = \gamma_1+\gamma_2 \cos(\omega t+\varphi)$

where $\alpha_i, \beta_i, \gamma_i$ $(i = 1,2)$ are constants.

2) Description of the communication and control processes.

Let us consider the deliberation from the view point of partner A(Q,W):
For a given value of the control vector $\underline{u} = (\rho,\varphi)$ immediately after a state transition at the level W, the time variation of α,β and γ is completelv specified. Having $(\alpha,\beta,\gamma,\rho)$ the sixteen elements for $P_{ij}(t)$ at the level Q are determined.
Let τ be the holding time at the current state of W. The succession of states $S_{i_1}, S_{i_2},\ldots,S_{i\tau}$ at the level Q (where i_ν = 1,2,3, or 4) during this holding time is governed by the

above time varying transitional probabilites and is simulated on the computer by a Monte-Carlo method. For a preselected homeostatic state S_h (h = 1,2,3, or 4) we calculate the relative frequency of occurence u of that state during the holding time τ. We also evaluate the crosscorrelation

$$r = 1 - \frac{\bar{D}}{3} = \frac{1}{\tau} \sum_{i=1}^{\tau} \{1 - \frac{D^{(i)}}{3}\}$$

between the state sequences of the level Q of partner A and the higher level W' of the partner B(Q', W'), during the same time interval τ. During this time interval a number of transitions may have occured at the level W'.

The pair of values (u,r) is reported afferently at the end of the holding time to the level W, thereby fixing the transition probabilities P_{ij} at that level as described in section IV. The higher level W plays the dual role of "transmitter" toward the level Q' of partner B and "controller" of its own underlying game at level Q. In this last role its mission is to modify - via efferent (feed forward) control commands-the control vector $\underline{u} = (\rho, \varphi)$ on the basis of the received signals (u,r). The objective of the control is to satisfy a properly defined criterion. The situation calls for a two-objective control procedure. Namely from the point of view of partner A the objective could be the maximization of a weighted sum of the average values $E[u]$ and $E[r]$ of u and r respectively, i.e. maximization of the "figure of merit".

$F = \lambda_1 E[u] + \lambda_2 E[r] = \max$

where λ_1 and λ_2 are nonnegative constants with unit sum.
From the point of view of partner B, we should have as an objec-

tive the maximization of another figure of merit, i.e.

$$F' = \lambda_1' E[u'] + \lambda_2' E[r'] = \max$$

where $0 \leq \lambda_1' \leq 1$ and $\lambda_2' \leq 1-\lambda_1'$

In such an antagonistic 2 -objective control problem the Partners involved could compromise by seeking a regime of mutual adaptability or Coexistence which amounts in maximizing a joint "figure of merit"

$$'F = \sigma F + \sigma' F'$$

Where σ, σ' are nonnegative constants with unit sum.

3) Selection of control mechanisms.

The values of the control variables (ρ,φ) as well as (ρ',φ') are selected by the hierarchical W and W' respectively according to the collective observables (u,r), (u', r') immediately after the respective transitions i.e.

$$\rho = f_1(r,u) \qquad \rho' = f_1'(r',u')$$

$$\varphi = f_2(r,u) \qquad \varphi' = f_2'(r',u')$$

The joint selection of the above functions or mappings: $(r,u) \rightarrow (\rho,\varphi)$ and $(r', u') \rightarrow (\rho',\varphi')$ in a way which will maximize the "joint figure of merit" F, constitutes the control problem. This is in general a very difficult stochastic control problem. In the present work we assume that the control vector $\underline{u}$ can take on only a finite number of values: $\underline{u}_1, \underline{u}_2, \ldots, u_N$. We also consider that the rectangular region $0 \leq r \leq 1$, $0 \leq u \leq 1$ of the "received" vector (r,u) or (D,u) is partitioned in M regions: $R_1, R_2, \ldots, R_M$, as shown in Fig.7 where for computer simulation purposes we have taken $M = N = 8$. In this specific case we have considered that ρ can take on two values and φ

four values:

$\rho \in \{\rho_1, \rho_2\}$; $\varphi \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{3}\}$

Outputs of the computing program are the additive joint figures of merit for thirty randomly selected maps with the corresponding redundancies at the lower levels for the two partners. The program also selects the two maps (one for each partner) which produce the best (suboptimum) joint figure of merit amongst the tried control laws.

For the determination of the "best" pair of homeostatic states (i.e. the pair for which the "maximum maximorum" of the joint figure of merit is obtain) we ran the program 16 times corresponding to the 4x4 combination of the selected homeostatic states. In Fig.8 and 9 we display the behavioral (W, W') state transitions as functions of time for the extreme values (out of 16x30 = 480 values) of the joint figure of merit. We also present in Figs10 and 11 some results for the case where the rhytms α, β, γ keep constant phase relationships. In the above simulation the element P_{18} has not been included in the formulation, i.e. the facility of "flipping over" form (000) → (111) wherever the redundancy drops below a given threshold has been disregarded.

Thus far the objective of this work was a search for the adoption of an optimum common code which would map in a hierarchical fashion each partner's experience and behavior. Such a code would optimize the "Joint Figure of Merit" or, it would optimize the sequential behavioral mode turnover under the (conflicting) constraints of the

special situation. To find such a code one should either go through an exhaustive search of all possible mappings $\{u,r\} \rightarrow \{\alpha,\beta,\gamma,\rho\}$ or speculate on heuristics (see next section) which would allow the apriori exclusion of unpromising mappings and limit the exhaustive search to the members of a very small subset of allowable codes. Such heuristics do not exist for the moment, which means: we do not know <u>in advance</u> how we should transduce our experiences in behavioral modes or interpret the partner's behavior so as to <u>best</u> (whatever this means) compromize between the conflicting drives: Self-preservation (homeostasis) and self - transcendance (via - learning). In the absence of heuristics or values (i.e. parameters a priori determining decision making) we have been suggesting a rather "existentialistic approach". We have confined our search to a randomly selected subset of 30 out of the $8^{16} = 2^{48}$ possible joint behavioral modes - under the constraint of adopted quantization.
In the next and final section we are going to suggest a possible mechanism of emergence" of a new hierarchical level "Y" on top of the hitherto highest level "X" of the organism concerned.
It is our thesis that feed forward control commands precipitated from this newly built level may under certain conditions constrain the existing dynamics at X, thereby providing some sort of heuristic rules for behavioral selection of the type mentioned above.

VIII/ A model on the emergence of a new hierarchical level in a self - organizing system.

For the material we intend to treat in this section it will be instrumental to come back to the continuous dynamic (phase space) description, briefly introduced in section III.
At each hierarchical level, we introduce a phase space description involving type and number of dynamic variables and parameters pertaining to the particular level. The motion and transactions that the system undergoes in this phase space at the level(s) concerned (such as sequential passage from steady state to steady state, "time of rest" or stability properties of each state, as well as flight time between successive steady states) depend essentially on the parameters of the system. These, when perturbed beyond some critical values, give rise to a series of transitions associated with the branching or bifurcation of the solutions of the equations of evolution. Such perturbations are the results of a concerted influence from a) the information transmission (afferent signals) coming from the level immediately below b) the information impinging from the external environment and c) the feedback control (efferent signals) coming from the level immediately above.
As a result of these critical changes, the (hitherto) stable reference state(s) in which the system resided become unstable. The system at the level considered has to move or accomodate itself to another steady state provided by the real solutions of the (nonlinear) set of coupled differential equations (modeling the system at this level) under the new values of the parameters.
The permanence of the system at the specific level (say χ) during these deliberations is ensured by the "sameness" of the variables

and of the major characteristics of the resulting states:
The system at X moves from steady state to steady state by breaking down and resynthesizing a set of ordered relationships between dynamic variables (e.g non-linear oscillators) the nature of which does not change during these transactions
As an example, consider a homeogeneous mixture of n chemical reactting substances with relative compositions x_i.
This homogeneous mixture can find itself at many homogeneous steady states, depending on the number of the possible stable combinations amongst the x_i's. All these states belong to the same hierarchical level. Suppose now that an external perturbation gives rise to a particular bifurcation as a result of which the system moves to a steady state which is non-homogeneous, ie. the concentration of some of the components $x_i(r)$ are now functions of space variables.
This new state does not belong to the previous hierarchical level x_i; the system has moved itself to another level where its nature differs fundamentally from the previous one.
Such symmetry - breaking transitions can be caused by diffusion as first pointed out by Turing (1952). Prigogine and coworkers (1971) have demonstrated the generality of this type of phenomena in non-linear systems far from equilibrium, and introduced the term <u>dissipative structures</u> to describe these new hierarchical levels.
The biological implications of such transitions are nowadays widely recognized (Nicolis and Prigogine, 1977).
The purpose of this section is to propose a general formulation for the emergence of new hierarchical levels in a complex system - such as the ones introduced in the previous sections.
In a quite general way we shall consider each hierarchical "platform" as a dynamic constraint where a "storage and integration"

process is taking place.

As we are primarily interested in the activity of the nervous system we will represent storage and integration by a crosscorrelation. This means that the signals say $q_\lambda(t)$ transmitted sequentially from the level Q, (each one corresponding to a certain steady state of the system at the level Q) <u>and</u> the ones arriving from the "environment"(W) $w_\lambda(t)$ are time averaged upon arrival at the receiving "platform" X (e.g. a particular post - synaptic membrane aggregation in the cortex) with the result that some of the variables q_λ are completely "washed out" at the higher level X while the others are nonlinearly crosscorrelated or matched as

$$< q_\lambda^\nu(t) w_\lambda^\rho (t+\tau) > \ , \quad 0 < \tau < T$$

where T is the transmission time interval either from the lower level Q or from the "environment" W. It should be clear that the description we adopt here is global in the sense that the dynamic variables q_λ, w_λ entering the analysis are quantities averaged over a large number of (physico-chemical) configurations of the elementary "units" constituting the system. Such global descriptions are necessary in dealing with complex situations like e.g. the central nervous system.

As we pointed out above, <u>along</u> an existing hierarchical level the system may proceed in time from steady state to steady state via a sequence of bifurcations which do <u>not</u> change the nature of the dynamical variables involved. <u>Across</u> a given level on the other hand (i.e. as we move from this hierarchical level to the next higher one) a <u>crosscorrelation operation</u> is necessary. The two procedures seem at first glance quite distinct. (see Fig. 12)

One could think however of a generalized bifurcation scheme built in the dynamics of the system which might incorporate a stochastic operator responsible for the formation of crosscorrelations between a collective property at the actual level X with some process representing the fluctuating environment (W). This operator should be inactive or "masked" as long as the system moves along the level X and should manifest itself as soon as a bifurcation leading to a change of the nature of the variables in inevitable, in much the same way as diffusion manifests itself in the bifurcation involving a symmetry breaking instability leading to an non-homogeneous steady state.

Under what sort of environmentally imposed constraint could such a bifurcation take place? In the present context, by appropriate environmental fluctuations with variance exceeding a certain threshold, which could seriously disturb the system at X.

Facing the prospect of irreversible disorganization at the level X the system might use its only alternative i.e "activate" the crosscorrelation operator or change variables and "fly" to a higher level Y from which by sending efferent (feedforward) control commands, it may also inhibit the excessive fluctuations at the level X. (Fig.12)

Once this stabilisation at the level X is secured the new states may be abandoned, and the level Y cease to exist altogether; it is beneficial nevertheless for the organism to "keep" the new levels as long as possible - initially as a biasing agency against possible reccurrence of similar perturbations and later as an incorporeal element of a more complex "self".

In order now to describe the system at the "disturbed" level X, we express the internal dynamics of the system at this level by suitable non-linear "rates" f_i (x_j, t) describing, for example,

all possible couplings between the pertinent variables.
In addition to the phenomena arising from these couplings, it is conjectured that a collective behavior of a new kind may establish itself via a "weighting procedure" described by a stochastic operator as follows:
The individual variables x_i are weighted with a stochastic matrix of environmental fluctuations $w_{ij}(t)$ e.g. the "sociocultural milieu"). Subsequently, this action feeds back on the evolution of the variables x_i and constrains the dynamics at the level X in a way that reflects explicitly the properties of the environment. (The existence of numerous feedbacks between neocortical and limbic levels adds credence to this view).
We may now write down a set of coupled nonlinear differential equations for the macroscopic dynamic variables x_i which in the absence of spatial differentiation ($\nabla^2 x_i = 0$) read:

$$\frac{dx_i}{dt} = f_i(x_j,t) + w_i(t) + \sum_{j=1}^{K} \int_0^t x_j(t')w_{ij}(t'+\tau)dt' \qquad (1)$$

Here $w_i(t)$ stands for the fluctuations induced by the environment and the third term on the right represents the influence excercised by the crosscorrelation operator on the hierarchical level X.
For simplicity, it has been assumed that the weighting action of this operator involves a linear dependence on x_j's although this need not be true in the most general case. Obviously, for a noisy environment modeled after a random process with zero mean and root mean square deviation (r.m.s) equal to σ, this term is zero when x_i is either constant (steady states at the level X) or oscillates with a random phase, or if the frequencies ω_κ of the K

(for κ sufficiently large) oscillators are uncorrelated.
Dynamically, the "crosscorrelation operator" can be imagined as a sequence of a "Master hard non-linear oscillator"(MHNLO) potentially existing at the level X with two stable steady states, one "at rest" (i.e when non-functioning) and one limit cycle when in the position "on".
This MHNLO in a pool of environmental (white) noise w(t) with zero mean and power spectral density $\sigma(\frac{\text{watts}}{\text{Hz}})$ can be represented as

$$\ddot{x}-2\varepsilon(1-4\alpha x^2+8\beta x^4)\dot{x}+\omega_0^2 x = w(t) \qquad (2)$$

It has been calculated (Nicolis et al. 1976) that the ratio of the (mean) times during which such an oscillator is on the nonexcited versus the excited state is given as:

$$\mathcal{E} \sim \frac{(\alpha\ -4\beta)^{1/4}}{2\sqrt{\pi\varepsilon}}\ \sqrt{\sigma}\ e^{-\frac{c}{\sigma}} \qquad \text{where}$$

$$c = \frac{\varepsilon}{12\beta}\ (\alpha+\sqrt{\alpha^2-4\beta})\left\{1-\frac{\sqrt{\alpha^2-4\beta}}{4\beta}\ (\alpha+\sqrt{\alpha^2-4\beta})\right\}$$

For α,β of the same order of magnitude (strong "hard" nonlinearity) we may have either $c>0$ or $c<0$.
The above result indicates that for given parameters α,β,ε

$\mathcal{E} \sim \sqrt{\sigma}\ \exp(\pm\frac{|c|}{\sigma})$. In the case $\mathcal{E} \sim \sqrt{\sigma}\ \exp(\frac{|c|}{\sigma})$

$\mathcal{E}$ initially decreases with increasing σ passes through a minimum and then increases. In the case $\mathcal{E} \sim \sqrt{\sigma}\ \exp(-\frac{|c|}{\sigma})$ $\mathcal{E}$ increases monotonically with σ. In the first case we can always determine a "favorable"set of parameters α,β,ε, so that ($c>0$) for a given (moderate) level of environmental fluctuations σ,$\mathcal{E}<<1$. This means that the oscillator will be activated with practical certainty whenever the r.m.s of the environmental fluctuations exceeds σ.

The (probabilistically) activated MHNLO will subsequently move into its limit cycle with a basic frequency ω_0 and an appreciably high amplitude.

What are the conditions under which the above described hard oscillator can influence the behavior of the system?

Suppose that initially some of the variables entering into eg.(1) are excited in such a way that they perform almost synchronous oscillations. Then, it is possible that the MHNLO will cause further frequency entrainment amongst a number k of such oscillating components at the level X so that $x_i \sim A_i(t)\cos(\omega_0 t+\varphi_i(t))$. The individual members of the entrained group of oscillators can subsequently enter into a sequential phase-locking relationships with spectral components of the environmental random phasor $w_i(t)$ and establish a set of ordered relationships $\varphi_{i+1} - \varphi_i \leqslant \omega_0\tau$. Under the above postulated dynamics we eventually will obtain a non-zero term

$$\int_0^t x_j(t')w_{ij}(t'+\tau)dt' \quad \text{(due to phase coherence)}$$

and a non-zero sum $\displaystyle\sum_{j=1}^{K}\int_0^t x_j(t')w_{ij}(t'+\tau)dt'$

(due to frequency entrainment).

In other words, the correlation operator appearing in eg.(1) will be "activated" very much like the diffusion operator is "switched on" if the initial condition corresponds to an non-homogeneous distribution of matter.

So, we readily visualise concrete cases where in principle at least a bifurcation implied by the last right hand term in (1) can be implimented.

The quantity $\int_0^t x_j(t')w_{ij}(t'+\tau)dt' \sim y_i$ will be regarded as the variable pertaining to new hierarchical level Y.

The term $\sum_{j=1}^{K} y_j$ gives then the amount of total efferent feedforward control exercised from Y back to X.

For $\sum_{j=1}^{K} y_j < 0$ we have negative feedforward i.e a tendency of restraining the excessive fluctuations on X. (Fig.12)

In order to substantiate these conjectures it would be necessary to establish at least on simple models the occurrence of bifurcations in the system of the stochastic equation(1). We should in other words be able to estimate in such cases the probability density functions for the real parts $R_e(\lambda_i)$ of the eigenvalues of the crosscorrelation operator and establish the conditions under which $R_e(\lambda_i) > 0$ can take place at least for one i with an appreciable probability.

A solution to this extremely complicated mathematical problem will not be attempted in this paper.

In conclusion the mechanism suggested in this section points toward some physical process via which a new, more abstract hierarchical level may emerge; via precipitated feedforward controls it constrains the dynamics on the hitherto highest level of the organism- thereby providing regulation of behavior or facilitation of the "Decision making" procedure.

IX /Concluding Remarks.

From the detailed study of the proceeding model it becomes clear that under the spell of environmental or the partner's "noisy" stimuli the organism may activate dormant potentialities i.e "switch - on operators, instrumental in the execution of a bifurcation which triggers the emergence of a more abstract hierarchical level.

So what we consider here to be the goal of decision making is an "evolving adaptation to the environment". Animals are by birth adapted but they do not evolve during their own lifetime.

Man, the Learner, strives for adaptation on ever more abstract levels of interactions with his environment.

This implies a dynamic process of sequential equilibria between the behavior (actions) of the individual and those of the environment. Intellectual development (isomorphically presented here as the emergence of higher hierarchical cognitive levels) is in the direction of more abstract and more stable equilibria.

The emergence of rules or constraints via which heuristics are created [which in turn rescue the individual from the ocean of "computational complexity" and behavioral paralysis resulting from an exhaustive search over all possible codes of mapping experience to behavior] appear to have a rather subtle origin:

There are not founded exclusively (as "analytical" schools would claim) either upon obedience to authority alone (e.g an "internalization" of the socio-cultural milieu) or upon submission to group pressure, but rather upon the intrinsic need to reconcile and balance one's own point of view with that of others.

For example, to resolve the inner conflict between homeostasis (self-preservation and image) and matching one's internal representations with environmental triggers (self-transcendance or identification of the world).

In short, the Decision making process seems once again to spring directly from the need to ameliorate the perennial conflict between the ontological and the epistemological paradigm.

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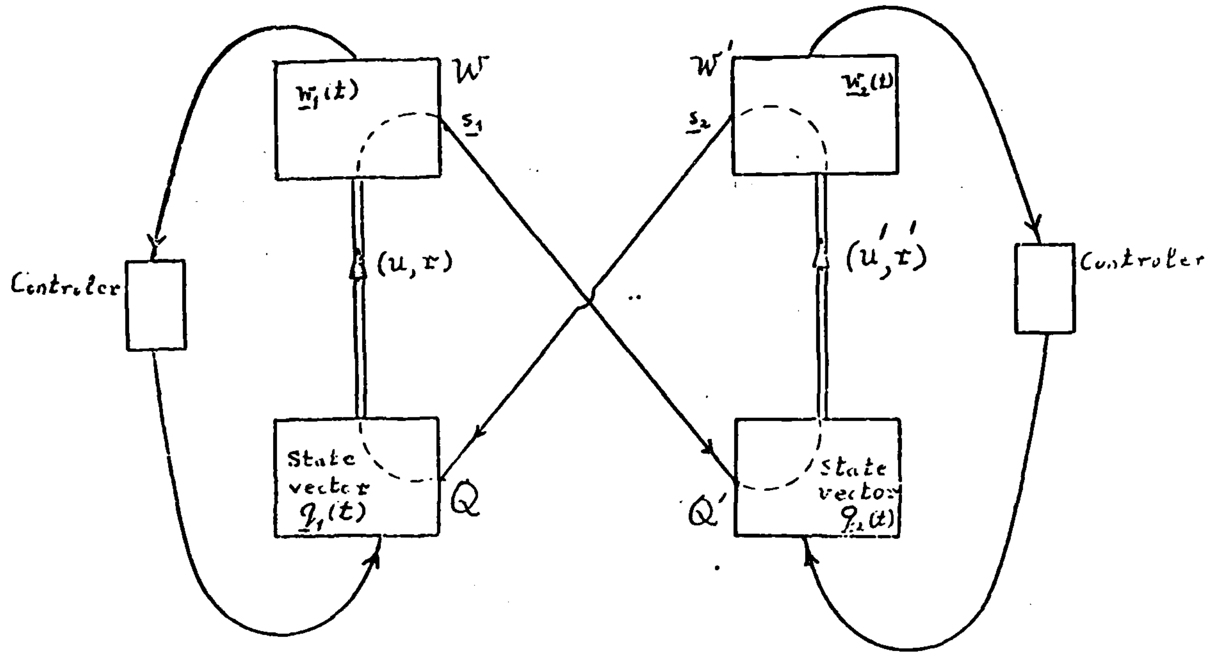


Fig.1a State variables model for two communicating multihierarchical Systems.

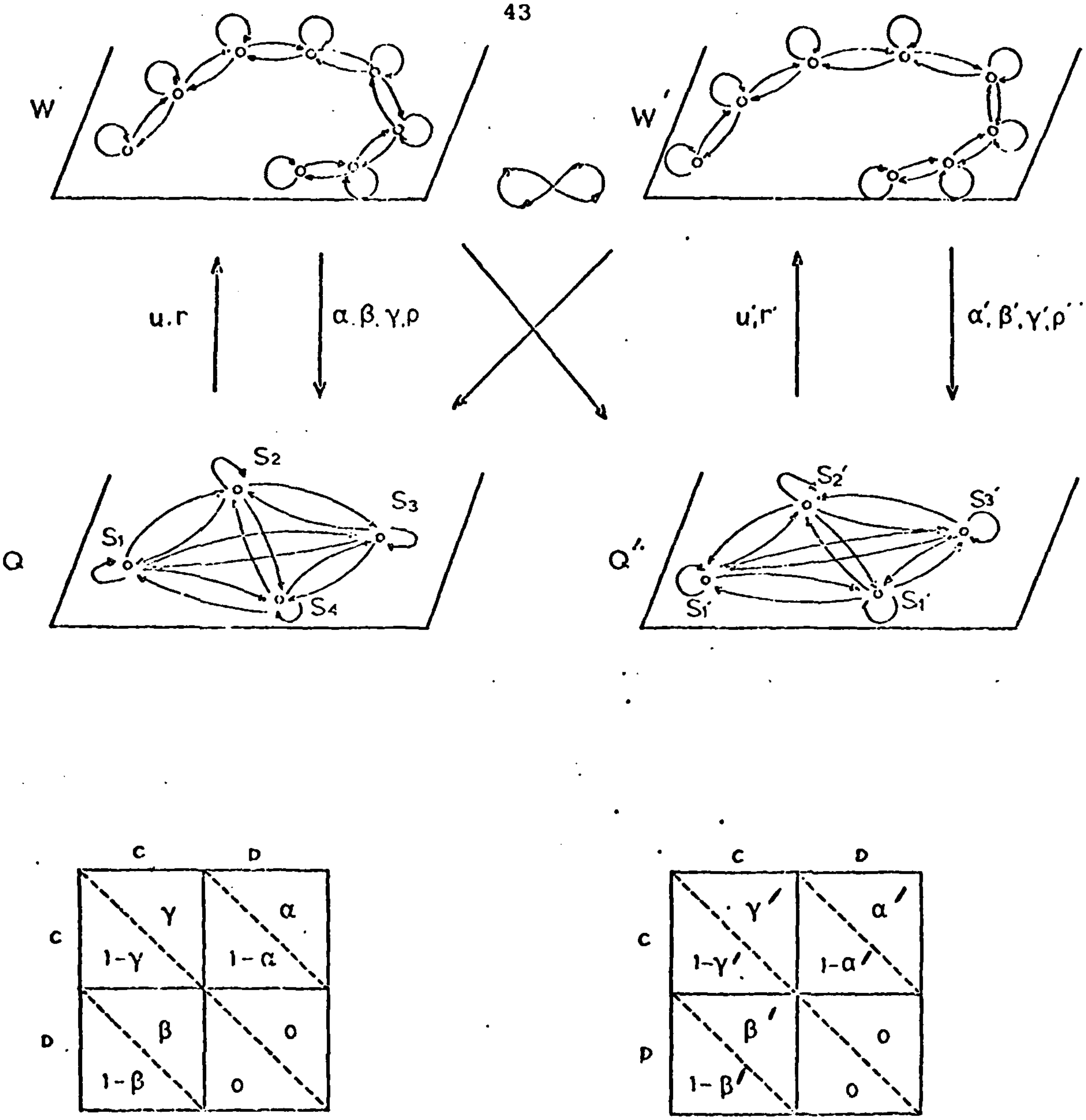
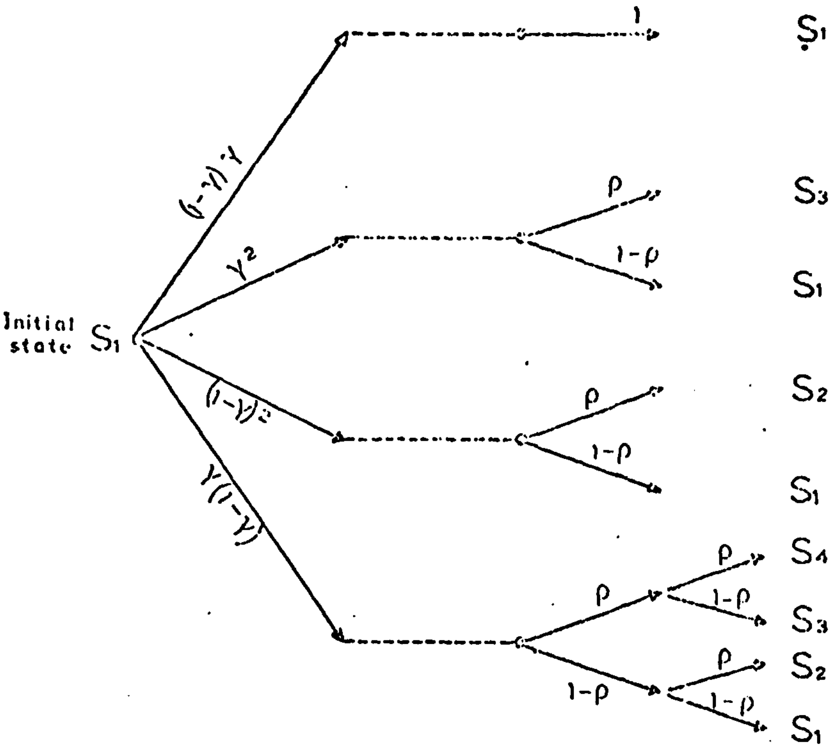


Fig.1B The general layout of the communication process between two hierarchical systems



$$P_{11}=(1-\gamma)\gamma+\gamma^2(1-\rho)+(1-\gamma)^2(1-\rho)+\gamma(1-\gamma)(1-\rho)^2=\{1-\gamma+\gamma(1-\rho)\}\{\gamma+(1-\gamma)(1-\rho)\}$$

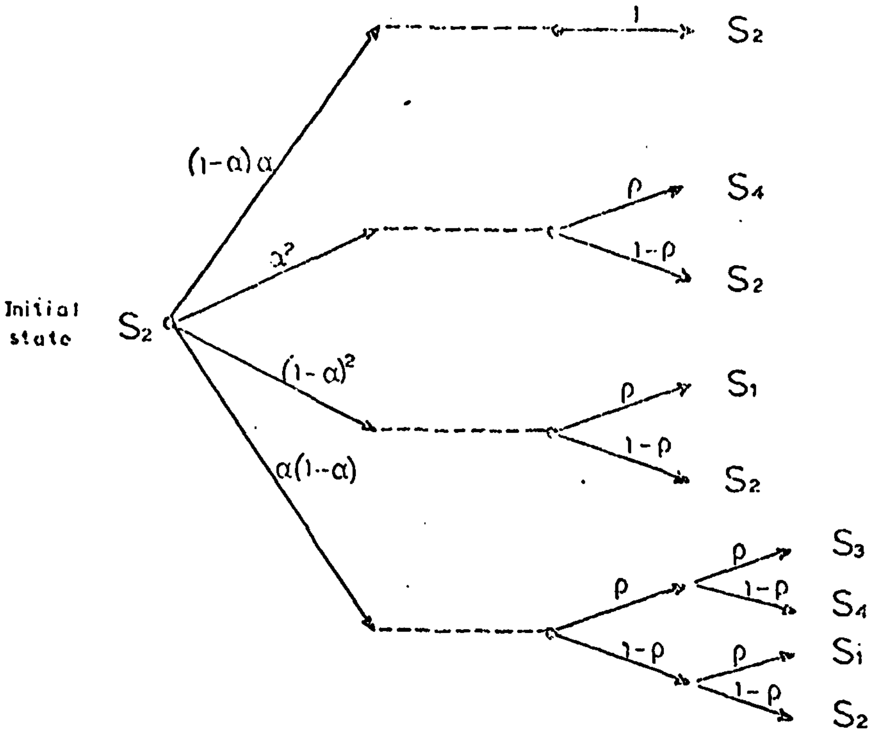
$$P_{12}=(1-\gamma)^2\rho+\gamma(1-\gamma)\rho(1-\rho)=\rho(1-\gamma)\{1-\gamma+\gamma(1-\rho)\}$$

$$P_{13}=\gamma^2\rho+\gamma(1-\gamma)\rho(1-\rho)=\gamma\rho\{\gamma+(1-\gamma)(1-\rho)\}$$

$$P_{14}=\gamma(1-\gamma)\rho^2$$

$$\sum_{j=1}^{4} P_{ij} = 1$$

Fig.2 Tree structure for transition probability with initial state S_1



$$P_{21} = (1-\alpha)^2\rho+\alpha(1-\alpha)\rho(1-\rho)=\rho(1-\alpha)\{1-\alpha+\alpha(1-\rho)\}$$

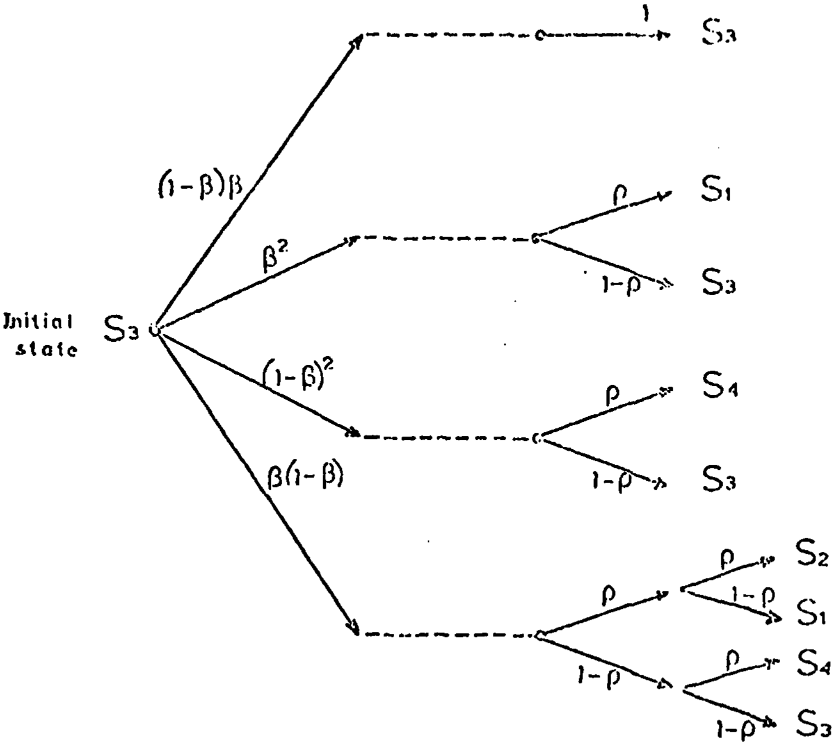
$$P_{22}=(1-\alpha)\alpha+\alpha^2(1-\rho)+(1-\alpha)^2(1-\rho)+\alpha(1-\alpha)(1-\rho)^2=\{1-\alpha+\alpha(1-\rho)\}\{\alpha+(1-\alpha)(1-\rho)\}$$

$$P_{23}=\alpha(1-\alpha)\rho^2$$

$$P_{24}=\alpha^2\rho+\alpha(1-\alpha)\rho(1-\rho)=\alpha\rho\{\alpha+(1-\alpha)(1-\rho)\}$$

$$\sum_{j=1}^{4} P_{ij} = 1$$

Fig.3 Tree structure for transition probabilities with initial state S_2



$$P_{31}=\beta^2\rho+\beta(1-\beta)\rho(1-\rho)=\beta\rho\{\beta+(1-\beta)(1-\rho)\}$$

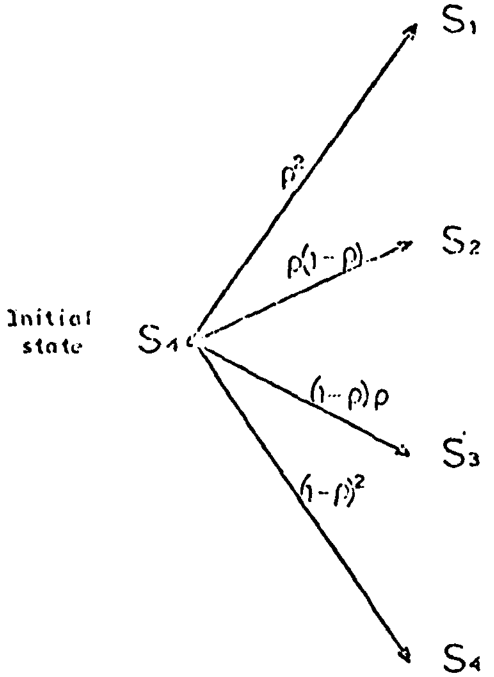
$$P_{32}=\beta(1-\beta)\rho^2$$

$$P_{33}=\beta(1-\beta)+\beta^2(1-\rho)+(1-\beta)^2(1-\rho)+\beta(1-\beta)(1-\rho)^2=\{1-\beta+\beta(1-\rho)\}\{\beta+(1-\beta)(1-\rho)\}$$

$$P_{34}=(1-\beta)^2\rho+\beta(1-\beta)\rho(1-\rho)=\rho(1-\beta)\{1-\beta+\beta(1-\rho)\}$$

$$\sum_{j=1}^{4} P_{ij} = 1$$

Fig. 4 Tree structure for transition probabilities with initial state S_3



$$P_{41} = \rho^2,$$

$$P_{42} = \rho(1-\rho),$$

$$P_{43} = \rho(1-\rho),$$

$$P_{44} = (1-\rho)^2,$$

$$\sum_{j=1}^{4} P_{ij} = 1$$

Fig. 5 Tree structure for transition probabilities with initial state S_4

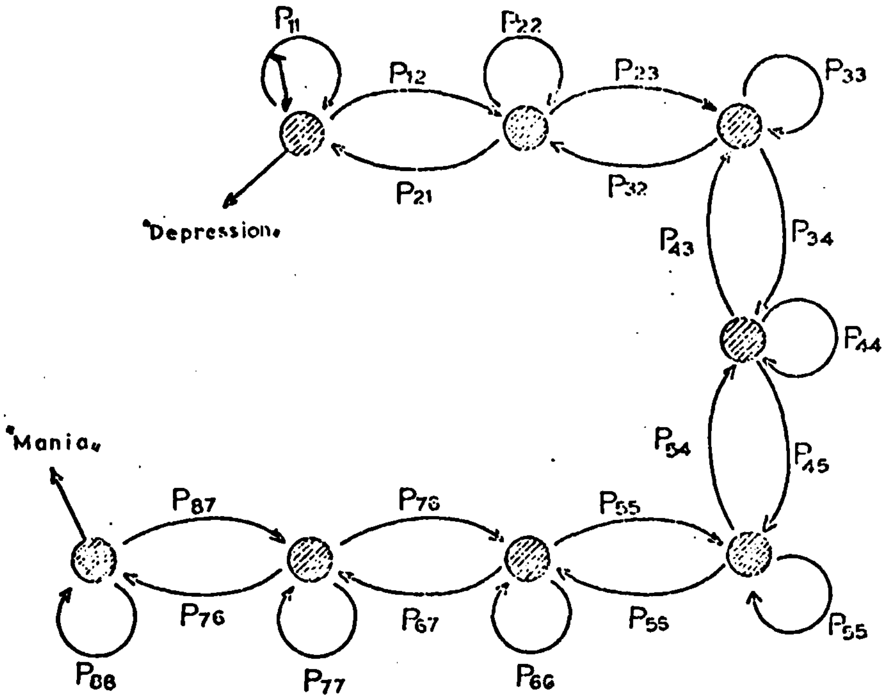


Fig.6 Layout of the Markovian chain (5) at the higher levels W(W')

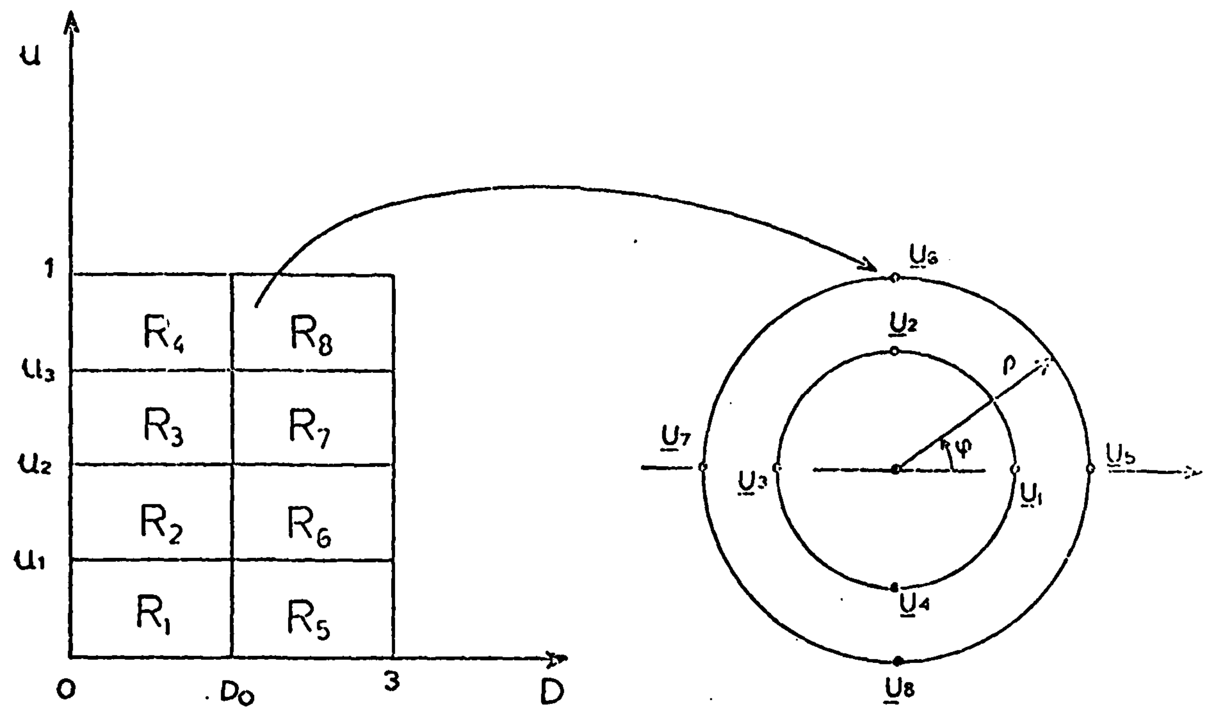


Fig.7 (a) Quantization on the plane (D, u)
(b) The discrete values for the control vector $\underline{v} = (\rho, \varphi)$

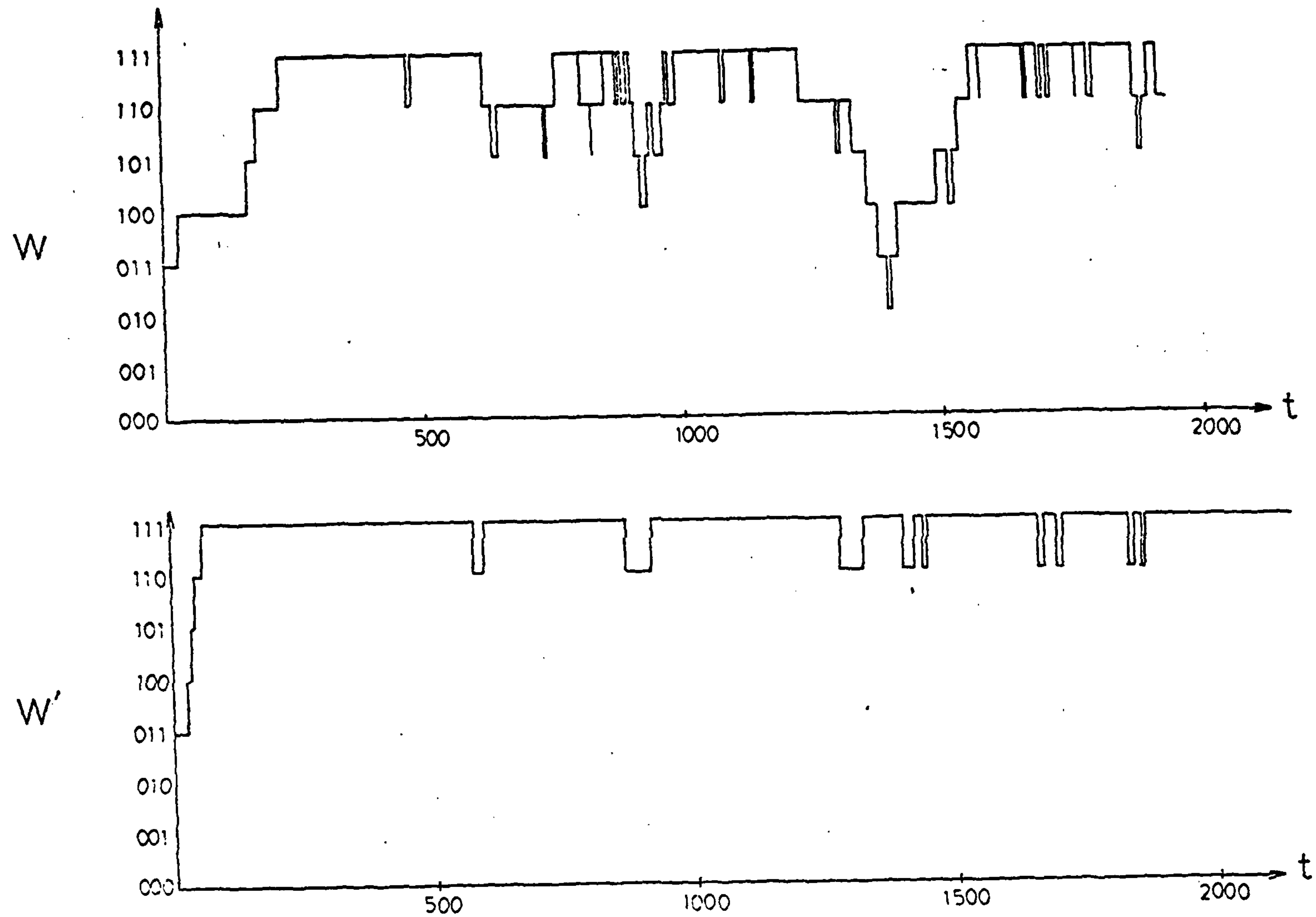


Figure 8. See caption on next page.

Evolution of the states of the higher levels W and W' when the underline *rhythms* are expressed by $\alpha(t)=\{(\alpha_{max}+\alpha_{min})+(\alpha_{max}-\alpha_{min})\cos(\omega t+\varphi)/\}2$, $\beta(t)=\{(\beta_{max}+\beta_{min})+(\beta_{max}-\beta_{min})\cos(\omega t+\varphi)\}/2$ and $\gamma(t)=\{(\gamma_{max}+\gamma_{min})+(\gamma_{max}-\gamma_{min})\cos(\omega t+\varphi)\}/2$. Values of the parameters for the computer simulation: $\alpha_{max}=0.9$, $\alpha_{min}=0.1$, $\beta_{max}=0.8$, $\beta_{min}=0.2$, $\gamma_{max}=0.7$, $\gamma_{min}=0.3$, $\alpha'_{max}=0.7$, $\alpha'_{min}=0.3$, $\beta'_{max}=0.8$, $\beta'_{min}=0.2$, $\gamma'_{max}=0.9$, $\gamma'_{min}=0.1$; $\omega=\pi/10$; higher level transition probability parameters $\pi_\varkappa=0.1$, $g_\varkappa=0.2$, $c_\varkappa=d_\varkappa=0.8$, $\pi'_\varkappa=0.2$, $g'_\varkappa=0.1$, $c'_\varkappa=d'_\varkappa=0.8$, $\lambda_\varkappa=\mu_\varkappa=\nu_\varkappa=\xi_\varkappa=$ $=\lambda'_\varkappa=\mu'_\varkappa=\nu'_\varkappa=\xi'_\varkappa=2$ for $\varkappa=1,2,\ldots,8$. Values of control variables ρ $\{0.2;0.8\}$, $\rho'\in\{0.1;0.5\}$ φ & $\varphi'\in\{0,\frac{\pi}{2},\frac{3\pi}{2}\}$.

The figure corresponds to the *maps* which yield the maximum joint figure of merit ($\mathfrak{J}=0.29$) amongst thirty computer runs with randomly *selected maps*. For details on the computer simulation see [8].

Caption to Fig 8

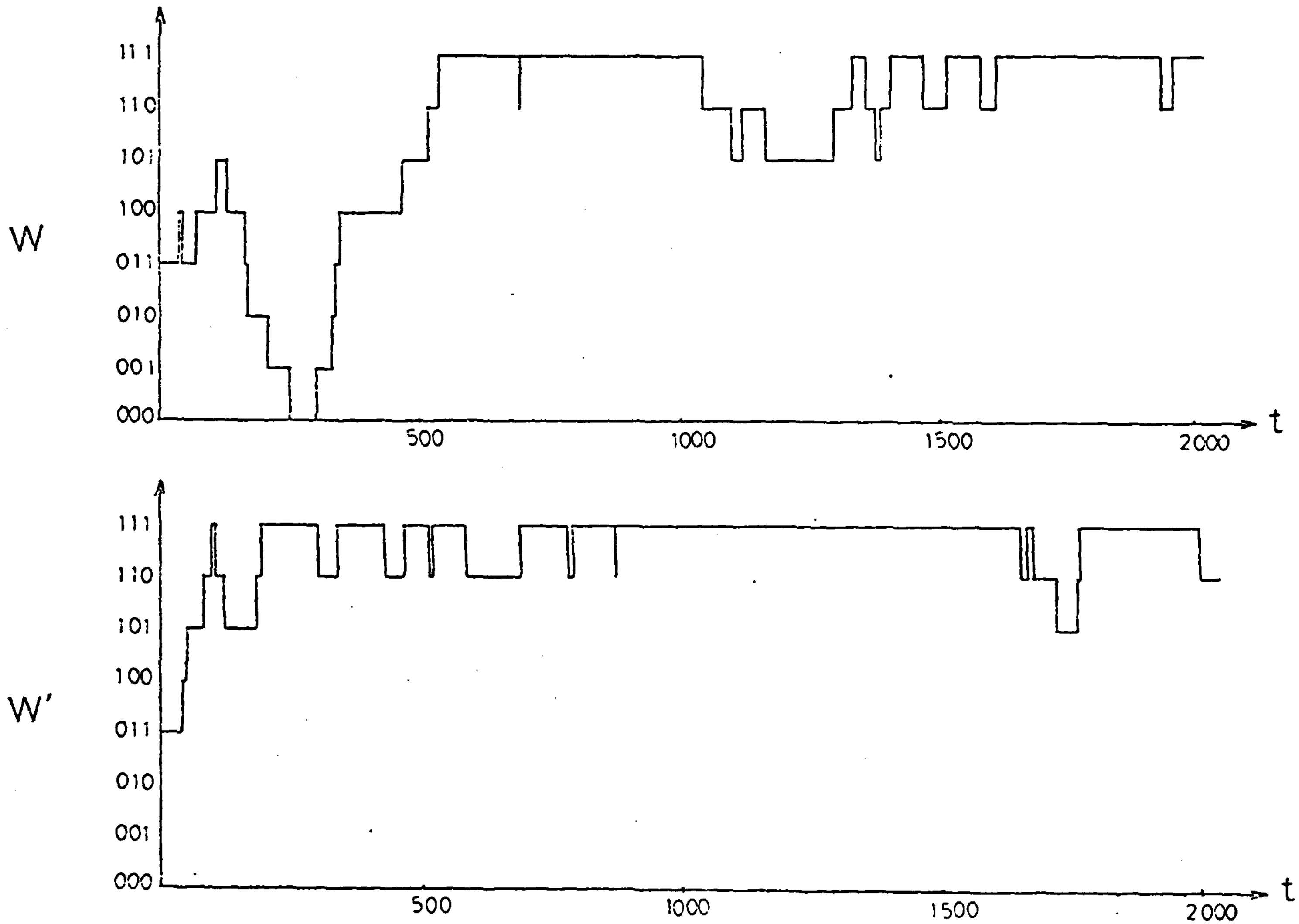


Figure 9. See caption on next page.

Caption for Fig.9

Evolution of the states of the higher levels W and W' with parameters as in Fig.8. The figure corresponds to the maps which yield the minimum joint figure of merit ($\mathfrak{F}$=0.19) amongst thirty computer runs with randomly selected maps.

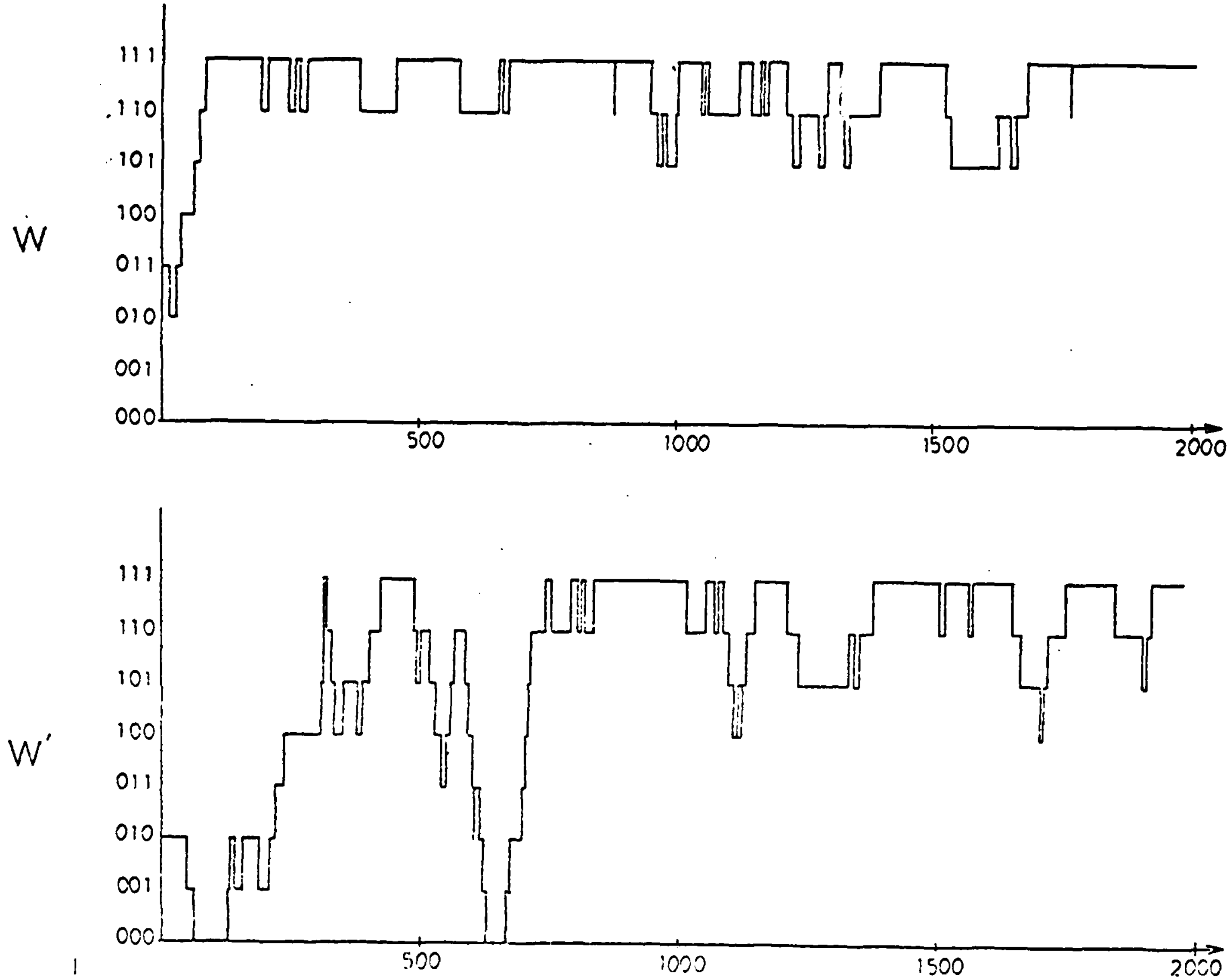


Figure 10. See caption on next page.

Caption for Fig.10

? Evolution of the states of the higher levels W and W when the underline rhythms are expressed by $\alpha(t)=\{(\alpha_{max}+\alpha_{min})+(\alpha_{max}-\alpha_{min})\cos(\omega t+\varphi)\}/2$, $\beta(t)=\{(\beta_{max}+\beta_{min})+(\beta_{max}-\beta_{min})\cos(\omega t+\varphi)\}/2$ and $\gamma(t)=\{(\gamma_{max}+\gamma_{min})+(\gamma_{max}-\gamma_{min})\cos(\omega t+\varphi)\}/2$. Values of the parameters for the computer sumulation: $\alpha_{max}=0.9$, $\alpha_{min}=0.1$, $\beta_{max}=0.8$, $\beta_{min}=0.2$, $\gamma_{max}=0.7$, $\gamma_{min}=0.3$, $\alpha'_{max}=0.7$, $\alpha'_{min}=0.3$, $\beta'_{max}=0.8$, $\beta_{min}=0.2$, $\gamma'_{max}=0.9$, $\gamma'_{min}=0.1$; $\omega=50/\pi$; higher level transition probability parameters: $\pi_{\varkappa}=g_{\varkappa}=0.2$, $c_{\varkappa}=d_{\varkappa}=0.8$, $\pi'_{\varkappa}=g'_{\varkappa}=0.3$, $c'_{\varkappa}=d'_{\varkappa}=0.7$, $\lambda_{\varkappa}=\mu_{\varkappa}=\nu_{\varkappa}==\xi_{\xi}=\lambda'_{\varkappa}=\mu'_{\varkappa}=\nu'_{\varkappa}=\xi'_{\varkappa}=2$ for $\varkappa=1,2,\ldots,8$. Values of control variables $\rho C\{0.2;0.8\}$, $\rho' C\{0.1;0.5\}$, φ & $\varphi' C\{0,\frac{\pi}{2},\pi,\frac{2\pi}{2}\}$.

The figure corresponds to the maps which yield the maximum joint figure of merit ($\mathfrak{J}=0.41$) amongst thirty computer runs with randomly selected mappings.

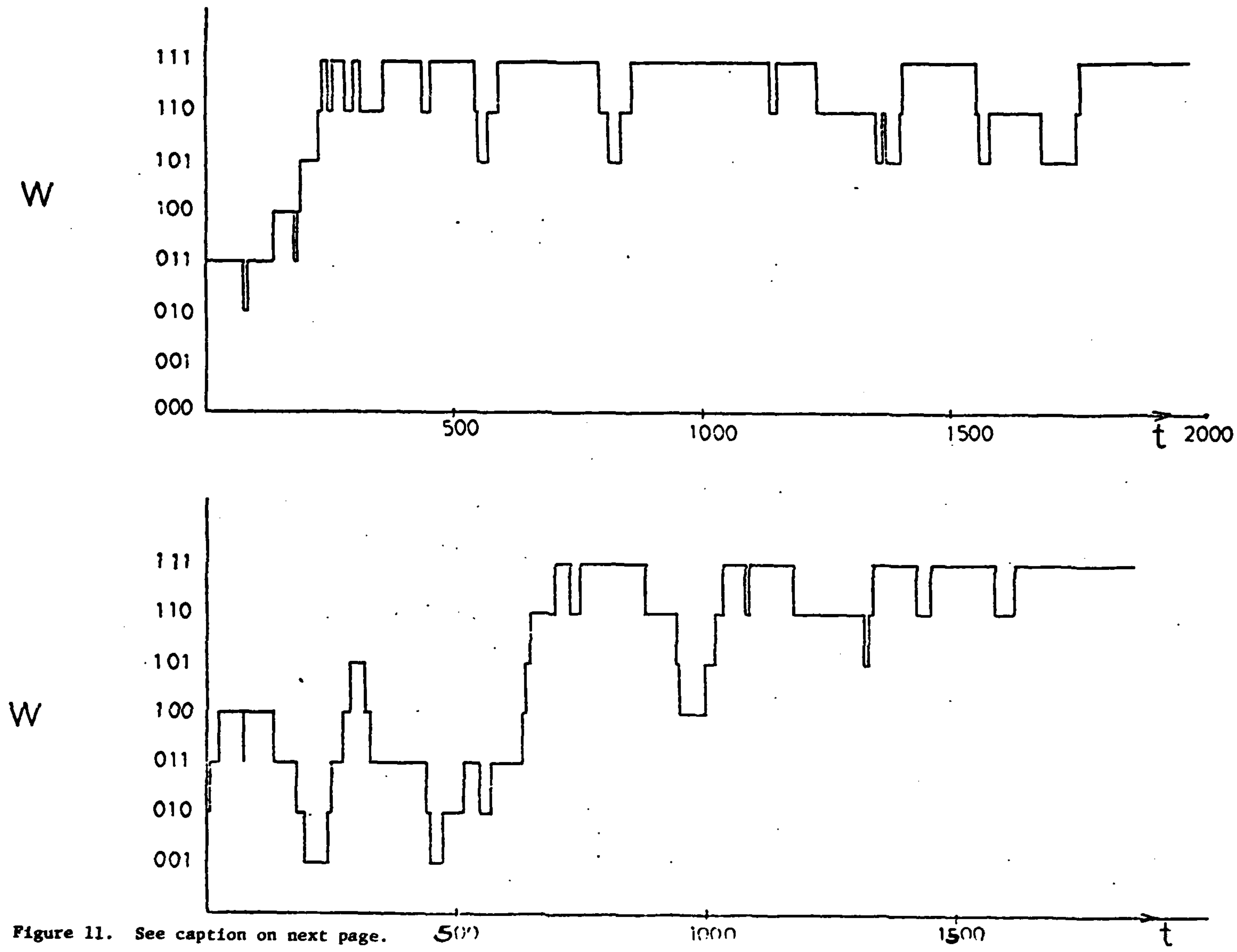


Figure 11. See caption on next page.

Caption for Fig. 11

Evolution of the states of the higher levels W and W′ with parameters as in Fig .10. The figure corresponds to the maps which yield the minimum joint figure of merit ($\mathfrak{J}$=0.23) amongst thirty computer runs with randomly selected mappings.

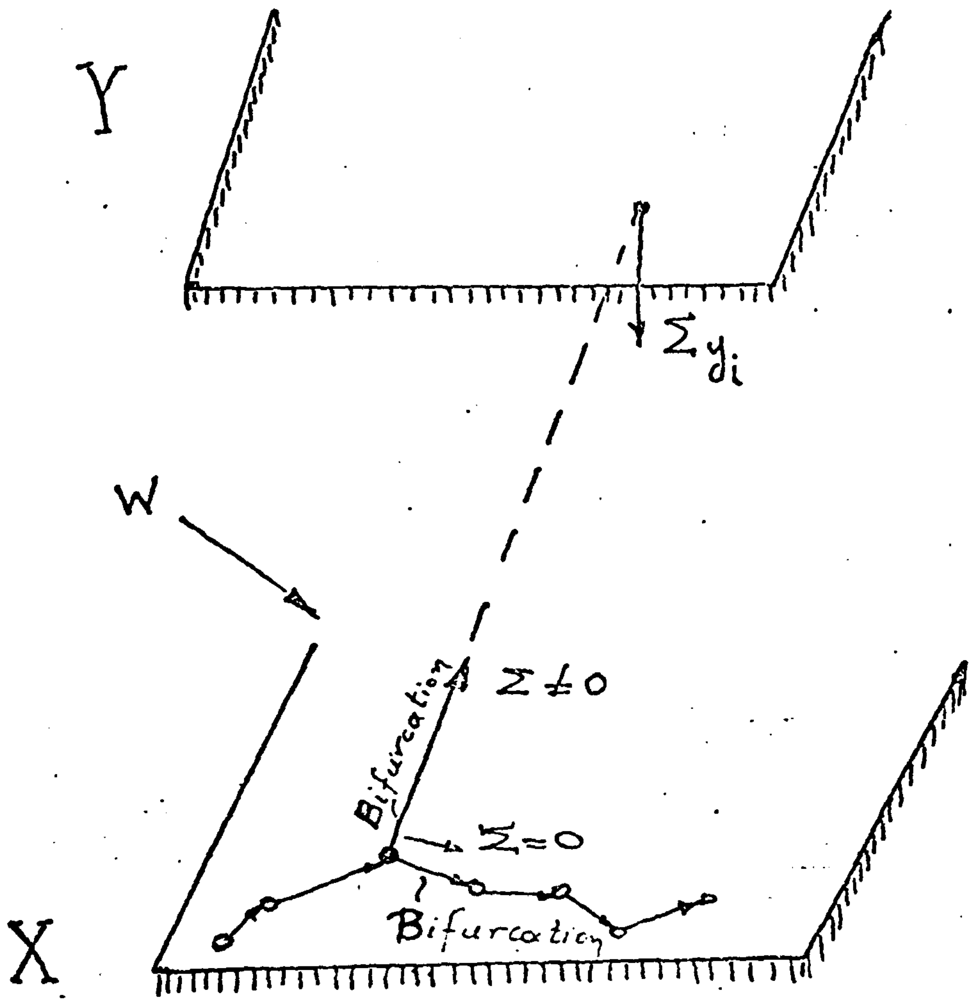


Fig.12: Building a new hierarchical level.

VOLUME 6

NEW DECISION MODELS

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New Decision Models

Statement

Generally, my field of interest is mathematical psychology (methodology and new models), formach approaches to science of science, formal action theory, natural languages and communication problems (linguistics, psycholinguistics, and more precisely: dialogues, texts and discussions) and formal semiotics.

As concerns the topics of this conference, my general position is that further development of decision theory and decision practice requires new models, involving new types of variables, and new inference schemes.

In particular, it appears necessary to extend decision theory so as to include pre-decisional situations, especially the motivational aspects of the person making the decision, his understanding of the situation, diagnosis and planning of action.

In this statement I shall present in some detail a new model of decision under risk, which for the first time combines two models treated thus far separately: SEU model and the model of achievement situation of Atkinson. After that, I shall present an overview of my other works relevant for the process of decision making.

A new model of decision in risky situations

1. The basic axiom of decision theory states that in choosing among several options, or alternatives, the decision maker (individual or collective, in large organization or not) will select that option which yields highest expected utility. This, I think, may always be assumed as tautological; the real question is then to define the utility in such a way that the resulting theory should either have predictive power, or may serve as a defensible normative guideline.

In other words, utility--call it reward, profit, etc.,--should be defined in such a way, that either of the following two alternatives should hold:

a) the definition should be accompanied by measurement procedures, enabling one to determine the personal utility of an individual. The results of the measurements ought to yield such values, that the option with maximal expected utility should coincide with the option which the person chooses. This is a requirement of predictive power of the theory.

b) the utility should be defined in some way--as profit, balanced with losses, discounted profit, etc.,--and a decision maker in doubt what to do, should be recommended to calculate the expected utilities of various options, and advised to choose that option which yields the highest expected utility. If the choice of utility function can be reasonably and convincingly defended, the theory may serve as a normative guideline. In this case, of course, the choices of the decision maker will automatically satisfy the principle of maximal expected utility.

2. One of the failures of decision theory is that neither the approach (a), nor the approach (b) seems to work very well in practice.

Firstly, except for some simplest cases, it is very hard (if not impossible) to find an adequate utility function for individual persons. Somehow it always appears that the real choices did not exactly match the predicted ones. Moreover, the elaborate experimental techniques used to test the simplest SEU model are of a very limited applicability.

On the other hand, it also appeared that the normative theory does not work--mostly because the persons who make the decisions, while agreeing <u>in theory</u> on the principles leading to optimality of certain choices, do not always make these choices <u>in practice</u>.

In my opinion, the latter fact--which will be the main issue of this note--is due to the failure in taking into account the psychological and sociological factors involved in utility.

3. Indeed, the theorists of decision making are very much attracted by the simplicity of principles involved in the SEU model. These principles, however, are quite deceptive, and it is easy for the decision theorist, who is--so to say--only an "outside advisor," to concentrate only on <u>economical</u> variables, that is, take into account only rewards and losses. The theorists of decisions <u>do not make</u> the decisions themselves (in the sense of accepting the responsibility), and only--at best--advise what is the optimal choice.

In real decision making, however, there appear quite powerful psychological and sociological components.

4. As the latter aspects (sociological) will be the object of my separate talk, in this note I will concentrate on psychological components.

5. First of all, let us observe the following "paradox" of decision making (I put the quotation marks, to indicate that this is only an apparent paradox).

Suppose (to use an example) that an oil company faces the problem of choosing a drilling site. For simplicity, assume that the decision situation can be reduced to the case of two options only: drill at location A, or drill at location B (but not both!).

Assume that after collecting all available evidence (geological surveys, expert opinions, etc.), it is impossible to evaluate the probability of finding oil at A and at B. Suppose these probabilities are such that there is more chance of finding oil at location A.

A decision theorist will say that (other conditions assumed equal) the optimal choice is to drill at A.

It is clear, however, that higher probability of finding oil does not make the oil--still, oil may be at B, and <u>not</u> at A, despite the probabilities to the contrary. Thus, it may happen that the <u>optimal</u> decision (drill at A) yields <u>bad</u> results, while the <u>suboptimal</u> decision (drill at B) leads to <u>better</u> results.

For the company director, the knowledge that the decision theorist is on the average more often right than wrong, is of limited significance: he is facing just one decision, to which the law of averages does not apply, and--unlike the decision theorist--he is personally interested in being right, as it is his money, career, etc., which are at stake.

6. To trace the psychological factors involved in such types of situations, let us simplify further the above example, leaving only the most essential features. Suppose, therefore, that a person faces a "tough" decision situation. He may either choose a "safe" option, with known outcome, or a "risky" option. The latter may lead to success--with great reward, economical or not (fame, promotion, Nobel Prize, etc.), or it may lead to failure, possibly disastrous, to the man himself, and/or to the institution he represents.

The man has to make the decision, and accept the sole responsibility for it, perhaps risking his career, if he fails, but also gaining a lot, if he succeeds.

A decision theorist would consider the rewards, losses, etc., and their probabilities, possibly even taking into account the "non-measurable" quantities, like the value of promotion, or ruining the career. He then determines the expected reward (utility) for "safe" and for "risky" options. Assume that he comes to the conclusion that the "safe" option is better, and so advises the decision maker.

The latter, however, may think along the following lines: "All right, I agree that the safe option is indeed better. But, if I take the risky option and succeed, I gain not only in terms of money, career, etc.,--imagine how proud I would be, that I had taken the risk!" So he may proceed to follow the risky path.

7. What happened here is that the decision theorist failed to take into account the "Atkinsonian" component in assessing the utility, namely the incentive value of success as such, regardless of the economical and other rewards which it brings, and the incentive value of avoiding failure as such--not only avoiding the economical or personal loss.

The main issue here is that--contrary to SEU model--these incentives depend on the probabilities of success and failure: the less likely is the success, the higher incentive of achieving it, and the more likely is the failure, the higher incentive of avoiding it.

8. I have developed a model which accounts for these two components of "psychological" character, in addition to the other components, of "economical" character. The essence of the model is as follows.

Let p be the (subjective or not) probability of success in risky option, so that $1-p$ is the probability of failure.

The "value," or utility of success, is then defined as

$$U_S = M(R + f(p))$$

where M is the strength of the general motive to achieve, R is the value of the reward in case of success, and $f(p)$ is some function monotonically decreasing with p (so that its value is highest at $p = 0$). This function reflects that component of utility, which is highest at the least likely successes.

On the other hand, the utility of failure is assumed to be of the form

$$U_f = F(L + g(p))$$

where F is the strength of the general motive to avoid failure (fear of failure), L is the loss resulting from the failure, and $g(p)$ is again a monotonically decreasing function (so that its value is lowest at $p = 1$). This function reflects the component of utility which is highest in absolute value, but negative, if one fails at an easy task (failure to succeed in case when the success is almost certain).

Consequently, the expected utility of the risky option is

$$E_r = pU_s + (1 - p)U_f,$$

and the last quantity should be compared with the reward for the "safe" option, say K.

9. At present, there are no reliable measurement procedures enabling one to determine the values M and F of the motive strengths. There exist, however, quite good techniques which lead to determining the sign of the difference $M - F$, i.e., allowing to partition people into those, for whom the motive to achieve exceeds the fear of failure, and those for whom the situation is opposite. Consequently, a workable research strategy is not to attempt determining the functions $f(p)$ and $g(p)$, but rather making some intuitively acceptable assumptions about these functions, and trying to develop some qualitative predictions.

Following Atkinson, I assumed that

$$f(p) = 1 - p, \quad g(p) = -p.$$

This assumption worked quite well in Atkinson's model, where, however, the "economical" variables R, L and K were neglected, and the research concentrated only on "psychological" variables, determining the choice of the task difficulty (choice among tasks with various probabilities of success p). It turns out that for persons with $M > F$, the optimal choice is the task with probability of success as close to 1/2 as possible, while for those with $M < F$, the optimal choice is to select a task with probability of success as close to either 0 or 1 as possible (i.e., choose either a very easy task, so as not to fail, or chose a very difficult task, so as not to be hurt by failure, which is almost certain anyway).

10. Substituting the above values for $f(p)$ and $g(p)$ into the expected utility, one comes to the conclusion that the "risky" option will be selected over the "safe" one, if

$$M(R + 1 - p)p + F(L - p)(1 - p) > K$$

which leads to a quadratic inequality for p:

(1) $$-(M - F)p^2 + (M(R + 1) - F(L + 1))p > K - FL.$$

11. For fixed M, F, R, L and K, let $C(M,F,R,L,K)$ denote the set of values p lying between 0 and 1, for which the inequality (1) holds; let us agree to call this set the risk area.

Intuitively, given the values M and F of psychological variables (motive strengths) and economic variables R, L and K, the person will choose the risky option only if the probability p of success is in the risk area; otherwise, he will choose the safe option.

12. The determination of the risk area, and its dependence on the parameters M, F, R, L and K is quite straightforward. The relevant results may be formulated as follows:

(a) In both cases, $M > F$ and $F > M$, the risk area may be empty, or of the form $p_0 < p \leq 1$. In other words, a person will either choose always the safe option, regardless of p, or he will choose the risky option only if the chances of success p are sufficiently high. Let us call the risk area in this case "normal."

(b) In addition to normal risk areas, the following risk areas, to be called "odd," are possible in some circumstances (determined by the mutual relations between the parameters.

(b1) In case $M > F$, i.e., for persons whose achievement motive exceeds their fear of failure, it may happen that the risk area is of the form

(i) $0 \leq p < p_1$

or

(ii) $p_0 < p < p_1$ with $p_0 > 0$ and $p_1 < 1$.

Thus, a person might choose the risky option only if the chances of success are sufficiently low (case (i)), or if they are neither too small nor too large (case (ii)).

Case (i) corresponds to the situation when the dominant component of utility is the "pure joy of success." In case (ii) we have the same situation, except that too small chances of success make the risk option inferior because of the prospect of a loss.

In both cases, the essential fact is that if the success is too certain, the risky option becomes inferior, because the "joy of success" does not counterbalance the reward for the "safe" option.

Normal risk area, and both cases (i) and (ii) of odd risk areas are presented on Fig. 1.

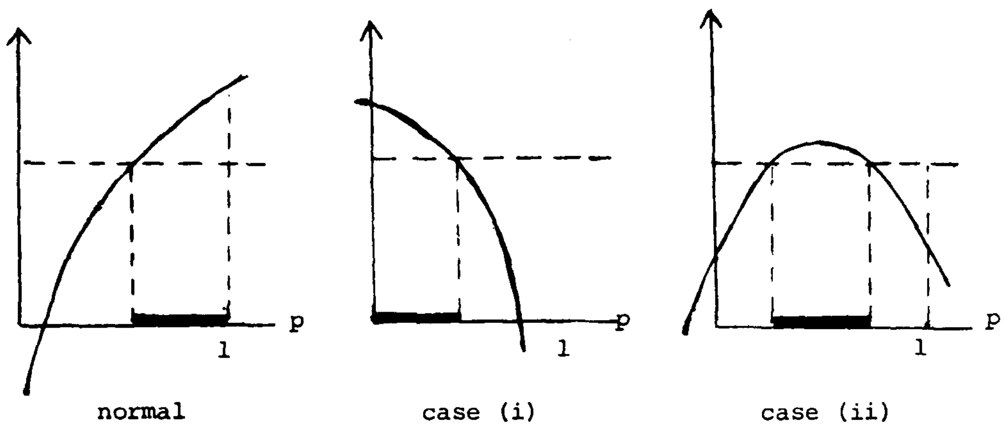


Fig. 1. Risk areas in case M > F.

(b2) In case of persons with M < F, i.e., for whom fear of failure dominates above the need of achievement, it may happen that the risk area is of the form

(iii) $0 \leq p < p_o$

or

(iv) $0 \leq p < p_o$ or $p_1 < p \leq 1.$

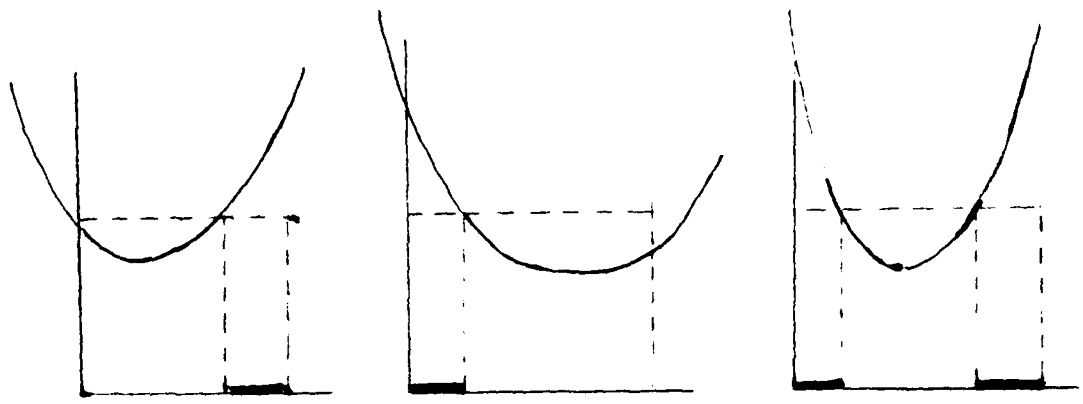
Case (iii) corresponds to the situation when a person, with high fear of failure, facing high loss when he fails, decides nevertheless of the risky option, if the probability of success is sufficiently <u>small</u> (i.e., he is risking almost certain failure).

Case (iv) is the mixture of the above case and the normal case.

Normal risk area, and both odd risk areas in this case are presented on Fig. 2.

14. It is not claimed that the effect of odd risk area is universal--in fact, such a phenomenon is rather rare, requiring special inequalities between the economical and psychological parameters of the model. However, the appearance of odd risk areas in some cases may account for apparently "irrational" behaviour of some persons. The point is that such a behaviour is irrational only if one adheres strictly to SEU model; for an individual, such behaviour does not appear irrational at all.

15. The second consequence of the model--which I think is of great importance for practice--is the discovery that the two main motives of Atkinson's theory, namely motive M to achieve success, and motive F to avoid failure, <u>do not play symmetric role</u>.



normal (iii) (iv)

Fig. 2. Risk areas in case $M < F$.

In Atkinson's theory--which is, in a sense, an extreme case of the suggested model, since it assumes $R = L = K \approx 0$ (no economical variables), the motives M and F always appear jointly, through their difference $M - F$.

However, if one takes into account the economical variables, the symmetry of M and F no longer holds: the risk areas depend on M in a different way than on F.

The practical consequences of it are the following. In order to control people's behaviour, for instance, induce them to risk taking, in form, for example, of trying innovations, one may either increase their motive to success, or lower their fear of failure (or both). The techniques used to increase the achievement motive are quite different in nature, efforts and costs, than the techniques used in decreasing the fear of failure. A careful study might show, therefore, how to allocate the resources between these two types of activities in order to achieve a given goal (e.g., induce as many employees as possible to suggest innovations).

16. One final remark concerning decisions in large organizations, and the relevance of the above considerations in such cases.

The decisions in large organizations are usually made after collective deliberations, seeking advice, and so on; the responsibility, however, rests usually on one person. He might be inclined, in some situations, to proceed against a person whom he dislikes, who poses a threat for him, etc., even if by doing so he chooses a more risky option, "worse" from the point of view of SEU model--simply because the success--<u>if occurs</u>--gives him additional advantage over his enemies. The psychological mechanism here is different than the one suggested in the model (satisfaction from success <u>as such</u>), but the result is the same--appearance of abnormal risk area.

The practical consequences of the above considerations are, as I mentioned, restricted by the difficulties in measurement of the relevant parameters. However, by manipulating with certain parameters, one can increase or decrease the likelihood of a person "entering" the normal, or abnormal risk area.

Beyond the mentioned already effect of manipulation with psychological parameters M and F, one can manipulate with economical variables. The direction depends, of course, on the goal: one may want to bring a person <u>into</u> a risk area, or one may want to push him <u>out</u> of it. For instance, I suppose that a good director of an enterprise should be a man with $M > F$, and it might even be best if his risk area in "odd." This is achieved by <u>decreasing</u> the reward R: the higher the component of "sheer pleasure from successing," the better. On the other hand, a decision maker in an air-traffic control room with "odd" risk area should probably be fired at once.

It appears that even if the quantitative predictions are impossible at the moment, the mere recognition of the possibility of the discovered phenomenon may be of practical importance, by directing the attention to some desirable or undesirable effects.

<u>Model of prosocial behaviour</u>

This model will be the object of my separate talk, so it will be only mentioned briefly. The model is based on the idea that in choosing an action, a person takes into account not only the expected "direct" reward, but also the predicted changes of attitudes towards him of persons of his "reference group." That is, his decision is influenced by *what he thinks the others will think of* him. The utility of such evaluation by others is assumed to be an increasing function of the evaluation (i.e., everybody prefers to be valued higher). One can now introduce a taxonomy of persons, depending on the shape of this utility function, in particular, on its behaviour on the positive (acceptance) and negative (rejection) side. The model leads to qualitative predictions of behaviour in given circumstances, described in terms of "acceptance-rejection" level of the person in his group: the prediction provides classes of situations in which a given person is likely to behave in an antisocial or prosocial way.

The novelty of the model, as compared with the existing theories of pro- and antisociality, lies in the total abandoning of the assumption that the tendency to such behaviours is an individual trait. It appears that there is no need for this assumption in order to explain the variety of the observed behaviours.

<u>Generalized decision model and linguistic representation of motivation</u>

Looking at the SEU model, or Atkinson's model, from a more general point of view, one can see that in each case the basis for the decision is the evaluation of a certain number of alternatives on a certain number of scales (dimensions, psychological continua). These can be dimensions such as subjective probability, incentive value, etc. Thus, one can consider generally, a space, spanned by such dimensions (see M. Nowakowska, "Language of Motivation and Language of Actions," Mouton, The Hague 1973).

In this book, the space in question was called "motivational space"; each of its dimensions corresponds to one evaluation criterion, so that the decision alternatives may be ranked separately on each dimension.

The motivational space is generated by the linguistic representation of motivation, or more precisely, by various classes of functors, such as epistemic (e.g., "I know that," "I am certain that," "I doubt that"), emotional (e.g., "I am glad that"), motivational (e.g., "I want," "I prefer") and deontic (e.g., "I must," "I ought to"). These functors, in turn, were obtained by abstracting from the content of expressions, in which one justifies, explains, evaluates, etc., the planned or performed actions. The linguistic representation of such scales as that of subjective probability, its discriminativeness and richness, were analysed for the first time in decisional context in the quoted book.

A decision criterion, by definition, must assign to each possible set of points in motivational space, i.e., to a set of orderings along particular continua, a final ordering, or at least the first element of such ordering (optimal decisions). In the quoted book it was shown that the formal structure is here the same as the formal structure of group decisions, and in consequence, one may apply Arrow's Impossibility Theorem. It turns out that such an interpretation leads to certain important psychological consequences, simply by observing that people must partition into categories depending on the Arrow's postulate which they break (e.g., lack of dictator, sovereignity, principle of independence of irrelevant alternatives, and so on). Each of these categories of persons can in turn be interpreted in psychological terms.

This conclusion from Arrow's theorem is of special cognitive importance, since it provides theoretical justification of the fact that in each decision situation, where the final decision is based on partial evaluations, one must resign from one of the conditions which one would like to have satisfied as "natural." In other words, if one has a transitive relation of preference on the set of alternatives, this is always attained at the cost of breaking a certain "natural" rule of aggregation of preferences.

Moreover, the theoretical and empirical significance of the identification of group and individual decision making, and application of Arrow's theorem, consists, firstly, on taking into account the psychological continua which have only ordinal character; such continua are traditionally neglected in decision theory, while they seem to play some role in decision making. Secondly, these considerations lead to a definition and taxonomy of internal conflicts; it is worth while to observe that (see Nowakowska, 1973) even in the simplest situation of four competing jointly unattainable motives, the number of different conflicts equals 113.

For empirical research, the approach mentioned suggests (a) analysis of classes of situations, in which the criteria cannot be aggregated to a transitive relation, and (b) in case when such a relation is obtained, observing which of the Arrow's postulates is broken. The ultimate goal would be to determine classes of decision situations, in which a given postulate would have highest chance of being broken; the knowledge here would allow restricting the classes of decision rules, which are applicable in a given situation.

Formal theory of actions. A model unifying verbal and nonverbal behaviour

It is obvious that a good decision model should always be accompanied by an extension including the structure of pre-decisional situation. The first such model was presented in the quoted book (1973). The novelty was not only analysing verbal and nonverbal behaviour jointly in a unified system, but also the dynamic analysis of different strings of actions, leading to various goals.

Another novelty was the study of various valuations (e.g., economical, ethical, etc.) of strings of actions, and also the analysis of their temporal aspects (times of occurrences of outcomes, periodicity, etc.).

In some simplification, the main primitive concepts of the system form a quadruplet

$$[D, L, S, R]$$

where D stands for the "vocabulary of actions," L is the "language of actions," i.e., the set of all admissible strings of actions, S is the set of results, and R is a relation connecting strings of actions from L, with outcomes of the set S, and the times of their occurrences.

Formal analysis of the system may be carried out quite independently of the different possible interpretations. The main lines of research are:

(1) Analysis of the structure of language L, through an application of some concepts borrowed directly from formal linguistics, such as generative grammars, distributional classes, parasitic strings, etc. These concepts acquire then an action-theoretic interpretation, and become useful tools in explicating the "grammatical" features of actional situations.

It is worth to mention, that to each decision situation there corresponds a different language L, or--more precisely, a family of languages L, depending on the type of admissibility, and possibly also on the level of admissibility.

(2) Analysis of attainability of some outcomes, or more generally--configurations of outcomes, that is, composite goals. If a given goal is attainable, that is, there is at least one string leading to it, the natural topics are various types of optimality. Here the analysis may proceed towards such action-concepts as decision moments, complete possibility, praxiological sets, etc.

If a given goal is not attainable, one obtains an internal conflict of motives, since one is forced to resign from some partial goals in order to attain others.

(3) The third line of research consists of studying the relations between verbal behaviour (utterances) and nonverbal actions. This line of research may be subdivided into the following topics:

a) the decision model, based on the concept of motivational space, as discussed above;

b) development of a "motivational calculus," by studying the rules of inference from those utterances in the natural language which involve motivational functors. These rules (based on the notion of semantic admissibility), allow in turn, to formalize the inference in motivation, by using the principles analogous to those known in logical theory of enthymemes (i.e., "tacit premises").

It ought to be mentioned, that the motivational calculus (see Nowakowska 1973), consists of some 40 implications, which are proved to be jointly consistent, by the construction of a set-theoretical model.

The notion of semantic implication is fuzzy in the same degree as the notion of admissibility. If the latter were given in form of membership function, one could define the strength of semantic implication $A \to B$ by postulating that the sum of this strength and the value of admissibility of "A and not B" is 1:

$$(\text{strength of } A \to B) + (\text{admissibility of } A \text{ and not } B) = 1$$

In effect, these semantic implications (in form of transformation rules from sentences involving motivational functors) constitute the rules of approximate reasoning, the approximation resulting from the fuzziness of the underlying concept of semantic admissibility.

The motivational calculus can be seen as important complementation of Zadeh's ideas of approximate reasoning.

c) One may also explore the behaviour which is consistent (or inconsistent) with a given utterance, e.g., promise. Here one can obtain a rather general result, asserting that under some conditions, the set of all strings of actions which break (or satisfy) a given promise is a context free language. This result characterizes the regularities of social norms.

Generally, the analysis of motivational consistency, motivational horizon, etc., leads to a formal theory of planning behaviour, where elaboration of a plan is identified with generation of a sentence (that is: a string of actions) in generative grammars.

(4) The fourth line of research is connected with ethical valuations of strings of actions leading to ethically diverse outcomes. A hypothesis here is that there are two basic strategies of such valuations, "puristic," when only the ethically worst outcome is taken into account, and "liberal" one, where the process of averaging takes place. Whether the second strategy is applicable, depends, of course, on the existence of an interval type scale of ethical valuations.

(5) The system may be extended so as to cover the actions of many persons, cooperating or conflicting. The research in this direction leads among others, to a formal theory of organizations. It may also be profitably applied to communication theory, in at least two different ways.

(a) by applying the system to analysis of dialogues, one obtains a formal theory, in which it is possible to explicate the structure and nature of various ties operating in a dialogue, i.e., semantic interrelations between consecutive utterances, ties due to changes of topics, etc.

This theory is of special importance for eristic dialogues, where the object is to use actions (argumentiation) in such a way so as to move the opponent into a desired region of semantic space ("convince him of something").

(b) by applying the system to analysis of verbal and nonverbal communication, one obtains a theory of cooperation of different media of expression (e.g., verbal medium, gestures, medium of facial expressions, etc.). The consideration of "multidimensional units" of such communication (simultaneous actions on different media) leads to explication of such concepts as generation, inhibition, cancellation, and supporting of meaning, and more generally--to creation of a sort of "pragmatic semantics."

(6) The analysis of group actions, leads also to a new theory of social changes, and in particular, to a formal theory of alienation.

The basic concept here is the distribution of social goods, and admissibility of such a distribution. Adding the notion of preference over the class of such distributions, one can explicate the concepts such as stability and fairness, and deduce an inherent dilemma of choice between fairness and stability of social systems.

One of the subsystems of the theory of social change, based on action theory, is a theory of alienation. The latter concept is treated dynamically, against the structure of the reference group, and against the structure of alienating group (monopole). In greatest simplification, the degree of alienation is highest when there is highest divergence between real and expected degree of participation in social goods.

(7) The recent development of the system of actions consists of "fuzzifying" its basic concepts. The central notions in the extended system are those of state, history (a sequence of states), and event (a set of histories), as well as the fuzzy relation of "causing," allowing partial control over the changes of states. The novelty here lies in introducing families of languages of actions, with various degrees of admissibility, allowing a rich analysis of the problems of attainability.

These considerations lead also to the problems of system synthesis, hence an approach in a sense dual to the traditional approach of system analysis.

It is worth to mention also that in all the above outlined developments of action theory, one has a great flexibility of interpretation of the basic primitive notion, namely that of admissibility of a string of actions. Here one can interpret this notion as (a) physical admissibility, (b) psychological admissibility for a given person, (c) social admissibility, i.e., consistency with a given social role, (d) organizational admissibility, (e) ethical admissibility, and so on. In each case, one gets a different type of constraints on actions, hence a different action language L, whose analysis leads to a formal explication of the nature and structure of various types of constraints.

To sum up, the actions theory is a formal system which concentrates on explanation of the connection between thought and action.

Indirectly connected with action systems, is another line of my research, that on semiotics. Here I have recently constructed the first in the literature formal theory of fuzzy semiotics, connecting the structure of the object with the structure of its sign, in particular special signs called verbal copies (descriptions).

The basic notions of this theory are: object and its different levels of representation, e.g., iconic, symbolic, verbal and so on. (Incidentally, the latter being the foundation of the formal text theory situation, and the algebra of situations, representation in form of verbal copy, the notions of adequacy of such a copy, its languages, etc.)

The introduction of the concept of fuzzy meanings of the object allows consideration of "levels of meanings," and the structure of the meaning space.

The theory has its empirical counterparts, and also leads to important cognitive consequences, in form of explication of such notions of Gestalt psychology as the notion of good figure, and various Gestalt laws.